A brief history of the astronomical theories of paleoclimates[1]

André Berger, Université catholique de Louvain, Louvain-La-Neuve, Belgium

Abstract. Paleoclimatic reconstructions help to discover the natural variability of the climate system over times scales ranging from years to hundreds of thousands of years. They are fundamental in climate research, especially now, because they provide a unique set of data to validate models over climatic situations largely different from those of the last 150 years. The climatic situations of the last century are indeed available in great detail, but with a very poor diversity. Among the different modes of climatic variations, the glacial-interglacial cycles have the advantage that they provide examples of extreme climates and that their primary astronomical cause is now pretty well known.

The Astronomical Theory of paleoclimates aims indeed to explain these climatic variations occurring with quasi-periodicities situated between tens and hundreds of thousands of years. Such variations are recorded in deep-sea sediments, in ice sheets and in continental archives. The origin of these quasi-cycles lies in the astronomically driven changes of the latitudinal and seasonal distribution of the energy that the Earth receives from the Sun. Milutin Milankovitch extensively published about this theory between 1920 and 1950, but the relationship between the astronomical parameters, insolation and climate, had already been suggested at the beginning of the nineteenth century. The evolution of ideas from these early times to the present day is briefly reviewed, but this introductory paper does not claim to be a full historical survey of what has contributed to structure the astronomical theory of paleoclimates over the last two centuries. It might rather be viewed as a pedagogical tool for teaching some of the basic concepts in a historical context. Written to be concise, it provides a sequence of references to authors who have contributed to what is now named

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(often improperly) the Milankovitch astronomical theory. It is also expected to be a corrective to distortion of credit and intellectual contributions.

As this astronomical theory aims to explain the glacial-interglacial cycles, the discovery of their existence is first summarized from the existing literature (e.g. Hann, 1903; Bard, 2004; Krüger, 2008). For the astronomical theory, a brief history of its early elaboration and of its gradual refinement up to the 1980's is provided. The last 20 to 30 years are characterized by an extremely large number of researches. They use the astronomical theory to attempt explaining the physical mechanisms through which the long-term variations of the energy received from the Sun is affecting the Earth's climate. For partial reviews of this literature, the reader is invited to refer to papers like Berger (1988 and 1995) and/or to proceedings of symposia discussing more recent applications (e.g. Schellnhuber et al., 2004; Sirocko et al., 2007).

The narrative style used in this paper cannot mask the fact that, in the early 19th century, authors lived in a dramatically different intellectual environment than Milankovitch and not to speak of more recent scientists. The "pool of ideas" on which they could draw their conclusions as well as their motivation for thinking about the ice ages were radically different in many respects. All the ideas were not part of some unified theory of paleoclimates and these early naturalists were not necessarily drawing on each other's work. It is however amazing to see, when reading their scientific papers, how much they were aware of their respective views, exchanging ideas and critics through scientific communications, papers and letters (like for example those between Tyndall, Lyell and Herschel, see Fleming, 1998, p. 68, 73-74).

Introduction to the astronomical theory

The seasonal and latitudinal distribution of the energy that the Earth receives from the Sun has long-term variations which are related to the orbit of the Earth around the Sun and to the inclination of its axis of rotation. These involve three well-identified astronomical parameters: the eccentricity, *e* (a measure of the shape of the Earth's orbit around the Sun), the obliquity, ε (the tilt of the equator with respect to the plane of the Earth's orbit), and the climatic precession, $e \sin \tilde{\omega}$, a measure of the Earth–Sun distance at the summer solstice ($\tilde{\omega}$, the longitude of the perihelion, is a measure of the angular distance between the perihelion and the vernal point that are both in motion). The present-day value of *e* is 0.016. As a consequence, although the Earth's orbit is very close to a circle, the Earth–Sun distance, and consequently the insolation, varies by as much as 3.2 and 6.4 per cent respectively over the course of one year. The obliquity, which defines our tropical latitudes and polar circles, is presently 23°27'. The longitude of the perihelion, $\tilde{\omega}$, is 102°, which means that the Northern Hemisphere winter occurs when the Earth is closest to the Sun. The average periods of variations of these eccentricity, obliquity and precession are respectively 95800 years (but with others, in particular one of 400,000 years), 41000 years (very stable) and 21740 years (with a large dispersion around this value) (Berger, 1976).

Early theories of Quaternary Glaciations

Earth's history has been characterized by periods, called Ice Ages (with capital letters), when the climate was markedly colder than at other times. The most well known are the Pre-Cambrian Ice Ages, the late-Ordovician Ice Age, the Permo-Carboniferous Ice Age and the present Ice Age. Our Ice Age, which the Earth entered 2–3 Ma ago is called the Quaternary Ice Age. This Ice Age is characterized by multiple switches of the global climate between cold periods (named glacials and sometimes ice ages with lower cases) when extensive ice sheets were present, and warm periods (named interglacials) when there was less ice over the Earth (or at least not much more) or when the climate was more or less similar or warmer than today (Tzedakis et al., 2009; Yin and Berger, 2010). The need to offer an explanation for the origin and recurrence of these glaciations has progressively led to suggest and finally demonstrate that they are related to celestial mechanics, that is, to the characteristics of the Earth's orbit around the Sun and of its axis of rotation.

Early in the eighteenth century, the presence of erratic blocks and of moraines in the Alpine Valleys drew the attention of travelers and naturalists (for a more detailed history of early scientific research on glaciations, see Bard (2004) and Krüger (2008) where references are available.) In 1744, perhaps for the first time, a geographer living in Geneva, Pierre Martel (1706-1767) reported that the inhabitants of the Valley of Chamonix in the Alps of Savoy attributed the dispersion of erratic boulders to glaciers that in the previous times extended further into the lowlands. Similar explanation was given in 1815 by the hunter Jean-Pierre Perraudin (1767-1858), based on his observations in the Val de Bagnes in the Swiss canton of Valais. Horace-Benedict de Saussure (1740-1799), a naturalist with a passion for high mountains and the Mont Blanc in particular, noticed that such erratic blocks were located along the axes of the valleys and that they must have been transported from the peaks of the Alps over long distances, possibly by catastrophic floods ("currents of incredible violence and magnitude"). This explanation was also suggested by the paleontologist Georges Cuvier (1763-1832), who assigned the extinction of species to environmental catastrophes. In 1795, the Scottish naturalist James Hutton (1726-1797) explained that the presence of erratic boulders in the Alps was due to the action of the glaciers whereas the Scottish geologist Charles Lyell (1797-1875) proposed the theory that transport of rock blocs can be accounted by the action of ice rafts, ice breaking off the poles, floating across submerged continents and carrying debris with it (Lyell, 1830, vol. 3, 1833).

In Scandinavian countries, the Swedish mining expert Daniel Tilas (1712-1772) suggested, in 1742, that the drifting sea ice could be the reason for the presence of the erratic boulders in Scandinavian and Baltic regions. In 1818, the Swedish botanist Göran Wahlenberg (1780-1851) published his theory of a glaciation of the Scandinavian peninsula that he regarded as a regional phenomena (from Krüger, 2008, p. 118) . A few years later, the Danish-Norwegian geologist Jem Esmark (1763-1839), for whom the transport of rocks by ice seemed evident, offered a sequence of worldwide ice ages. In 1824, he proposed that the changes in climate were the cause of these glaciations and tried to show, perhaps for the first time, that they originated from changes in the

Earth's orbit (Esmark, 1824, 1827). We may consider Esmark’s ideas as the first distinctly modern formulation of the astronomical theory of long-term climatic change. His chronology differed from the one accepted today because it was based on the contemporary theories that assumed that the Earth developed from an old comet with the ice ages corresponding to the periods when the terrestrial comet was at the aphelion (i.e. at the greatest distance from the Sun).

In 1829, the Swiss engineer Ignaz Venetz (1788-1829) explained the dispersal of erratic boulders in the Alps, the Jura Mountains and the North German Plain as being due to huge glaciers which variations in the location of the fronts would have been due to climatic changes but not related to astronomical cause. This idea was adopted by the German Professor of forestry Albrecht Reinhard Benhardi (1797-1849) who, in a paper published in 1832, speculated about former polar ice caps reaching as far as the temperate zones of the globe. At about the same time, Jean de Charpentier (1786-1855), using Venetz's explanation but restricted to the Alps, introduced the hypothesis of diluvian glaciers (de Charpentier, 1836), the ice equivalent of the de Chaussure-Cuvier floods, to explain the erratic boulders. He presented his paper in 1834 before the Schweizerische Naturforschende Gesellschaft, referring to astronomical causes but only in very general terms "such as for example a change in the ecliptic, the precession of the equinoxes, the progression of the planetary system in space, the asteroids…" (de Charpentier, 1841) . In the mean time, following several excursions into the Bavarian Alps, the German botanist Karl Friedrich Schimper (1803-1867) came to the conclusion that ice was most likely the explanation for the movement of local boulders. In 1837, he introduced the term “ice age (Eiszeit)” (Schimper, 1837a) and assumed that the Earth’s history had been characterized by periods of cold climate and frozen water. This idea was similar to Esmark’s suggestion of worldwide ice caps. At this time he convinced his former university friend, the Swiss geologist Louis Agassiz (1801-1873), that reality of such periods must be granted on the grounds of empirical findings (Schimper, 1837b). In the same year, de Charpentier gave Agassiz a course of field work on glaciers. Subsequently, Agassiz and Schimper developed a theory of a sequence of glaciations based upon the work of Johan Wolfgang von Goethe (1749-1832), Venetz, de

Charpentier and on their own field work. In the mind of Agassiz, the concept of the ice ages was a Cuvierian catastrophe, in which the water of the biblical Flood was being replaced by a gigantic glacier. It is with such theory that Agassiz convinced Charles Lyell and that in 1837, at the opening of the Helvetic natural History Society at Neuchatel, he shocked his audience by delivering his address entitled ‘Upon Glaciers, Moraines and Erratic Blocks’ (Agassiz, 1838). Because of the high similarity of Agassiz’s hypothesis with Schimper’s idea, Evans (1887) claimed that "Agassiz actually 'borrowed' his conception of glacial theory, usually attributed to him, from K. Schimper" and that, "aware of his indebtness, he also most carefully concealed it". A similar complaint came from de Charpentier. Excluded from Agassiz's book (1840), he felt indeed that Agassiz should have given him credit as he was introducing him to in-depth glacial research.

In parallel, research on the scientific concept of the greenhouse effect was going on with the French Physicist Joseph Fourier (1786-1830) and later with the Irish chemist John Tyndall (1820-1893) who was also greatly interested by the high mountains in the Alps and the movement of glaciers (Tyndall and Huxley, 1857). Along with these studies on the Alpine glaciers, investigations of the polar ice caps were going to contribute to the debates. The first descriptions of the Antarctic ice cap remained however largely uncertain. For example, the French Joseph Alphonse Adhémar (1797-1862) evaluated its average thickness at more than 100 km! Realizing that such an ice cap is difficult to accept and following his brilliant intuition that the glaciations must have been periodic, he turned towards astronomy not only to test his calculations about Antarctica but also to determine the cause of the ice ages and of their recurrence. In 1842, Adhémar published his book explaining Agassiz’s hypothesis on the existence of ice ages on the basis of the known precession of the equinoxes, thereby implying that there had likely been more than one. Convinced that the Southern Hemisphere was currently under an ice age (because his estimate of the thickness of Antarctica) and influenced by the current explanation of the mechanisms of seasons and tides at that time, Adhémar invoked the difference of duration of the seasons between the two hemispheres as a possible cause of the ice ages. By combining the astronomical precession (calculated from the value of 50.1" per year of the

French astronomer Jean-Baptiste Joseph Delambre 1749-1822) and the rotation of the terrestrial orbit (calculated from the value of 11".83 per year of the French mathematician Louis Benjamin Francoeur 1773-1849), he further concluded "that a period of 21000 years must exist between the present time and the moment when the seasons will correspond to the same point of the orbit". From his calculations, he concluded (i) that the astronomical winter in the Southern Hemisphere is 7 days longer than the summer, (ii) that consequently there must presently be an ice age in the Southern Hemisphere and (iii) that 11000 years from now that there will be an ice age in the Northern Hemisphere. According to Adhémar theory, the great accumulation of ice around the pole having its long winter displaced the Earth's centre of gravity resulting in a partial displacement of the ocean waters, and a flooding which further increased the cooling. Similarly, to attempt explaining the deglaciation, he introduced large scale oceanic currents to link the two hemispheres. His meridian circulation of the ocean at the surface and at depth (Adhémar, 1842, figures 2 and 6 and page 304) is actually a simplified representation of what is called nowadays the thermohaline circulation (a conveyor belt transporting heat from the south to the north in the Atlantic Ocean, with the Gulf Stream being part of it). He was certainly not aware that this concept of oceanic current would play, one century later, a fundamental role in the explanation of the energy transport from the southern to the Northern Hemisphere. His prediction about the climatic effect of the Gulf Stream is also remarkable (his page 366): "One might at the maximum conclude that our hemisphere would cool more rapidly if the Gulf Stream would not exist; what is not that certain because it would not be impossible that the vapour produced by the warm water currents would contribute itself to increase the polar ice". In 1979, Ruddiman and McIntyre invoked the same mechanism to explain the last Glacial Maximum: "The juxtaposition of an "interglacial" stream alongside a "glacial" land mass is regarded as an optimal configuration for delivering moisture to the growing ice sheets".Many criticisms were raised against Adhémar ideas, like his glaciations affecting the two hemispheres in opposite ways and, more importantly, his hypothesis about the difference in the length of the seasons between the two hemispheres. As underlined by Charles Lyell and the German Alexander von Humboldt (1769-1859) more important than the length of the seasons is indeed the total energy received during a season or the whole year. The computation of such total irradiation was actually possible based on calculations already made by the French Jean Le Rond d'Alembert (1717-1783) for precession and the English Sir John Frederick William Herschel (1792-1871) for insolation. John Herschel (1832) claimed (page 298): “it is demonstrable that, whatever be the ellipticity of the Earth’s orbit, the two hemispheres must receive equal absolute quantities of light and heat per annum, the proximity of the Sun at perigee exactly compensating its swifter motion”. This is a consequence of the second law by the German astronomer Johannes Kepler (1571-1630), a law that Herschel reformulated as: "The amount of heat received by the Earth, while describing any part of its orbit is proportional to the angle described round the Sun's center". Actually the irradiation received at a latitude of the Northern Hemisphere during a given Northern Hemisphere season is equal to the irradiation received by the same latitude of the Southern Hemisphere during the same local Southern Hemisphere season (Berger et al., 2010). - Being given the role played by John Herschel in the astronomical theory, let us stress three of his fundamental statements (Herschel, 1832). The first one concerns: "the total quantity of heat received by the Earth from the Sun in one revolution is inversely proportional to the minor axis of the orbit *(and consequently depends upon eccentricity)*". The second one is a direct consequence: "since the major axis is invariable and therefore the absolute length of the year *(through the third law of Kepler)*, it follows that the mean annual average of solar radiation is dependent on the eccentricity of the orbit". These statements implicitly lead to conclude that the so-called solar constant (defined as the energy received at the mean distance from the Earth to the Sun) is actually dependent on the eccentricity (see Berger and Loutre, 1994 p.118-119). In Milankovitch however no difference is made between the semi-major axis and the mean distance. At the bottom of page 212 (of the 1969 English translation of the 1941 Milankovitch's book), one finds: "the uniform flow of energy across the sphere whose radius represents the mean distance of the Earth to the Sun *or* the semi-major axis of the Earth's orbit is called the solar constant J_0". The third one concerns the application to the Earth's climate, first noting a present-day agreement between astronomy and geological observations:"The eccentricity of the Earth’s orbit is actually diminishing…the annual average of solar radiation is actually on the decrease. So far, this is in accordance with the testimony of geological evidence…",

then turning to: " the extreme effects which a variation in the eccentricity may be expected to produce in the summer and winter climates in particular regions of its surface…It will appear that a (*large*) amount of variation may operate during great periods of time to mitigate or to exaggerate the difference of winter and summer temperatures …but the actual diminution of the eccentricity is so slow that the transition from a state of the orbit to the present nearly circular figure would occupy upwards of 600,000 years (*we know now that the average period of eccentricity is much smaller*)"

and referring to Lyell: "adopting the very ingenious idea of Mr Lyell, would suffice, by reason of the combined effect of the precession of the equinox and the motion of the apsides of the orbit itself, to transfer the perigee from the summer to the winter solstice, and thus to produce a transition from the one to the other species of climate, in a period sufficiently great to give room for a material change in the botanical character of a country…"

he finally concludes in confirming his reluctance to accept the astronomical theory: " But if on executing the calculations, it should appear that the limits of the eccentricity are really narrow, it should appear that the mean as well as the extreme temperature of our climates would not be materially affected…the obliquity of the ecliptic being confined within too narrow limits for its variation to have any sensible influence."

Pioneers of the Astronomical Theory

Within the next decades, largely because of the discovery of the repetitive aspect of global glaciation (for example, in the Vosges, in Wales, and in the American records), glacial geology became strongly tied to astronomy (for a detailed list of the papers from the 19th century to 1980, see Berger, 1988). An astronomer in Paris, Urbain Le Verrier (1811-1877) , famed for having discovered the planet Neptune, calculated the planetary orbital changes of the Earth over the last

10^5 years (Le Verrier, 1855), although he did not seem to be interested by the astronomical theory of paleoclimates (Lequeux, 2009). In parallel, L.W. Meech (1855) published the first detailed determination of the instantaneous, daily and seasonal amount of energy received by any latitude of the Earth from the Sun. This calculation was based on the elliptic integrals introduced in 1825-1828 by the French mathematician André-Marie Legendre (1752-1833). Meech further analyzed the influence of Le Verrier's secular values of eccentricity (mainly the extreme values) on the Sun's annual intensity and calculated it for 10,000 years BP. After discussing it in relation to the Northern and Southern hemispheres, he concluded (page 30): "this wide fluctuation of winter and summer intensities, in relation to the two hemispheres, scarcely affected the aggregate annual intensities". Taking into account the impact of the maximum variation of obliquity given by the French astronomer Pierre-Simon Laplace (Marquis de, 1749-1827), he finally concluded (page 41): " that Great geological Changes must be referred to other causes than the secular inequalities of the Earth's orbit and might result from the motion of the whole Planetary System in Space", a conclusion for which he referred to the work of the French astronomer and physicist Siméon-Denis Poisson (1781-1840) (Poisson, 1835). Twenty years later, Chr. Wiener (1876) published a very similar calculation, but which included the total irradiation received over different parts of the year.

It was also at that time that the Scottish scientist James Croll (1821-1890) initiated a series of important works that would continue to bear much fruit into modern times (for details, see Imbrie and Imbrie, 1988; Fleming, 2005). Croll's theory (Croll, 1875) depends upon the inequality in the length of the seasons, but unlike Adhémar he considered it at a time of a great eccentricity of the Earth's orbit. Three major astronomical factors were recognized in his model: precession, orbital eccentricity and axial tilt. The importance of Croll is that he approached the glaciation problem from the synergistic standpoint of the combined effects of these three major astronomical factors on seasonal insolation during perihelion and aphelion. Moreover, a specific characteristic of his model lies essentially in his hypothesis that the critical season for the initiation of a glacial is the Northern Hemisphere winter. He argued that a decrease in the amount of sunlight received during the winter favors the accumulation of snow, and that any small initial increase in the size of the area covered by snow would be amplified by the snowfields themselves (positive feedback). After having determined which astronomical factors control the amount of sunlight received during winter, he concluded that the precession of the equinoxes must play a decisive role (Croll, 1864). But his main contribution was to show that changes in the shape of the orbit which were unknown to Adhémar determine how effective the precessional wobble is in changing the intensity of the seasons (Croll, 1867a). Croll’s first theory predicts therefore that one hemisphere or the other will experience an ice age whenever two conditions occur simultaneously: a markedly elongated orbit and a winter solstice that occurs when the Earth is far from the Sun. This would produce a climate so severe that the snow falling during the long cold winter would be heavy enough to persist through the short hot summer and thus develop ice sheets. According to Croll's calculations based on the planetary orbital changes of the Earth determined by Urbain Le Verrier in 1855 for the last 10^5 years, these conditions were reached 240,000 years ago and ended 80,000 years ago (We know now that the cold conditions ended much later, about 12,000 years ago, the Last Glacial Maximum having culminated 20,000 years ago (CLIMAP, 1976)). Later, Croll hypothesized that an ice age would be more likely to occur during periods when the axis is closer to vertical, for then the Polar Regions receive a smaller amount of heat (Croll, 1867b). However, Croll did not have accurate data on the variations of obliquity which will become available only a few years later with Stockwell (1873) and much later with the Milankovitch complete formulation of the astronomical theory. In the mean time, Croll received the support of the English naturalist Charles Robert Darwin (1809-1882) who wrote in his book "On the Origin of Species" (Darwin,1872): "Mr Croll in a series of admirable memoirs, has attempted to show that a glacial condition of climate is the result of various physical causes, brought into operation by an increase in the eccentricity of the Earth's orbit… and its influence on the oceanic currents" (it is surprising that reference to climate and Croll disappeared in the 1900 popular impression of the "Origin of Species"). The Scottish geologist Archibald Geikie (1835-1924- and his brother James Geikie (1839-1914) showed convincingly that several glacial phases follow one after the other separated by interglacial periods with a moderate climate similar to present or warmer (Geikie, 1874). The German geographers, Albrecht Penck (1858-1945) and Eduard Brückner (1862-1927) also came with their multiple glaciations in the Alps (Günz, Mindel, Riss and Würm; Penck and Brückner, 1909) whereas, at about the same time, the American geologist Thomas Chowder Chamberlain (1843-1928) presented the classification of the American glaciations (Nebraskan, Kansan, Illinoian, Wisconsinian; Chamberlin, 1882).

But, as time went on, many geologists in Europe and America became more and more dissatisfied with Croll’s theory, finding it at variance with new evidence about the last ice age. His calculations lead indeed to great climatic changes opposite in the two hemispheres (e.g. Howorth, 1890) whereas geological investigations were showing that the glacial periods are practically synchronous in the two hemispheres. Moreover, the duration of post-glacial time estimated by the Swedish geologist Gerald de Geer (1858-1943) was much smaller than the 80,000 years necessary in the framework of Croll's theory.

The theory of glacial periods was then taken up by the Irish astronomer Sir Robert Stawell Ball (1840-1913) who claimed to refine the Croll's theory and calculations. Actually, he calculated only the quantity of heat received by a whole hemisphere in winter and summer, using the ratio between the two to argument about glaciations (this ratio is independent of eccentricity, Berger and Yin, 2012). Not only these calculations were already made by Chr. Wiener (1876) but they also refer only to one hemisphere as a whole preventing application to one latitude in particular. Similar critics of Ball's "astronomical theory" were drawn by the Irish Edward P. Culverwell (1894-95) who, using Meech's results, calculated the heat received by the latitudes 40° to 80° for winter at aphelion (i.e. at the height of the ice age). These calculations allowed him to show that for a winter longer than now, the present day latitude of 54° has the same solar climate as latitude 50° during the supposed ice age, with a conclusion that no ice age could be produce with such a small climate displacement. It must be stressed however that Culverwell omitted to take the variation of obliquity into account which explains why the amplitude of his variations in insolation was reduced.

Facing all these discussions, it is not surprising that scientists turned to other theories. For example, Sir Charles Lyell (1872) considered that the astronomical theory was exaggerated and turned towards the rearrangement of land and sea to show that this might produce extremes of heat and cold in global climates. Other propositions to explain the glaciations were also made in relationship with the greenhouse theory which was starting to be scientifically elaborated. It is actually the Frenchman Jacques Joseph Ebelmen (1814-1852) who was probably the first to suggest that past changes in the carbon cycle could have changed the climate of the Earth through changes in the atmospheric concentration of "carbonic acid" (Ebelmen, 1845, cited by bard, 2004 p. 626). Some years later, Tyndall (1861) came with the same idea that changes in the atmospheric concentration of greenhouse gases, like carbon dioxide and water vapour, could produce "all the changes revealed by the geologists". Such working hypothesis was also advanced by Chamberlin (1899) who assumed that "the changes in atmospheric carbon dioxide result from the weathering of rocks and through the agency of organisms". Based on his investigation of the effect of the atmospheric composition on climate, Luigi de Marchi (1895) concluded that neither the astronomical nor the geological theories can lead to a plausible explanation of the ice age. His analysis of the dependence of the air temperature upon the ratio of the radiant energy received to that lost by the Earth, as well as the distribution of land and sea water, convinced him that “a slight change in the transmission of the atmosphere for solar rays and heat would suffice to produce an ice age in the middle and high latitudes”. He was however not prepared to give the causes of such changes in the atmospheric transmission (water vapor or carbon dioxide?). It is the Swedish chemist Svante Arrhenius (1859-1927) who considered that the ice ages were caused by falls in the atmospheric content of carbon dioxide, amplified by an increase of the snow covered areas and the oceanic currents (Arrhenius, 1896). Citing both de Marchi and Arrhenius: " From the point of view of climatology and meteorology, in the present state of sciences the hypothesis of Croll seems to be wholly untenable as well as in its principles and his consequences…. It becomes more and more impossible to reconcile the chronology demanded by Croll's hypothesis with the facts of observations". By supplementing the notion of the carbon cycle developed by Arrhenius, Thomas Chowder Chamberlin (1843-1928) suggested in 1899 that the rhythmic action of the carbon cycle could partly explain the glaciations cycle.. It became therefore clear that the astronomical theory of the glacial period was unable to explain the new geological facts and that improvements were definitely necessary.

Such revival started with attempts to improve the astronomical calculations. Hargreaves (1896) estimated the impact of obliquity on the insolation of different latitudes but only for the annual average. He was not interested in the variability induced by the seasonal cycle. Analyzing the impact of extreme values of obliquity on insolation, Nils Ekholm (1901) missed the extreme values of insolation itself as he did not account for the influence of precession and eccentricity. The Austrian astronomer Rudolf Ferdinand Spitaler (1849-1946) tried in 1907 to calculate which values of the three astronomical parameters are the most favorable for glaciers formation and extend. Unfortunately his work was based on the calculation made in 1907 by the Austrian geophysicist Friedrich Hopfner (1881-1949) who missed to take into account the discontinuity of insolation in the Polar Regions. Based on John Nelson Stockwell (1822-1920) work (1873), the German mathematician Ludwig Pilgrim (mostly known as a pioneer in colorimetry) calculated in 1904 the combined effect of the eccentricity of the orbit, the obliquity and the precession of the equinox and tabulated the variations in the solar radiation for about one million years prior to 1850, but the values of the planetary masses used by Stockwell were not necessarily the most recent ones (see Milankovitch below). Over the last decade of the 19th century and the beginning of the 20th century, there were therefore many calculations done and hypotheses offered in relationship with the astronomical theory of paleoclimates. However, these studies were dealing only with part of the problem, like discussing the annual mean but not resolving the seasonal behavior, or suffering from a lack of precision in particular in the long-term variations of the astronomical parameters, and/or being incomplete namely in using only one of the three astronomical parameters. But principally, no real attempt to model the response of the climate system to the astronomical forcing could be found. This was about the situation when, at the beginning of the 20th century, Milutin Milankovitch started to be interested by the astronomical theory of paleoclimates or, in his own words, by the "mathematical climate of the Earth" (see Milankovitch contribution and life below).

The Milankovitch era

It was during the first decades of the twentieth century that Rudolf Spitaler (1921) rejected Croll’s theory that the conjunction of a long, cold winter and a short, hot summer provides the most favorable conditions for glaciations. He adopted the opposite view, following the idea first put forward by Joseph John Murphy (1869), that a long, cool summer and a short, mild winter are the most favorable. Under these conditions, the cool summer prevents the winter snow from melting and allows, with time, its accumulation to build the ice sheets. In a landmark paper published half-a century before Spitaler, Murphy pointed out his agreement but also his disagreement with Croll as to the cause of the glacial climate. He argued that "a glacial period occurs when the eccentricity of the Earth's orbit is at its maximum and only one hemisphere is glaciated at the same time" but contrary to Croll, he believed that "the glaciated hemisphere is that of which the summer occurs at aphelion". In addition, Murphy (1876) used the more recent calculation by J.N. Stockwell (1873) taking into account the disturbance of the planet Neptune, the existence of which was not known when Urbain Le Verrier computations were made and used later by Croll. The diminution of heat during the summer half year resulting from this new hypothesis was later recognized by the Austrian climatologist Eduard Brückner (1862-1927), the German Russian-born geographer Wladimir Peter Köppen (1846-1940), and the German geophysicist Alfred Wegener (1880-1930) as the decisive factor in glaciations (Brückner et al., 1925). The hypothesis put forward by Murphy in the middle of the 19th century was going to appear, one century later, as being one of the most brilliant proposals made for explaining the generation of the ice sheets.

However, this idea of a cool Northern Hemisphere summer became popular mainly because it was also taken back in the early part of the 20th century by the Serbian engineer, astronomer and geophysicist Milutin Milankovitch (1879-1958). Milankovitch was actually the first to complete a full astronomical theory of Pleistocene ice ages, using the available astronomical elements to compute the subsequent changes in the insolation and climate. Milankovitch's main contribution was to explore the solar irradiance at different latitudes and seasons in great mathematical detail and to relate these in turn to the planetary energy balance as determined by the albedo and by the reradiation in the infrared according to Stefan's law. The basis at the heart of Milankovitch's argument is that "under those astronomical conditions in which the heat budget around the summer solstice falls below average, so will summer melt, with uncompensated glacial advance being the result". This theory requires therefore that the summer in northern high latitudes must be cold enough to prevent the winter snow from melting. This leads to a positive value in the annual budget of snow and ice which initiates a positive feedback cooling over the Earth through a further extension of the snow cover and a subsequent increase of the surface albedo. On the assumption of a perfectly transparent atmosphere and of the northern high latitudes being the most sensitive to insolation changes, that hypothesis requires a minimum in the Northern Hemisphere summer insolation at high latitudes. It is therefore not surprising that the most used product of the Milankovitch theory is his curve that shows how the intensity of summer sunlight varied over the past 600,000 years at 65°N. It is on such curves that he identified certain low points with four European ice ages (non-periodic and without hemispheric alternation) reconstructed 15 years earlier by Albrecht Penck and Eduard Brückner (1909).

One of the ideas originally introduced by Milankovitch was the concept of caloric season. These seasons are exactly half a year long, the caloric summer half-year comprising all the days receiving more irradiation than any of the winter half-year. This avoids taking into account the variations of the length of these seasons as is the case for the astronomical ones (Berger and Yin, 2012). Although this does not solve the difficulty of taking insolation into account because the beginning and end of these seasons vary in time, it remains a very interesting concept in natural sciences where the environment has no calendar.

A brief survey of Milankovitch's life might help now to better understand the personality of this great scientist and to follow more easily the development of his work which culminated in his 1941 "Kanon der Erdbestrahlung".

Milankovitch‘s life

Milutin Milankovitch was born in Dalj (Austria-Hungary, today Croatia) in 1879 and died in Beograd (Capital of Serbia) in 1958. He was a contemporary of the Alfred Wegener (1880–1930), with whom he became acquainted through Wladimir Köppen (1846–1940), Wegener’s father-in-law (Schwarzbach, 1985).

The father of Milankovitch died when Milutin was only 7 years old. His uncle, Vasilije Muacevic took then care of him and continued to support him all his life (in gratitude, Milankovitch gave his name Vasilije to his only son and dedicated him his work). Milankovitch graduated in 1896 from the Realka High School in Osijek where his Professor of Mathematics Vladimir Varicak had a great influence on his vocation for science. He then left for the University of Vienna where he found a strong inspiration from his Professor of mechanics, Johann Brick. He graduated in civil engineering in 1902. After one year of military service in the Habsburg Monarchy, Milankovitch returned to Vienna in 1903 and earned his Ph.D. in 1904 with a thesis on "Beitrag zur Theorie der Druck-kurven". At the beginning of 1905, he started to work in the construction company of Adolf Baron Pittel Betonbau-Unternehmung in Vienna where he gained a high reputation among the engineers for the quality of his work and his innovation in building of dams, bridges and factory halls.

In 1909, he was invited by the Philosophical Faculty of Belgrade University where he became a Professor at the Department of Applied Mathematics teaching rational and celestial mechanics and theoretical physics, what he did during 46 years up to 1955. It is during the first decade of the 20th century that he decided to concentrate on fundamental research. During his time in Belgrade, Milankovitch remained in close contact with numerous scientists and institutions, but also with engineer Petar Putnick with whom he was going to build bridges of reinforced concrete for the Railways Company.

As early as 1912, his interest turned to solar climates with a first work on "Contribution to the mathematical theory of climate" (Milankovitch, 1912). It is also a time when ice ages became one of his major research topics. In 1914, he married Christine Topuzovic in Dalj, his native village. Unfortunately, because of the crisis between Serbia and Austria-Hungary, he was arrested as a Serbian citizen and put in prison in Osijek. Benefiting from the help of Professor Emmanuel Czuber, he was liberated but had to exile to Budapest. During the four years that he had to spend in Budapest, he had access to the Library of the Hungarian Academy of Sciences owing to his Director, Kolomon von Celia, another lover of mathematics. This gave him the opportunity to work on the mathematical theory of climate change on Mars, which laid the foundations of modeling the climate of the Earth and of the other planets (Milankovitch, 1916). He returned to Belgrade with his family in March 1919 and was promoted a full Professor at the University of Belgrade.

His main contribution to science dates from this time with his first monograph, written in French and published in 1920 in the Publications of the Yugoslavian Academy of Sciences and Arts of Zagreb by Gauthier Villars in Paris: "Théorie Mathémathique des Phénomènes Thermiques produits par la Radiation Solaire". It is the need to clarify and critically analyze all the calculations available at that time which led him to write such a bible for the astronomical theory and insolation. It is amazing to see that most of the fundamental concepts of the astronomical theory by Milankovitch are already present in great detail in this monograph. In the first part, he formulates the way to compute the instantaneous and daily insolation (incoming solar radiation) and the irradiation received over a season and for each hemisphere. In addition to these formulas already available in L.W. Meech (1855), he introduced those for calculating the irradiation for any interval of the year. Very surprisingly he did it only through series expansion without using a much better mathematical tool, the elliptic integrals, introduced by L.W. Meech in 1855 and developed extensively by Chr. Wiener in 1876. But a large part of the monograph is devoted to the impact of the atmosphere of insolation and climate including the problem of the albedo-temperature feedback for which he introduced the idea of snowline. His development of one of the very first climate models (if not the first one) based on physical principles is probably the most original contribution of Milankovitch to science, but unfortunately the least cited. In the second part, tables with numerical values are given for the interval from 500,000 years BP to the present. These were based on Stockwell for the orbital elements and Pilgrim for the numerical values of the three astronomical elements. From these numerical values, he counted the number of cycles over 500,000 years to identify the average period of climatic precession: 20,700 years which was already estimated theoretically by Adhémar about one century before, of obliquity: 40,040 years and of eccentricity: 91,800 years. No attempt was made however to find any analytical expression which lead to the list and origin of all the spectral components characterizing the long-term variations of these astronomical parameters (this became available much later in Berger, 1978a). From these numerical values he also produced tables and figures for insolation from 130,000 years BP to the present, including his "equivalent latitudes", i.e. the latitudes which presently receive, at the summer solstice, the same amount of energy as 70°N in the past. Finally, one section is devoted to the secular motion of the poles and one to the climate of the Planets. Significant of the scrupulous honesty of Milankovitch is a 13-page list of references on which he was basing his work.

It is also in 1920 that he was elected a member of the Serbian Academy of Sciences and Arts. A few years later, he was invited to participate to the ecumenical congress of the Eastern Orthodox Churches to be held in Constantinople (Istanbul today) on May 1 1923. On this occasion he calculated a new calendar which appeared to be the most accurate at that time (Milankovitch, 1923). It was accepted by the congress, but never implemented in practice. His fundamental research on incoming solar radiation did not prevent him to continue to work as a civil engineer and to start writing popular books, like "Through the Distant Worlds and Times", a collection of letters written to a young (virtual?) friend and published in 1928 (Milankovitch, 1928).

Captivated by the Milankovitch monograph, Wladimir Köppen offered him to collaborate on the study of past climates. This was a key step for the publicity of Milankovitch's work. His insolation curves started indeed to be much better known after Wladimir Köppen and Alfred Wegener introduced them for the equivalent-latitudes 55°, 60° and 65°N, and the caloric summer insolation at 65°N and 65°S, in their work "Die Klimate der geologischen Vorzeit" (Climate of the Geological Past) published in 1924. In 1927, he was invited to contribute to two important publications. One was the "Handbuch der Klimatologie" for which he wrote the introduction: "Mathematical science of climate and the astronomical theory of climatic variations" published in 1930 in German (Milankovitch, 1930). In this paper, we find, as in his 1920 monograph, his calculations of the daily insolation and of the energy received from the Sun over the hemispheres and the whole Earth for all latitudes and for different seasons, and a chapter on modeling the influence of the atmosphere on surface air temperature and climate. It is also in this publication that a chapter is devoted to the climate of the past 600,000 years with his famous curve of the 65°N equivalent-latitude comparing his calculation based on Pilgrim and Stockwell used in his 1920 monograph to those obtained when using Le Verrier and Miskovitch (1931). The second in which he was asked to contribute was the " Bontraeger Handbuch der Geophysik" published in 1931. There, his paper: "Position and Motion of the Earth in the Universe" shows his skill and great passion for the theory of the planetary motion which is at the basis of his astronomical theory of paleoclimates (Milankovitch, 1931). In the following years, Milankovitch concentrated on the impact of snow on the summer insolation. His results (Milankovitch, 1938a) were very helpful for the geologists because they allowed calculating the long-term variations of the snow line over the last 600,000 years.

In 1939, pulling together his earlier papers in a single work, he decided to write his "Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem" published in 1941 by the Königlich Serbische Akademie (Milankovitch, 1941). The last page on the Kanon was printed 2 April 1941, but the bombing of Belgrade of 6 April almost destroyed the proofs of the whole book. The only copy left and kept by Milankovitch allowed finally the first sample of copies of the Kanon to reappear in the autumn of 1941. The German edition was translated in 1969 into English: "Canon of Insolation and the Ice Age Problem" by Israel Program for Scientific Translation and published for the Department of Commerce and the National Science Foundation, and re-published in 1998 by the Zavod za udzbenike i nastavna sredstva in Belgrade with an additional 35-page biography of Milankovitch by Nikola K. Pantic.

This book, being more complete than his 1920 monograph, is a real compendium on the astronomical theory. It is divided into six parts. Parts One and Two, devoted to the planetary motion around the Sun and the rotation of the Earth, provide all the necessary information to compute the numerical values of the eccentricity, obliquity and climatic precession. Part Four deals with the terrestrial insolation and its secular changes and allows not only computing the daily insolation, the seasonal irradiation and the caloric season insolation, but also their long-term variations. In paragraph 86 (of the 1969 English translation), Milutin Milankovitch stresses again the influence of obliquity on insolation, as it was done by his predecessors, but with many more details: “The variations of the quantities of radiation at an increase of obliquity by 1°, already published in 1914, were of fundamental importance because they showed for the first time the influence of the variations of obliquity upon insolation in full detail….an increase of obliquity slightly reduces the annual irradiation of the equatorial zones while those of the polar zones are notably increased… boundary lies at 43°33’….it reduces the geographical contrast. The summer radiation is reduced with an increase of obliquity only up to 11°23’, otherwise it is increased. The winter radiation is reduced at all latitudes.” For the irradiation over a season (paragraph 85), we find: “ Wiener in his treatise….. His results agree exactly with mine, though his method of computation is different. The same is also true of the results obtained by Lambert, Meech, Angot and Hargreaves.” As in his preceding papers, he did not attempt to use the elliptic integrals, although their numerical calculation was available (King, 1924). This might have helped him to resolve the problem raised by the convergence of his series expansion for the insolation at high latitudes about which he wrote (end paragraph 76): "For the higher latitudes (*above 55°*), for which a greater number of coefficients would be necessary, we would have to set up a greater number of equations…)". It is Fempl (1957, 1958), Milankovitch's assistant and colleague, who has started using the elliptic integrals for computing the long-term variations of insolation up to the latitudes of 80° and 85°. It is also in this chapter of his book that Milankovitch came back with the average and extreme periods of the astronomical elements that he deduced from their numerical values calculated either from Pilgrim-Stockwell (Table VIII) or from Le Verrier-Miskovitch (Table IX): “The secular variations of precession has a rather irregular behaviour…with an average interval of 21,000 years (16,200 to 25,800 years). The average period of the oscillations of eccentricity amounts to 92,000 years (77,000 to 103,000 years). The average between two consecutive maxima of obliquity was on the whole about 40,000 years (38,000 to 45,000 years)". We find however no indication about the split of the 21,000-year precession period into 19,000 and 23,000 years (Berger, 1978a). This split is playing a fundamental role in the explanation of the 100,000-year cycle found in geological record, as this period is often assumed to originate from a non-linear response of the climate system to these two precessional periods (Wigley, 1976, Berger, 1989). The same is true for the Berger (1978a) 400,000 –year cycle which is a key period in the search for analogues of our interglacial and its future (Berger and Loutre, 1996, 2002). The 72 pages of Part Five are devoted to his mathematical climate research, it means to the influence of insolation on the Earth's temperature and atmosphere. Part Six deals extensively (117 pages) with the Ice Age, its mechanism, structure and chronology. In Part Two, Milankovitch develops also the secular motion of the poles, another subject which fascinated him.

During World War II, Milankovitch decided to write his memoirs, not because "he considered himself as an important scientific, but because nobody knew him and his contributions better than him". This autobiography, "Memories, experiences and knowledge, written in Serbo-Croatian was published by the Serbian Academy of Sciences and Arts (Milankovitch, 1950, 1952, 1957, 1979) but never translated. This is one of the reasons which encouraged Vasko Milankovitch, Milutin's son, to write the history of his father's life (Milkankovitch V., 1995). The most important human and scientific features of Milutin Milankovitch's life are described in a lively and lovely way. This book will remain the most important contribution to Milankovitch's biography. To my knowledge, there are mainly three out in Serbian: Berger and Andjelic (1988) in French, Pantic (1998) in English and Petrovic (2002) in both English and Serbian. However there are many short notes about Milankovitch's life and the astronomical theory; some are excellent summaries, others do not necessarily provide an objective view of Milankovitch's contributions to science.

Following World War II, Yugoslavia became a federal state firmly held behind the Iron Curtain which made Milankovitch becoming more and more disappointed. In 1947, his son Vasko and wife Vera left the communist country to finally settle in Australia. Milankovitch "tried to bridge the enormous gap which separated them, looking to their future and offering advice…" In January 1953, he wrote to Vasko: "One consolation is that my Astronomical Theory of Climatic Changes is appearing more and more in the scientific literature around the World. My scientific authority has given me, even here, a unique independent position, so no one bothers me and I live in peace". Unfortunately, his experience at the I.N.Q.U.A. conference in Rome in September 1953, the last meeting he attended, was very unfortunate and most regrettable: he was forced by the president of the session, Richard Flint, to leave the floor after delivering only half of his paper (Jovanovic et al., 2004), although he was doing his best by delivering his lecture in French, not his native language (Milankovitch, 1954, 1956). This often mentioned incident is based mainly on Milankovitch's own recollection and feelings, but it was also reported to me by the Belgian climatologist Étienne Bernard who was present. I would like to see whether there is another independent account, either from Flint's papers or memoirs, if such exist, or from any other attendee of the Congress. Milankovitch might have overreacted, although this is purely hypothetical. His last publication was on the Astronomical theory published by the Serbian Academy of Science in 1957.

Milutin Milankovitch died in Belgrade on 12th December 1958. He was initially buried in the Topuzovic family grave in Belgrade, but later in the Milankovitch family grave in Dalj.

The Milankovitch debate

If we consider the Milankovitch insolation curve, however, we are left in no doubt that Milankovitch's success was partial, because the Quaternary has had many more glacial periods than was claimed during the first part of the twentieth century (Kukla, 1975a). In fact, until roughly 1970 the Milankovitch theory was largely disputed because the discussions were based on fragmentary geological sedimentary records and on inaccurate time scales, and because the climate was considered too resilient to react to 'such small changes' in his summer half-year caloric insolation (Simpson, 1940). Moreover, the accuracy of the long-term variations of the three astronomical parameters and of the related insolation (namely in polar latitudes) had also to be evaluated.

The first criteria used to test the astronomical theory was the visual or statistical relationship between minima and maxima of geological and insolation curves, the Milankovitch summer radiation curve for 65°N being used more frequently because of the more extensive nature of Pleistocene glaciation in the Northern Hemisphere.

These qualitative coincidences of the principal maxima and minima of both curves would, however, have remained somewhat illusory until the ambiguities stemming from *a priori* assumptions about sensitive latitudes and response mechanisms were resolved. As an attempt to solve this problem, many insolation values for different seasons and latitudes or combination of them (Broecker and van Donck, 1970; Kukla, 1972, 1975b; Kukla et al., 1981) were used up to the late

1970s. For a more extensive review of the publications of this epoch and the following ones, the reader should refer to Berger (1988).

In the meantime, climatologists (Shaw and Donn, 1968; Budyko, 1969; Sellers, 1970; Saltzman and Vernekar, 1971) started to approach the problem theoretically, but found that the climatic response to orbital change was too small to account for the succession of Pleistocene ice ages. However, if these early numerical experiments are viewed narrowly as a test of the astronomical theory, they are open to question because the models used much too simple parameterizations of important physical processes.

The Milankovitch renaissance

In the late 1960s, judicious use of radiometric dating and other techniques gradually clarified the details of the time scale, better instrumental methods came on the scene for using oxygen isotope as an indicator of ice volume and ocean temperature (Shackleton and Opdyke, 1973) but also salinity (Duplessy, 1970), ecological methods of core interpretation were perfected (Imbrie and Kipp, 1971), global climates in the past were reconstructed (CLIMAP Project Members, 1976), and atmospheric general circulation models and climate models became available (Alyea, 1972). Owing to these improvements, Hays et al. (1976) showed, for the first time, that quasi-periods of 100,000, 41,000, 23,000 and 19,000 years are significantly present in proxy records of the past climate. Independently, Berger (1973, 1976, 1978a) had already found these periods in the long-term variations of eccentricity, obliquity and climatic precession that he calculated using a new more accurate solution of the planetary system. This definitely confirmed the astronomical origin of the periodicities found in geological records. The existence, in particular, of a double precessional peak both in the geological record and in the astronomical solution has been, according to John Imbrie himself, one of the first most delicate and impressive tests of the Milankovitch theory and critical for its validation.

These results were at the origin of a revival of the astronomical theory of paleoclimates. New researches were going to be initiated in the four main branches of any astronomical theory, namely: (1) the computation of the astronomical elements, (2) the computation of the appropriate insolation parameters, (3) the development of suitable climate models, and (4) the analysis of geological data in both the time and frequency domains in order to investigate the physical mechanisms which are responsible for the long-term climatic variations and to calibrate and validate the climate models. An extensive list of the main contributions to this revival published up to 1980 is given in Berger (1988).

The large amount of papers related to the astronomical theory over the last 30 years show updates of the Milankovitch calculations and theory, but also new proposals of how astronomical elements of the Earth's orbit and axis of rotation might impact climate. The list here below does not include modeling the response of the climate system to astronomical forcing (for a review see e.g. Berger, 1995; Stocker and Marchal, 2001; Claussen et al., 2002; Sirocko et al., 2007; PMIP-Paleoclimate Modeling Intercomparison Programme) but focuses only on research dealing with the astronomical parameters and the related solar irradiation which were at the basis of Milankovitch’s work. The purpose is not to produce an extensive review of what has been done since Milankovitch’s last publication (this would be out of the scope of this introductory survey), but rather to give a feeling of how much fertile was the entire work of Milankovitch.

Milankovitch follow-up

1. About the astronomical solutions

New analytical astronomical solutions for the Quaternary appeared at the end of Milankovith’s life with Brouwer and Van Woerkom (1950) and later with Sharaf and Budnikova (1967), Anolik et al. (1969), Bretagnon (1974), Berger (1976, 1977, 1978a), Berger and Loutre (1991), Laskar (1988) and Laskar et al.(2004).

Milankovitch was concentrating on how to obtain the best numerical values for the long-term variations of the three astronomical parameters and the insolation. Its primary aim was to produce curves for climate or proxies (like his 65°N equivalent-latitude) that, in collaboration with Penck, Brückner, Köppen and Wegener, he correlated with the first geological record covering the last million years. Apparently, probably because of the limited number of data and techniques available at that time, he did not draw attention to the spectral characteristics of the astronomical parameters. He could have done it from a complete analytical solution of the system of equations which governs the motions of the Moon and the planets. Such analytical solution, calculated by Berger in the early 1970s, allows indeed the numerical values of precession, obliquity and eccentricity to be directly expressed in trigonometric form as quasi-periodic functions of time:

$$e\sin\tilde{\omega} = \Sigma P_i \sin\left(\alpha_i t + \eta_i\right)$$

$$\varepsilon = \varepsilon^* + \Sigma\, A_i \cos\left(\gamma_i t + \zeta_i\right)$$

$$e = e^* + \Sigma\, E_i \cos\left(\lambda_i t + \phi_i\right)$$

where the amplitudes P_i, A_i, E_i, frequencies α_i, γ_i, λ_i and phases η_i, ζ_i, ϕ_i were calculated in the 1970s by Berger (1978a) and later by Berger and Loutre (1991) using the development of the orbital elements by respectively Bretagnon (1974) and Laskar (1988) and the analytical expansions of obliquity and precession by Anolik at al. (1969). Such expressions for $e\sin\tilde{\omega}$, ε and e can be used over one to three million years (Berger and Loutre, 1992), but for more remote times numerical solutions are necessary (see below, Laskar et al., 2004).

These formulae show that ε and e vary quasi-periodically only around the constant values ε^* (23.32°) and e^* (0.0287). This implies that, in the estimation of the order of magnitude of the terms in the insolation formulae where ε and e occur, they may be considered as a constant to a first approximation. Moreover, in the insolation formulas, the amplitude of $\sin\tilde{\omega}$ is modulated by eccentricity in the term $e\sin\tilde{\omega}$. The envelope of $e\sin\tilde{\omega}$ is therefore given exactly by e, allowing the frequencies of e to be expressed as combinations of the frequencies of $\tilde{\omega}$; for example: $\lambda_1 = \alpha_2 - \alpha_1$, $\lambda_2 = \alpha_3 - \alpha_1$, $\lambda_3 = \alpha_3 - \alpha_2$, $\lambda_4 = \alpha_4 - \alpha_1$, $\lambda_5 = \alpha_4 - \alpha_2$, and $\lambda_6 = \alpha_3 - \alpha_4$ (Berger, 1978a; Berger and Loutre, 1990). This leads to the conclusion that the periods characterizing the expansion of e are nonlinear combinations of the precessional periods (and vice-versa) and, in particular that the eccentricity periods close to 100,000 years are originating from the periods close to 23,000 and 19,000 years in precession. This also shows that the frequencies of a given parameter are not all independent of each other. For example, $\lambda_3 = \lambda_2 - \lambda_1$, a relationship which can also be deduced directly from the frequencies of the fundamental orbital elements when creating the series expansion of the eccentricity. Similar relationships between the periods of eccentricity, obliquity and precession are available in Berger and Loutre (1990).

The full spectral characteristics of the astronomical elements and of the insolation date back only from the 1970s. Emiliani (1955), like Milankovitch (1920) 35 years earlier, estimated the mean periods of the astronomical parameters by counting the number of peaks from the Milankovitch curves which gave him about 92,000, 40,000 and 21,000 years for e, ε and $e\sin\tilde{\omega}$, respectively. These were confirmed in 1973 when Berger had completed his calculation of the long-term variations of precession, obliquity and eccentricity. Besides its high accuracy, the Berger calculation provided indeed, for the first time, a full list as well as the origin of the periods characterizing the theoretical expansion of e (with periods of 413,000, 95,000, 123,000, 99,000, 131,000 and 2,305,000 years in decreasing order of amplitude of the terms), of ε (with periods of 41,000, 53,600 and 29,700 years) and of $e\sin\tilde{\omega}$ (with periods of 23,700, 22,400, 18,900 and 19,200 years) (see also Berger, 1978a, and Berger and Loutre, 1991). Among these periods, those of 413,000, 2,305,000, 54,000, 23,000 and 19,000 years were new, their existence having never been even suspected before.

In their Science paper, Hays et al. (1976) used a spectral analysis technique which they applied on the numerical values of the astronomical parameters calculated by Brouwer and Van

Woerkom (1950) and Vernekar (1972) and found also 125,000 and 96,000 years for e; 41,000 years for ε; and 23,000 and 19,000 years for precession.

Because of new techniques available (like the wavelet transforms), the complex structure of the long-term variations of the astronomical parameters became possible (Berger et al., 1998). For the eccentricity, it can be shown that the 100,000 years period is not stable in time, being remarkably shorter near the present. Actually, the most important theoretical period of eccentricity, 400,000 years, is weak before 1 Ma BP, and becomes particularly strong over the next 400,000 years, with the strength of the components in the 100,000 years band changing in the opposite way. It is worth pointing out that this weakening of the 100,000 years period started about 900,000 years ago when this same period began to appear very strongly in paleoclimate records. This implies that the 100,000 years period found in paleoclimatic records is definitely not linearly related to eccentricity. We are now approaching a minimum of e at the 400,000 years time scale: at 27,000 years AP (after present), the Earth's orbit will be circular. Actually, transitions between successive strong 400,000-year cycles (as it is the case now) are characterized by very small eccentricity and short eccentricity cycles with a low amplitude of variation. At the 400,000-year time scale, the amplitude and frequency modulations of precession are inversely related: when the amplitude is small, the period is short. The reverse is observed at the 100,000-year time scale where a large amplitude is accompanied by a short period and vice versa. For obliquity, the main period is pretty stable, but there is an amplitude modulation with time duration of about 1,300,000 years (Mélice et al., 2001). At that time scale, a large amplitude corresponds to a short period, the reverse being observed at the 170,000-year time scale. The spectra of both the amplitude and frequency modulations of obliquity display significant power at 171,000 and 97,000 years (Mélice et al., 2001). Although this last period might look close to the so-called 100,000 years eccentricity period, these periods are not related.

Because of the great interest devoted to the 100,000-year cycle (Crucifix, 2011), the most important period in ice volume and CO_2 record, it was interesting to look for the presence of the

100,000-year cycle in the astronomical data first (Berger et al., 2005a). In addition to the already mentioned 100,000-year cycles in the eccentricity and amplitude modulation of obliquity, this cycle can also be found in the rate of change of eccentricity where it becomes stronger than the 400,000-year cycle, contrary to what happens in the eccentricity. It is also present in the inclination of the Earth's orbit on the invariable plane (plane perpendicular to the total angular momentum of the planetary system), but its origin prevents it to be associated to the 100,000-year cycle present in geological record.

Geological record are now available with a high accuracy over tens of millions of years (e.g. Lourens et al., 2001), allowing to calibrate the astronomical solutions by Laskar et al. (1990, 1999, 2004) who was the first to calculate them over such long time scales (a possibility foreseen by Deprit et al. at the Milankovitch symposium in 1984).

Because of the huge ice sheets present during the Pleistocene glaciations, their influence over the spectral characteristics of the astronomical elements was estimated by Dehant et al. (1990). For the much earlier geological Periods (e.g. Hinnov and Goldhammer, 1991), a similar sensitivity of the astronomical frequencies was performed to the changes in the Earth's rotation rate, the distance from the Earth to the Moon and the dynamical ellipticity of the Earth (Berger et al., 1989, 1992; Peltier and Jiang, 1994), showing a shortening of all of them back in time.

For much recent times, the variations of the astronomical parameters at the time scale of decades to millennia became available owing to the work of Bretagnon (1982) and its application to the astronomical theory by Loutre et al. (1992) and Bertrand et al. (2002).

2. About insolation

In addition to the critical high northern latitudes proposed by Milankovitch, other latitudes were suggested, as for example the tropics (Bernard, 1962) or the equatorial latitudes (McIntyre and

Molfino, 1996; Berger and Loutre, 1997), as well as seasons other than Northern Hemisphere summer, as for example fall and winter (Kukla, 1975b).

In addition to the caloric insolation of Milankovitch (up-dated by Vernekar, 1972 and Berger, 1978b), the seasonal and latitudinal distribution (Berger, 1979) of the daily solar irradiance (Berger, 1978) started to be used to force the climate models (Berger et al., 1990; Gallée et al., 1991, 1992; Ganopolski and Calov, this volume). This leads to consider precession as a main driving factor of the climate system as it is for the daily irradiance everywhere on Earth (except close to the polar night). Such behavior is fundamentally different from the behavior of the Milankovitch caloric insolations where precession and obliquity controls respectively the low and high latitudes (Berger and Pestiaux, 1984). This is the reason why, more recently, the total energy received during the astronomical seasons, which depends exclusively upon obliquity (Berger et al., 2010), is tentatively used to explain climatic changes of the lower Pleistocene and the glacial-interglacial cycles of the last 900,000 years. Huybers and Wunch (2005) argue that the 41,000-year cycle has always been dominat and the 100,000-year cycle is created by averaging groups of 2 and 3 obliquity cycles (80,000 and 120,000 years). This theory is consistent with the multi-state model by Paillard (1990) and the model by Ditlevsen (2008), leading to the 100,000-year cycle being a non-linear response to the 41,000-year obliquity cycle, but remains controversial.

Taking back the idea of Milankovitch about the important role played by the albedo of the Earth's surface, analysis of such impact on the spectral characteristics of solar energy absorbed at the surface of the Earth (Blatter et al., 1984; Tricot and Berger, 1988) showed that the gradient of insolation between the tropics and the polar regions has a spectrum which depends upon the kind of insolation used; the 40,000-year periodicity dominates in the extraterrestrial insolation whereas in the absorbed insolation by the surface, a 23,000-year signal is also present. This difference is due to the attenuation by the atmosphere and the surface albedo which reduce more strongly the variations of insolation in the high polar latitudes than in the tropics.

Much can be said about the astronomical signals found in the paleoclimatic record and about modelling the response of the climate system to the astronomical forcing, but that would be out of the scope of such a short introduction to the history of the astronomical theories over the last 200 years.

Conclusions

The purpose of this short note was to describe the scientific environment in which Milankovitch lived and developed his astronomical theory of paleoclimates. It was also a good opportunity to give credits to those scientists who introuded the key concepts of the astronomical theory of paleoclimates. These early scientists are often forgotten in the references list of papers where some authors are crediting Milankovitch for most of the ideas which in fact have been discussed much before his time or after . Milankovitch would have probably object to this overshight as he was known for his meticulous referencing on which he based his research and his integrity. For example, in his conference at the Charles University on 11 November 1937, he thanked not less than "34 eminent scientists for their fundamental contributions to geology and climate on which he could base his own work", Milankovitch, 1938b).

The most important contributions of Milankovitch are:

1. writing such a compendium where all chapters related to the astronomical theory are clearly written with all the details necessary for an in-depth understanding – masterfully written lecture notes.

2. introducing a new insolation parameter, the caloric season insolation. Although it does not completely solve the problem of using the irradiation over a given interval of the year (the beginning and end are changing with time), it has the advantage to have a fixed length accumulating the energy requested by many natural living species.

3. introducing the concept of physical models based on the principles of physics to try approaching the real climate much better than by using only the energy available at "the top of the atmosphere". Milankovitch actually wrote many more papers on modelling the impact of atmosphere and of the surface of the Earth on the insolation and climate than on insolation itself. He must definitely be considered as the "father" of climate modelling. Unfortunately very few people acknowledge his important contribution to this research field. For example, using only the daily insolation "at the top of the atmosphere" presents a real danger, namely because the latitudinal distribution of this parameter, which magnitude is much larger at the summer poles than at the equator, leads to a latitudinal gradient of insolation which has a sign opposite to the sign of the latitudinal gradient of temperature, usually cited to control the strength of the general circulation of the atmosphere, a key point for climate.

Contrary to what is often claimed, Milankovitch, as he recognised himself, cannot be credited for:

1. the calculation of eccentricity, obliquity and precession, referring to Stockwel-Pilgrim and to Le Verrier-Miskovitch respectively in his 1920 monograph and his 1941 Canon.

2. the calculation of daily insolation and the irradiation received during the astronomical seasons, which were already published decades before by Meech and Wiener. However Milankovitch has introduced a clear analysis of the impact of obliquity variation on insolation and the calculation of the energy available for a giving time interval of the year.

3. the very fundamental hypothesis that the occurrence of the Northern Hemisphere summer at the aphelion is the cause of glaciations. This is probably the greatest mistake done in the present-day literature because this is exactly what people refer to as the "Milankovitch theory of paleoclimates". This theory must definitely be attributed to Murphy who introduced the idea four decades before Milankovitch took it back.

4. some periods characterizing the long-term variations of the astronomical parameters. Not only Milankovitch did not seem to be much interested by these periods, but the precessional period of 21,000 years was well known since Adhémar at least and the periods of about 400,000, 54,000, 30,000, 23,000 and 19,000 years as well as of 2,305,000 and 1,300,000 years appeared for the first time with Berger's work in the early 1970s.

Before ending this short note, I like to stress again that my remarks just above have been clearly underlined by Milankovitch himself, rendering unto Caesar that which is Caesar’s, and does not minimize, in no way, his fundamental contribution to the scientific understanding of long-term climatic variations. Finally, let us point out that this great scientist is also one of the very few, even now, who cared lecturing and publishing not only in his native language, but also in German, French and Russian.

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Glaciations

1706-1767	Martel	erratic boulders due to glaciers
1767-1858	Perraudin	
1740-1799	de Saussure	floods to transport erratic blocs
1763-1832	Cuvier	environmental catastrophes
1797-1875	Lyell	ice raftsto transport erratic blocs
1712-1772	Tillas	drifting sea ice to transport
1780-1851	Walhenberg	glaciations
1763-1839	Esmark	1824 Astronmical theory of glaciations
1788-1829	Venetz	huge glaciers front to transport
1797-1849	Benhardi	polar ice cap
1786-1855	de Charpentier	diluvian glaciers
1803-1867	Schimper	1837 Eiszeit
1801-1873	Agassiz	sequence of glaciations
1749-1832	Goethe	
1786-1830	Fourier	Greenhouse effect
1820-1893	Tyndall	Greenhouse and glaciers
1858-1945	Penck	Glaciations in the Alps
1862-1927	Brückner	
1843-1928	Chamberlain	Glaciations in America

Pioneers of Astronomical Theory

1797-1862	Adhémar	periodic glaciations, southern hemisphere
		precession, length of the seasons
1769-1859	von Humboldt	season irradiation
1821-1890	Croll	1864 precession, NH Winter at aphelion
		1867 climatic precession, Le Verrier
		1867 obliquity
1809-1882	Darwin	supported Croll
1835-1924	Geikie A	
1839-1914	Geikie J	1874 glacial-interglacial cycles
1890	Howorth	against antisymmetry between hemispheres
1858-1943	de Geer	dated post glacial
1840-1913	Ball	claimed to refine Croll
1855-1931	Culverwell	small variations but obliquity missing
1857-1936	de Marchi	atmospheric transmission
1859-1927	Arrhenius	CO_2
1797-1875	Lyell	astronomical theory exaggeated
1896	Hargreaves	obliquity annual insolation
1901	Ekholm	extreme obliquity on insolation no precession
1849-1946	Spitaler	astronomical values for glaciation
1881-1949	Hopfner	missed discontinuity insolation high latitudes

Astronomers

1749-1822	Delambre	astr precession
1773-1849	Francoeur	perihelion
1717-1783	Le rond d'Alembert	precession
1792-1871	Herschell	insolation, total energy
1811-1877	Le Verrier	eccentricity and obliquity
1855	Meech	insolation, total energy
1752-1833	Legendre	elliptic integrals
1749-1827	Laplace	obliquity max
1781-1840	Poisson	motion through planetary system
1876	Wiener	insolation, elliptic integrals
1822-1920	Stockwell	astronomical parameters
		Neptune included
1924	King	numerical values of elliptic integrals

Milankovith Era

1921	Spitaler	suggested idea of Murphy
1869	Murphy	NH summer at aphelion
1876	Murphy	insolation based on Stockwell
1925	Penck et al.	adopted Murphy

1879-1958 Milankovitch life

1904		Ph D
1912		Mathematical theory of climate
1920		monograph in French
1923		new calender
1928		popular book
1924	Koppen-Wegener	Milankovich curves in their book
1930		Handbook Klimatologie
1931		Handbook Geophysik
1931	Miskovitch	calculated astr parameters based on Le Verrier
1938		snow line
1941		Kanon
1950		Memoirs
1953		INQUA regrettable experience
1957		last paper

1957	Fempl	extend Milankovitch to high polar latitudes
1995	Milankovitch V.	biography of Milutin