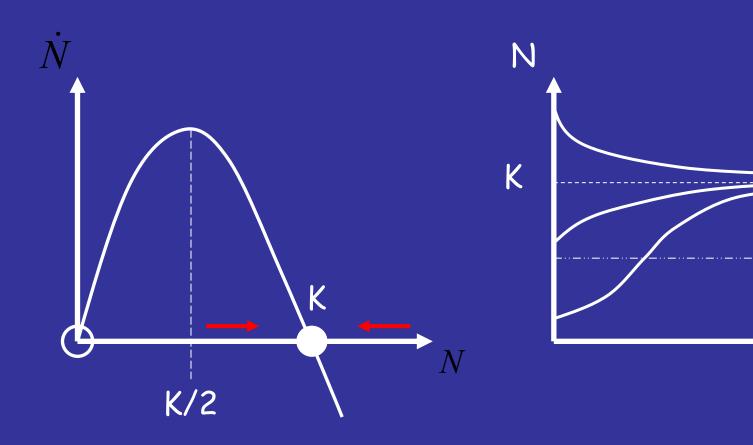
# Logistic Differential Equation

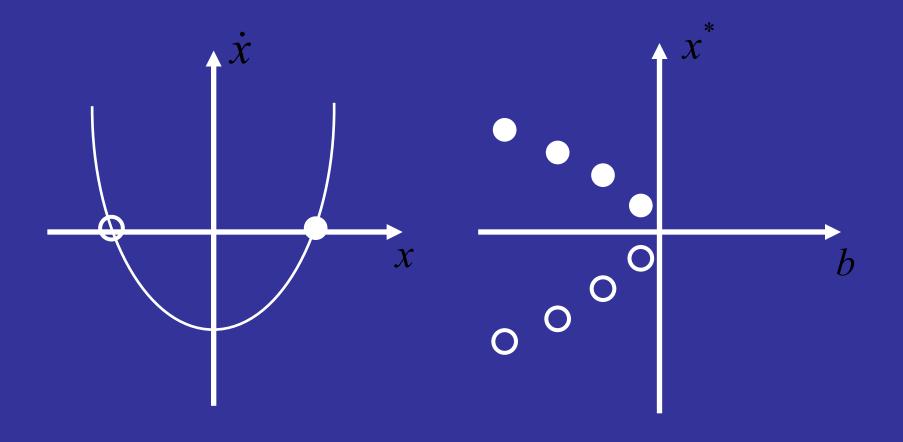
$$\dot{N} = rN \left( 1 - \frac{N}{K} \right)$$



### Saddle Node Bifurcation (1-dim)

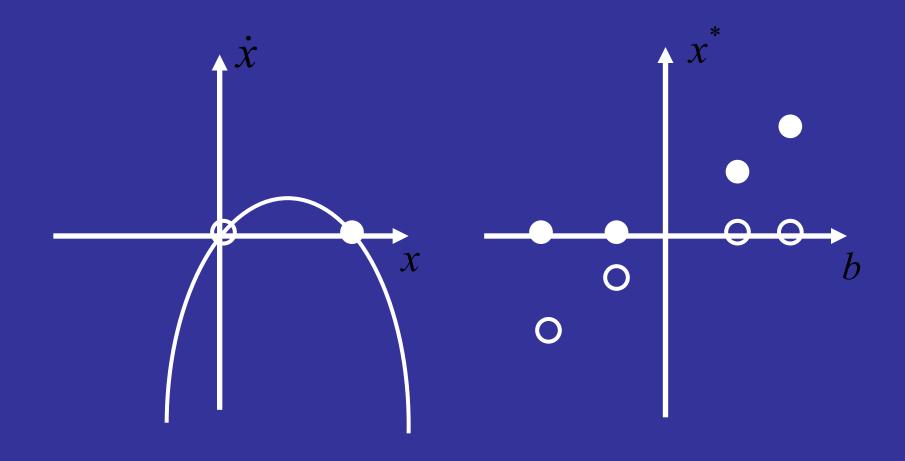
Prototypical example:  $\dot{x} = b + x^2$ 

$$\dot{x} = b + x^2$$



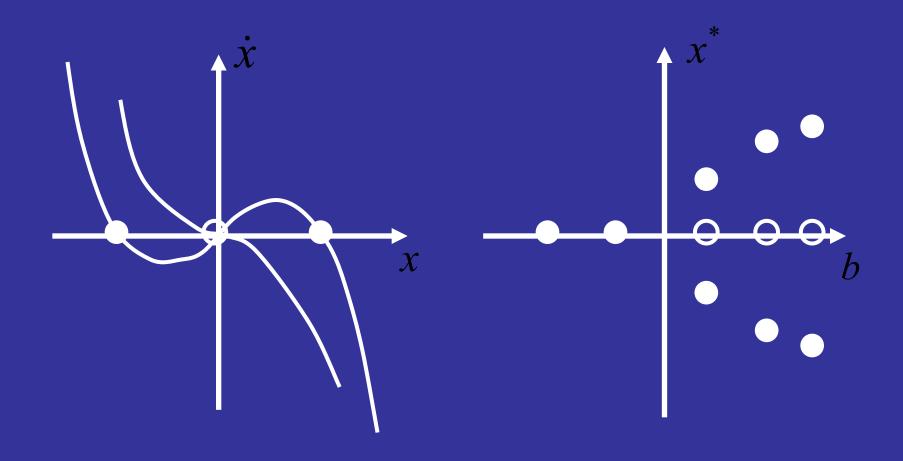
### Transcritical Bifurcatoin

Prototypical example:  $\dot{x} = bx - x^2$ 



## Pitchfork Bifurcation

Prototypical example:  $\dot{x} = bx - x^3$ 



#### Dynamics of Two Dimensional Systems

- 1. Find the fixed points in the phase space!
- Linearize the system about the fixed points!
- Determine the eigenvalues of the Jacobian.

Eigenwerte λ	Normalform von A	Name des Fixpunktes	Trajektorienverlauf
•	$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_0 \end{pmatrix}$	stabiler Knoten	
	$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$		***
	$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$	entarteter stabiler Knoten	2
	$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$	Sattel	
	$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$	instabiler Knoten	*
•	$\begin{pmatrix} x & -\omega \\ \omega & x \end{pmatrix}$	stabiler Strudel	(e)
6	- (a10)	instabiler Strudel	(a)
+	(0 -w)	Wirbel (Zentrum)	6
-	(\lambda \ 0 \ 0 \ 0 \ 0 \)	-	