Dynamics II:

Stochastic climate model

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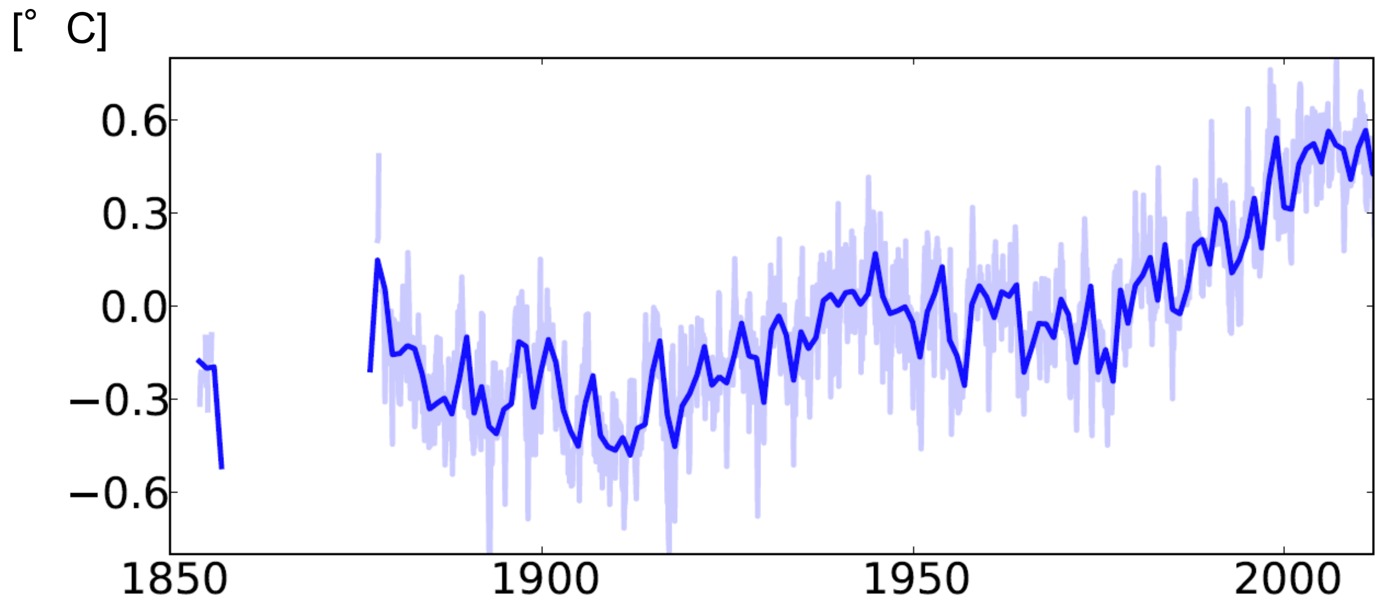
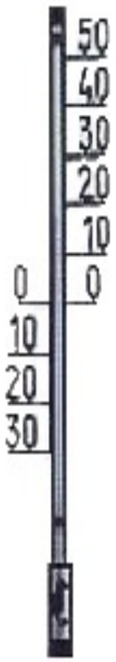
Statistical data analysis identifying mechanisms of climate change

Temperature of the last **150 years** (instrumental data)

Northern Hemisphere Temp. anomaly

HadCRU

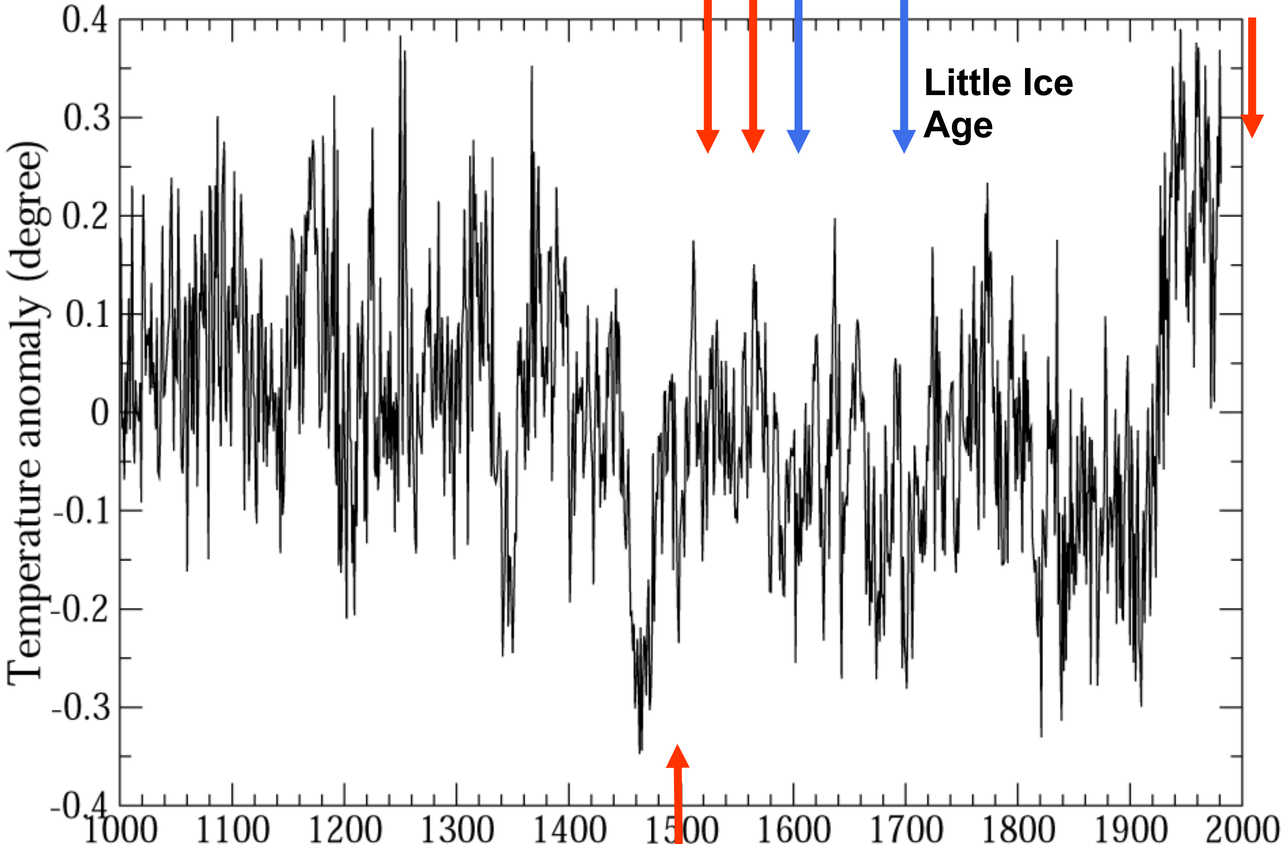




History

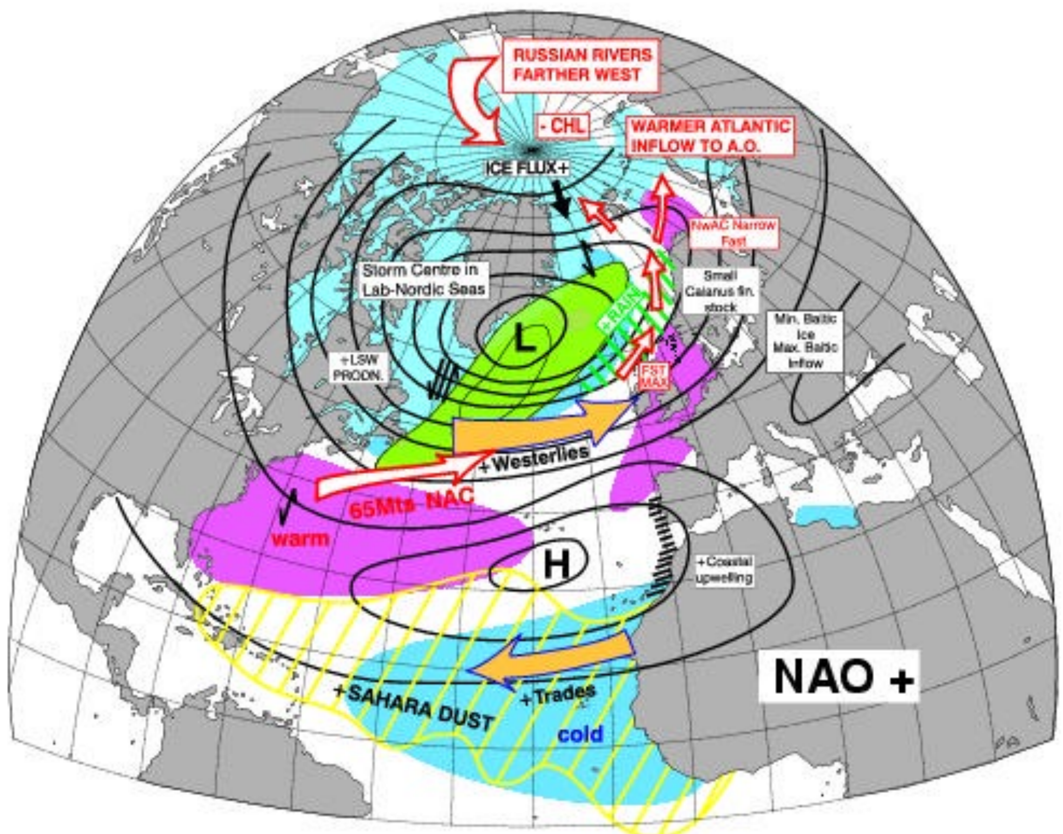
last **1000 Years**

Pieter Breughel d.Ä.



Analysis of spatio-temporal pattern

The Phases of the North Atlantic Oscillation



During the high phase of the NAO westerlies in the North Atlantic are enhanced, resulting in mild and wet winter conditions over Northern Europe. (Courtesy of CEFAS, UK)

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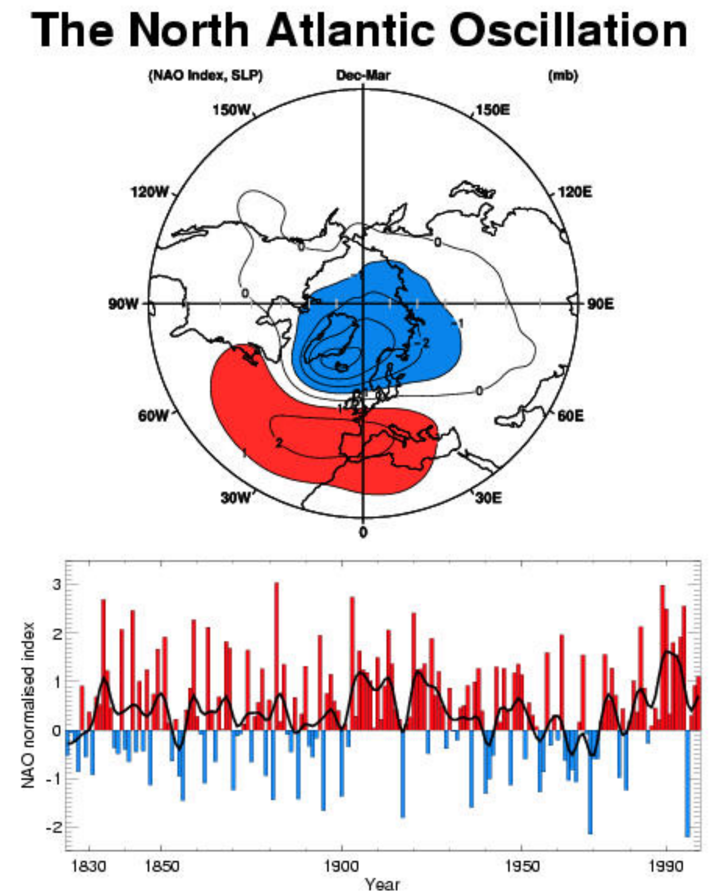
Analysis of spatio-temporal pattern

- NAO, ENSO
- Definitions



Upper panel: Observed Dec-March change in SLP associated with a 1 standard deviation change in the NAO index (after Hurrell, 1995, Science, 269, 676-679).

Lower Panel: Winter (December to March) index or the NAO based on the difference of normalized pressure between Lisbon, Portugal and Stykkisholmur, Iceland from 1864 to 1995. The SLP anomalies at each station were normalized by division of each seasonal pressure by the long-term mean (1864-1995) standard deviation. The heavy solid line represents the meridional pressure gradient smoothed with a low pass filter with seven weights (1,3,5,6,5,3, and 1) to remove fluctuations with periods less than 4 years (after Hurrell, 1995, Science, 269, 676-679, this version: courtesy of T. Osborn, CRU, UEA.

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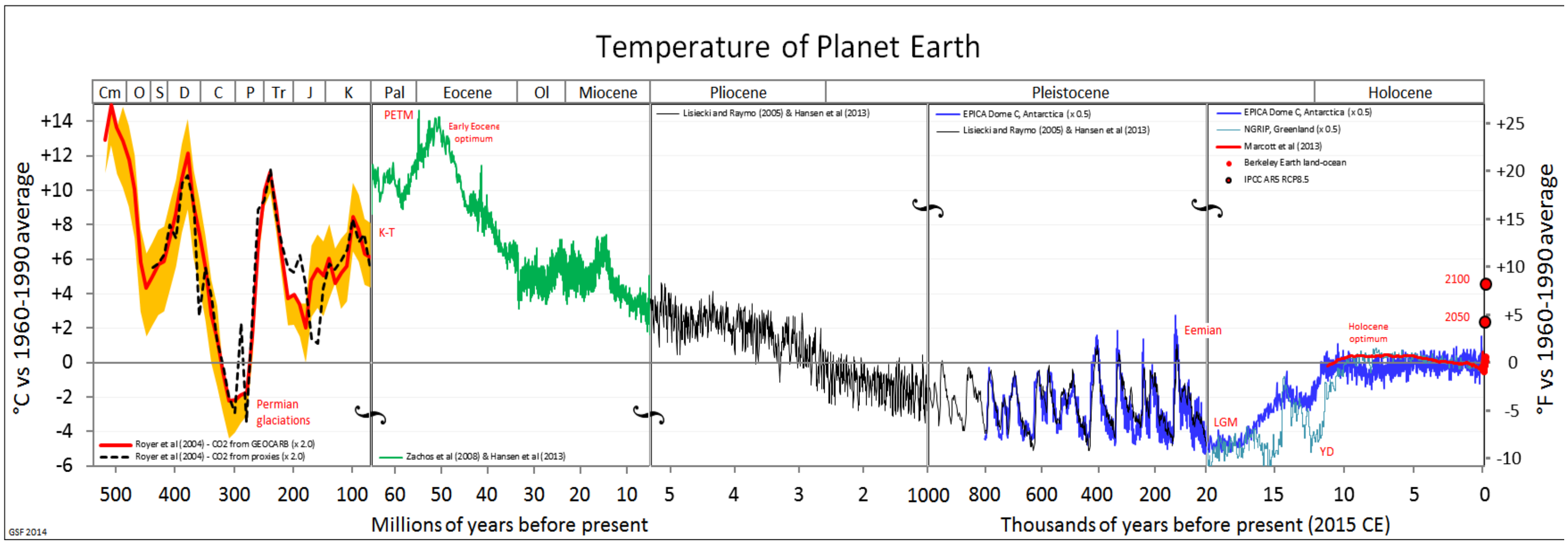
Long-term temperature evolution on Earth

Not only do **global temperatures** move up & down on geological time scales, nor do they just switch from one long-term mean to another:
They clearly show **changes in dynamic regime** — from high to low variability, from one dominant periodicity to another, from high to low drift, and so on.



Overall, to model this **complex behavior** we do need to consider both **chaotic** & **random** ingredients, both **intrinsic** & **forced** variability.

Compiled by Glen Fergus, https://commons.wikimedia.org/wiki/File%3AAll_palaeotemps.png

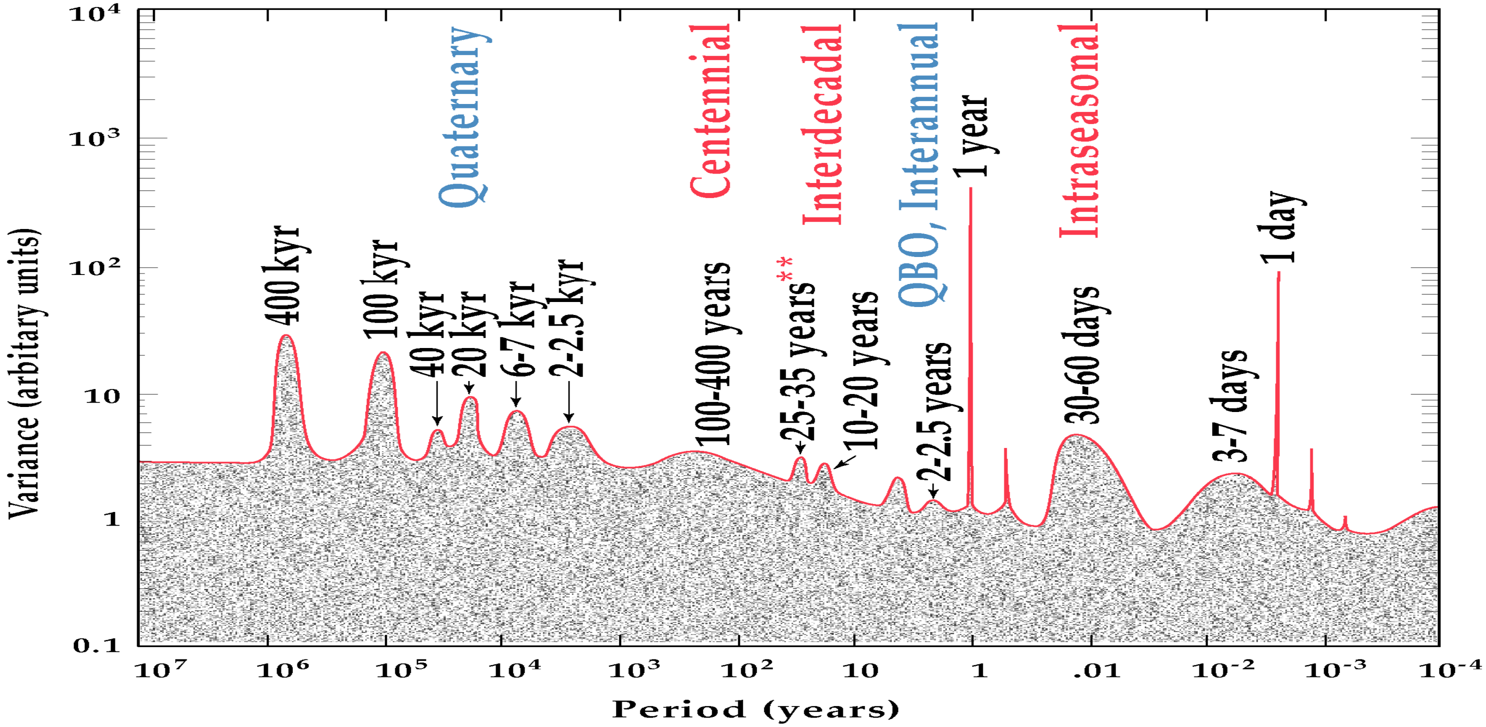
N.B. Plot is ~"log-linear": time axis is logarithmic+linear, temperature axis is linear.

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. High frequencies – white noise (or “colored”)

2. Low frequencies – slow evolution of parameters



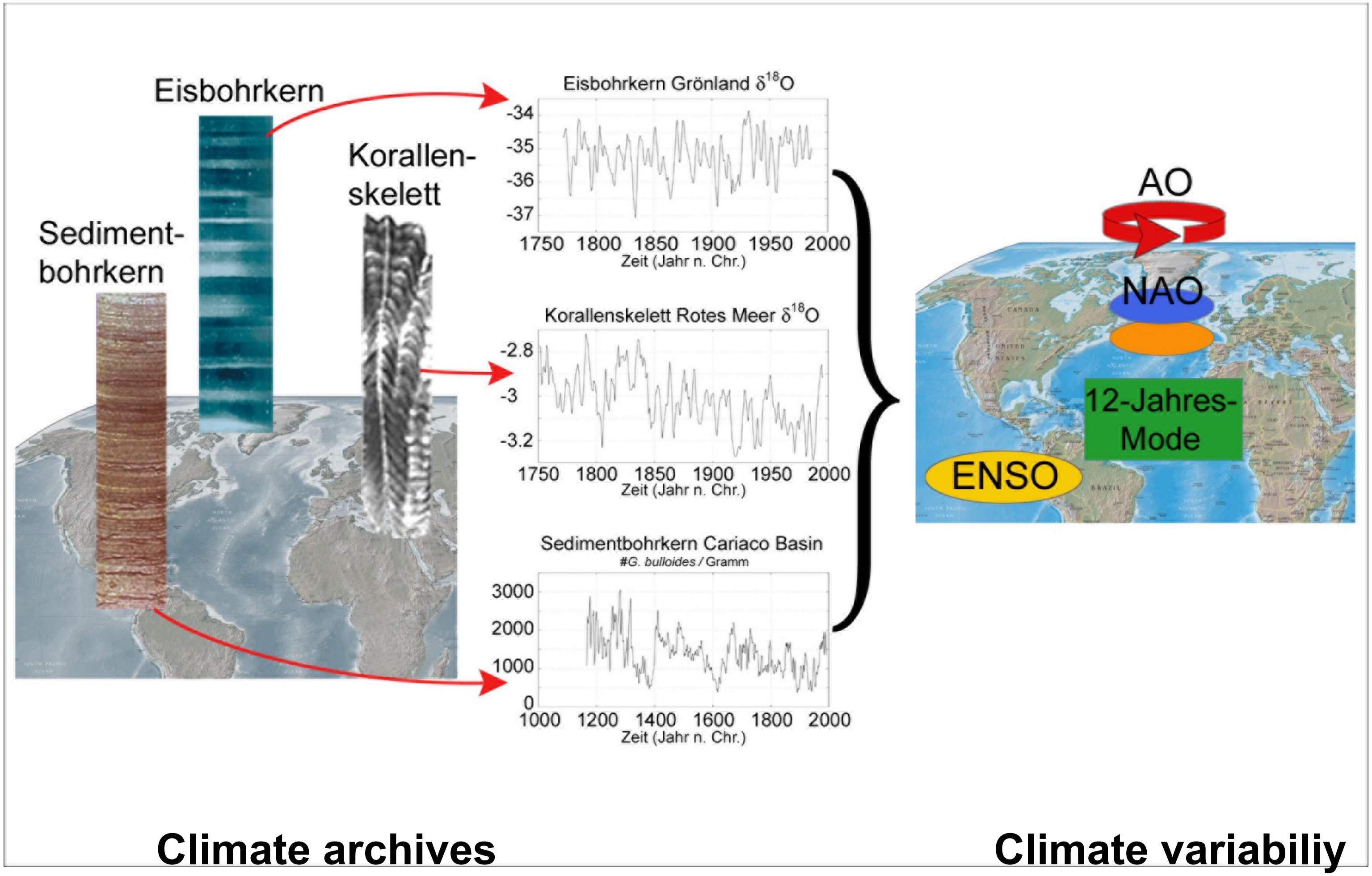
From **Ghil (2001, *EGEC*)**, after **Mitchell* (1976)**

*** “No known source of deterministic internal variability”**

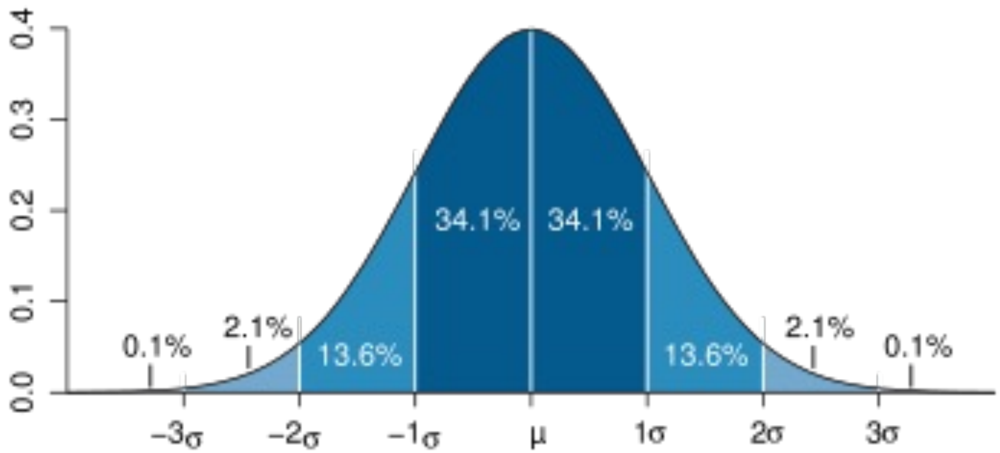
** **27 years – Brier (1968, *Rev. Geophys.*)**

Upscaling

Interpretation of Climate Data



Rules for normally distributed data



Dark blue is less than one standard deviation from the mean. For the normal distribution, this accounts for 68.27 % of the set; while two standard deviations from the mean (medium and dark blue) account for 95.45 %; and three standard deviations (light, medium, and dark blue) account for 99.73 %.

Spatial pattern

- Regression
- Correlation
- Composite maps

Statistic

Covariance (cross, auto)

$$\gamma(\Delta) = E\Big((x(t) - \bar{x})\,(y(t+\Delta) - \bar{y}) \Big)$$

e.g. coral e.g. meteorol. data

Correlation (cross, auto)

$$\rho_{xy} = \frac{\gamma(\Delta)}{\text{normalized}}$$

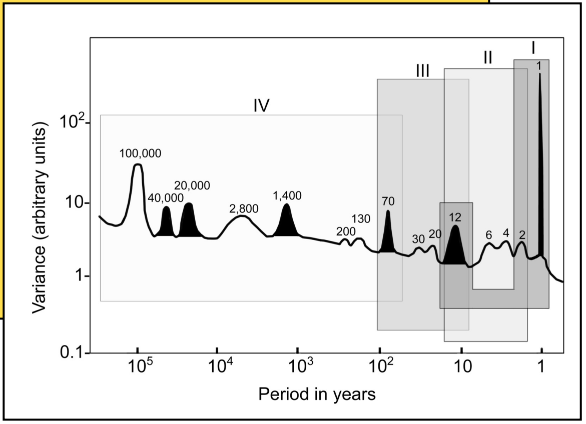
measures the tendency of x (t) and y (t) to covary

Spectrum (cross, auto)

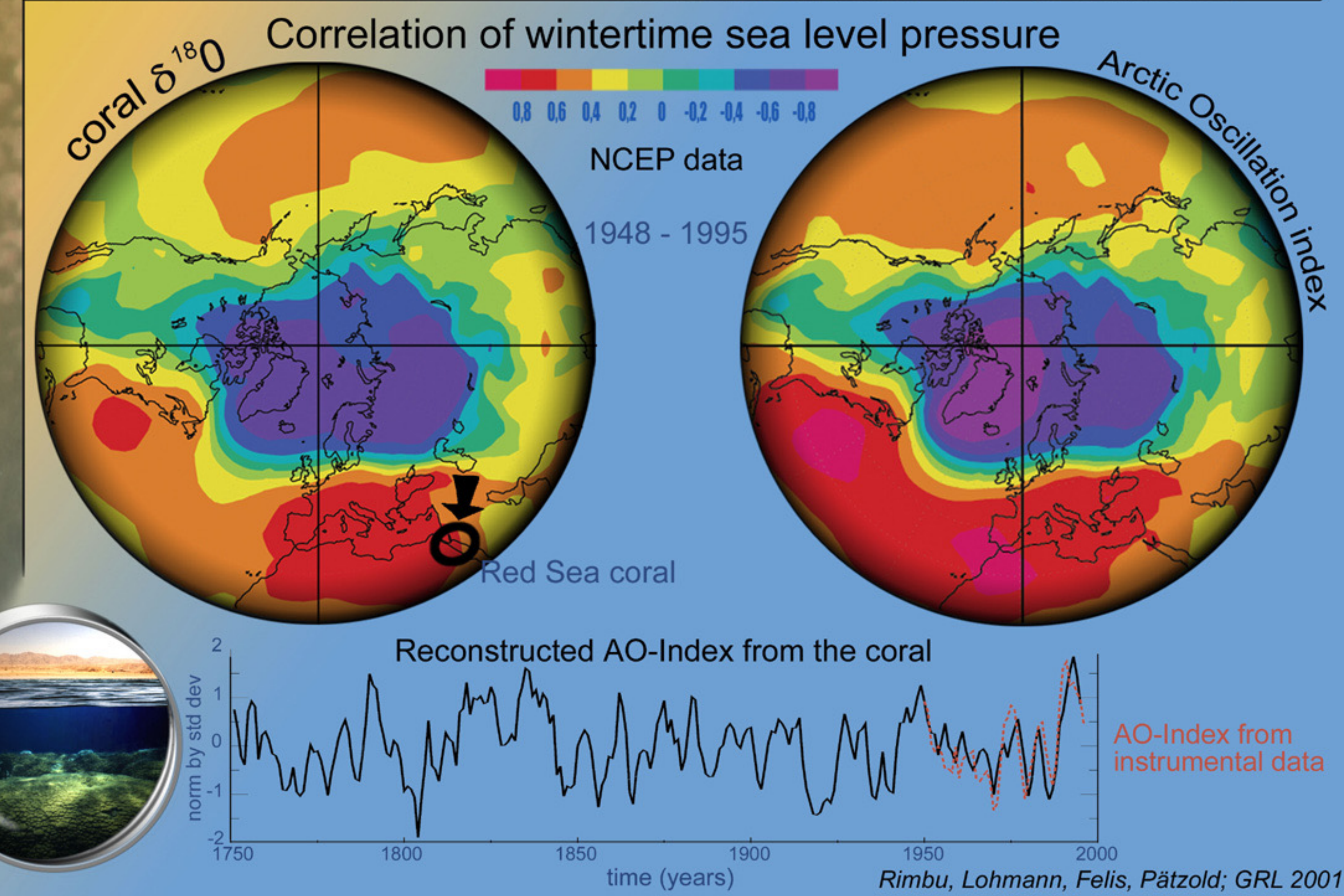
(spectral density)

$$\Gamma(\omega) = \sum_{\Delta=-\infty}^{\infty} \gamma(\Delta)\, e^{-2\pi i \Delta}$$

measures variance



ARCTIC OSCILLATION SIGNATURE IN A RED SEA CORAL



Example

```
#Correlation and significance

a<-1:1000
noise<-rnorm(1000)*500
b<-a + noise

plot(a,b)
cor(a,b)   #correlate a,b
cor.test(a,b)  #correlation + significance test
```

Statistical significance

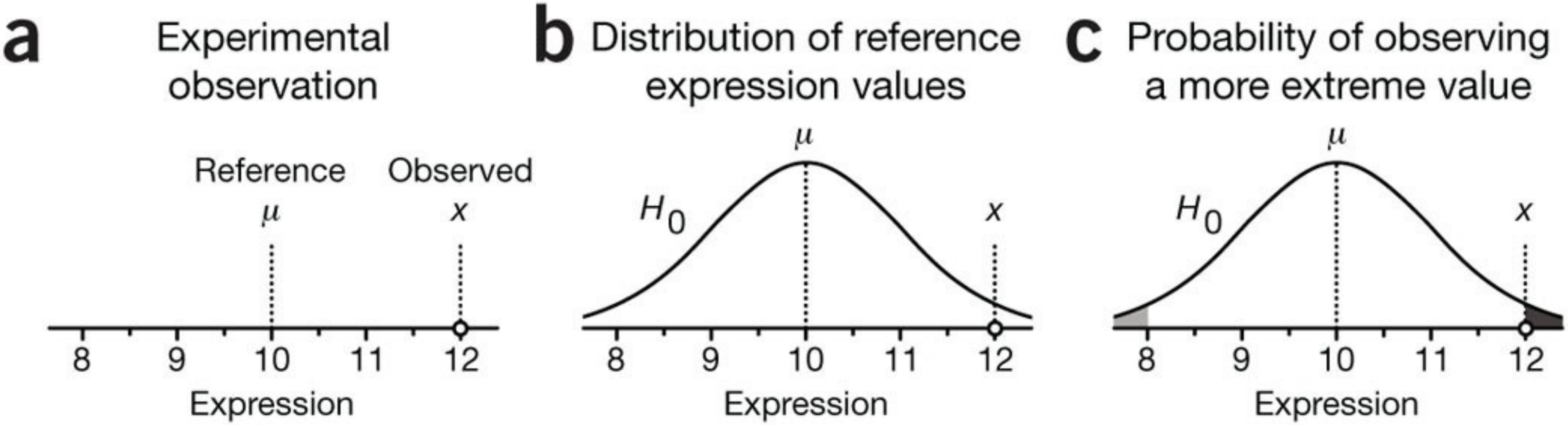
A result is called **significant** if it is unlikely to have occurred by chance.

"A statistically significant difference" means there is statistical evidence that there is a difference

it does not mean the difference is necessarily large

Student (1908): *The Probable Error of a Mean.* In: *Biometrika.* William Sealy Gosset

Significance of the difference between observed (x) and reference (μ) values



calculated by assuming that observations are sampled from a distribution H_0 with mean μ.

The statistical significance of the observation x is the probability of sampling a value from the distribution that is at least as far from the reference, given by the shaded areas under the distribution curve (**c**). This is the P value.

T-test

Null hypothesis: means of two normally distributed populations are equal. Given data sets: mean, standard deviation and number of data points, determine whether the means are distinct, provided that the underlying distributions can be assumed to be normal.

t value is above the threshold chosen for statistical significance (usually the 0.05), then the null hypothesis that the two groups do not differ is rejected in favor of an alternative hypothesis, which typically states that the groups do differ.

T-test

R-example
> mean(a)
[1] 500.5
> mean(b)
[1] 492.4706

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}} \text{ where } s_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_1^2 + s_2^2}{n}}$$

ratio of the difference and the variation within the sample sets

- A large t-score indicates that the groups are different.
- A small t-score indicates that the groups are similar.

t = 19.0316

compared against a value obtained from a critical value table (T-Distribution Table).

T-test

Significance is usually represented by the Greek symbol, α (alpha). Popular levels of significance are 5%, 1% and 0.1%. If a **test of significance** gives a p-value lower than the α-level, the null hypothesis is rejected.

If the *t* value is above the threshold chosen for statistical significance (usually the 0.05 level), then the null hypothesis that the two groups do not differ is rejected in favor of an alternative hypothesis, which typically states that the groups do differ.

R-example
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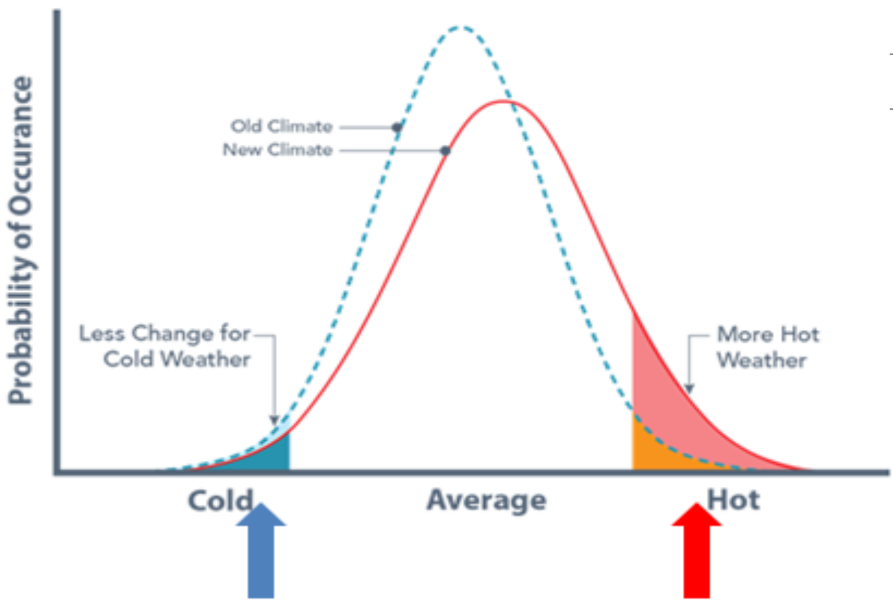
t = 19.0316

df = 998, p-value < 2.2e-16

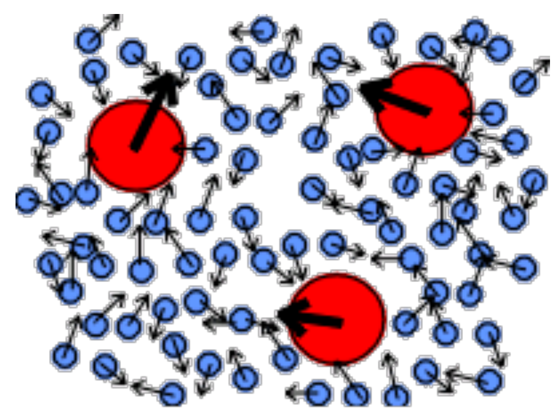
Climate & Weather

Probabilities



Brownian Particles: Climate

Blue Molecules: Weather



Climate variability

Lorenz (*JAS*, 1963)
Climate is deterministic and autonomous,
but highly nonlinear.Trajectories diverge exponentially,
forward asymptotic PDF is multimodal.

Hasselmann (*Tellus*, 1976)"
Climate is stochastic and noise-driven, but linear.
Trajectories decay back to the mean,
forward asymptotic PDF is unimodal.

Externally driven climate variability
Deterministic and stochastic
Non-linear
Internal and external variability can interact