Test Exam Dynamics II (Summer term 2018)

Lecturer: Prof. Dr. G. Lohmann

Instructions before you start: *The perfect score for the exam is* 100 *points, although the sum of the problems is* 150*. Therefore, you can choose among the problems to solve.* 50 *points are necessary for the course. Keep in mind that each problem has a different number of points.*
1-11 are for the dynamics of the atmosphere-ocean,
10-18 are for tools, statistics, and stochastic climate model,
19 and 20 are for fluid mechanics.

You are allowed to use a calculator & pen. Collaboration or use of alternative sources of information is not allowed. Good luck!

1. **Questions about atmosphere-ocean dynamics** (*20 points*, **for each Q** *2 points*).

   Q1: Please clarify: On the Northern Hemisphere, particles tend to go to the right or left relative to the direction of motion due to the Coriolis force?

   Q2: a) How is the Coriolis parameter f defined ?
   b) How is the second Coriolis parameter f_1 be defined ?

   Q3: Make a sketch of the Foucault pendulum and explain the horizontal dynamics of the Foucault pendulum

$$\ddot{x} = 2\Omega \sin\varphi \dot{y} - \frac{g}{L} x \tag{1}$$
$$\ddot{y} = -2\Omega \sin\varphi \dot{x} - \frac{g}{L} y \tag{2}$$

   with the Coriolis, Gravity forces using the sketch !

   Q4: Explain the climate variability modes NAO and ENSO !

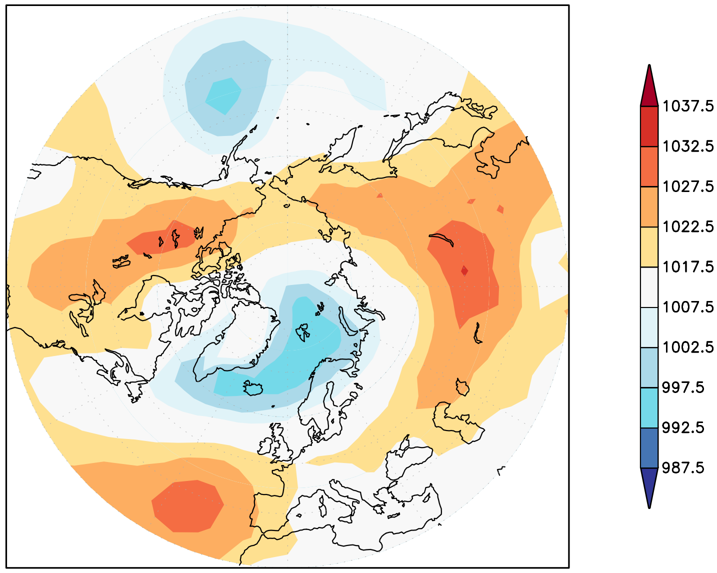

Figure 1: Sea level pressure (hPa) field for February 2015. In February, the circulation is characterized by a low pressure over the Greenland-Iceland-Norwegian Sea, and a surrounded high pressure. Data are from Trenberth and Paolino (1980).

Q5: What is the hydrostatic approximation in the momentum equations?

Q6: Explain the Taylor-Proudman Theorem! (remember $f = f_0$, barotropic circulation)

Q7: Please write down the barotropic potential vorticity equation for large-scale motion!

Q8: What are the two dominant terms in the horizontal momentum balance for the large-scale dynamics at mid-latitudes? Write down the geostrophic balance !

Q9: Draw the direction of large-scale motions in the atmosphere in Fig. 1 using the geostrophic balance.

Q10: Draw a schematic figure of the Atlantic Ocean meridional overturning! Include the directions (N,S), (E,W), depth in your sketch.

2. **Elimination of the pressure term** *(6 points)*

Assume a 2D flow without non-linear terms and friction, where the equations reduce to:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \tag{3}$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} \quad . \tag{4}$$

a) Eliminate the pressure in (3,4) .

b) Show: Defining the stream function ψ through

$$u = -\frac{\partial \psi}{\partial y} \quad ; \quad v = \frac{\partial \psi}{\partial x} \tag{5}$$

(mass continuity being unconditionally satisfied), the incompressible Newtonian 2D momentum and mass conservation degrade into one equation:

$$\partial_t \left(\nabla^2 \psi \right) = 0 \tag{6}$$

c) We now consider the rotating framework and add the Coriolis terms $-\rho f v$ and $\rho f u$ to the left hand side of (3,4). Subtract $\partial / \partial y$ (3) from $\partial / \partial x$ (4) to eliminate the pressure terms to derive the vorticity equation! Show that (6) changed into

$$\partial_t \left(\nabla^2 \psi \right) + \beta v = 0 \tag{7}$$

	Quantity	Atmosphere	Ocean
horizontal velocity	U	$10\ \ ms^{-1}$	$10^{-1}\,ms^{-1}$
horizontal length	L	$10^6\,m$	$10^6\,m$
vertical length	H	$10^4\,m$	$10^3\,m$
horizonal Pressure changes	δP (horizontal)	$10^3\,Pa$	$10^4\,Pa$
time scale	T	$10^5\,s$	$10^7\,s$
Coriolis parameter at 45°N	$f_0 = 2\Omega\sin\varphi_0$	$10^{-4}\,s^{-1}$	$10^{-4}\,s^{-1}$
density	ρ	$1\ \ kgm^{-3}$	$10^3\,kgm^{-3}$
viscosity (turbulent)	ν	$10^{-5}\,kgm^{-3}$	$10^{-6}\,kgm^{-3}$

Table 1: Table shows the typical scales in the atmosphere and ocean system.

3. **Scaling of the dynamical equations in the atmosphere and ocean** *(5 points)*

We work in the rotating frame of reference of the Earth. The equation can be scaled by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. We can estimate the relative contributions in units of m/s^2 in the horizontal momentum equations:

$$\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{U/T\sim 10^{-8}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{U^2/L\sim 10^{-8}} = \underbrace{-\frac{1}{\rho}\nabla p}_{\delta\mathrm{P}/(\rho\mathrm{L})\sim 10^{-5}} + \underbrace{2\Omega\times\mathbf{v}}_{f_0\mathrm{U}\sim 10^{-5}} + \underbrace{fric}_{\nu U/H^2\sim 10^{-13}} \tag{8}$$

where fric denotes the contributions of friction due to eddy stress divergence (usually $\sim \nu\nabla^2\mathbf{v}$). Typical values are given in Table 1. The values have been taken for the ocean.

a) Please repeat the estimate for the atmosphere using Table 1.

b) The Rossby number Ro is the ratio of inertial (the left hand side) to Coriolis (second term on the right hand side) in (8): terms

$$Ro = \frac{(U^2/L)}{(fU)} = \frac{U}{fL} \quad . \tag{9}$$

Ro is small when the flow is in a so-called geostrophic balance. Please calculate Ro for the atmosphere and ocean using Table 1.

4. **Wind-driven ocean circulation** *(8 points)*

the Sverdrup transport V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \tag{10}$$

where τ_x and τ_y are the components of the wind stress.

The Ekman transports V_E, U_E describe the dynamics in the upper mixed layer:

$$f V_E = -\tau_x/\rho_0 \qquad , \qquad f U_E = \tau_y/\rho_0 \tag{11}$$

where $U_E = \int_{-E}^{0} u dz$ and $V_E = \int_{-E}^{0} v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface. Denote w_E as the Ekman vertical velocity at the bottom of the Ekman layer. Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

$$-\int_{-E}^{0} \frac{\partial w}{\partial z} dz = w_E = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0 \, f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0 \, f}\right) \quad . \tag{12}$$

a) Assume that the windstress is only zonal with

$$\tau_x = -\tau_0 \cos(\pi y/B) \tag{13}$$

for an ocean basin $0 < x < L, \quad 0 < y < B$. Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case. Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

b) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}\,\mathrm{s}^{-1}$ and $\beta = 1.8 \cdot 10^{-11}\,\mathrm{m}^{-1}\mathrm{s}^{-1}$ and $B = 5000\,\mathrm{km}, \tau_0/\rho_0 = 10^{-4}\,\mathrm{m}^2\mathrm{s}^{-2}$.

c) Using the values in b), calculate the maximum of w_E for constant f.

5. **Rossby, gravity, and Kelvin waves** *(8 points)*

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \tag{14}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \tag{15}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{16}$$

with H=const. as mean depth and η as surface anomaly.

a) With the elimination of the fast gravity waves in equation (16)

$$\frac{\partial \eta}{\partial t} = 0$$

derive the dispersion relation for divergence-free Rossby waves! Ansatz: Introduce a stream-function for u,v: $\Psi \sim \exp(ikx + ily - i\omega t)$

b) With the assumption of $f = f_0 = 0$ derive the dispersion relation for gravity waves! The restoring force is related to gravity.

Ansatz: Start with the equation (16) and derive the solution.

c) Kelvin waves:

Derive the dispersion relation for Kelvin waves?

Why are Kelvin wave trapped?

6. **Rossby wave formula (long waves in the westerlies)** *(7 points)*

   Consider the vorticity equation

$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) = 0 \tag{17}$$

   a) Assume a mean flow with constant zonal velocity $u = U = const > 0$ and a varying north-south component $v = v(x,t)$ which gives the total motion a wave-like form. Furthermore, h =const. Write down the vorticity equation for this specific flow!

   b) Use a) and the ansatz

$$v(x,t) = A\cos[(kx - \omega t)] \tag{18}$$

   to determine the disperion relation $\omega(k)$, group velocity $\frac{\partial\omega}{\partial k}$, and the phase velocity $c = \omega/k$.

   c) Derive the wavelength $L = 2\pi/k$ of the stationary wave given by $c = 0$.

7. **Potential vorticity:** *(4 points)*

   An air column at $53°$N with $\zeta = 0$ initially streches from the surface to a fixed tropopause at 10 km height. If the air column moves until it is over a mountain barrier of 2 km height at $30°$N, what is its absolute vorticity and relative vorticity as it passes the mountain top?

   Assume: $\sin 53° = 0.8; \sin 30° = 0.5$

   The angular velocity of the Earth $\Omega = 2\pi/(1 \text{ day})$. Use

$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) = 0 \tag{19}$$

8. **Angular momentum and Hadley cell** *(10 points)*

Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0, \tag{20}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\,(u + \Omega r) = \Omega R^2 \cos^2\varphi + uR\cos\varphi \quad . \tag{21}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

a) Show, for inviscid zonally symmetric flow, that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = 0 \tag{22}$$

in (x, y, z) coordinates, where $y = R\varphi$. We assume that $-\frac{1}{\rho}\frac{\partial p}{\partial x} = 0$

b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

c) If the Hadley circulation is symmetric about the equator, and its edge is at $20°$ latitude, determine the strength of the subtropical jet. Use (20, 21).

9. **Analytical EBM** *(8 points)*

The temperature is described as $T(y)$ and the heat transport (sensible, latent and ocean) is modelled as diffusion:

$$C_p\, \partial_t T \;+\; k\, \partial_y^2 T \;=\; (1-\alpha) Q_S^{top} - (A + B\; T) \tag{23}$$

a) Show the solution if the planetary albedo α is chosen as a constant parameter. Use the ansatz with a global component and a latitude component

$$T(y,t) \;=\; T_0(t) + T_1(t) \cdot \cos\left(\frac{2\,y}{R}\right) \tag{24}$$

$$Q_S^{top} \;=\; Q_0 + Q_1 \cdot \cos\left(\frac{2\,y}{R}\right) \tag{25}$$

with $y = R\varphi$, R is the Earth radius, φ the latitude.

Separate the dynamics for T_0 and T_1. Use the orthogonal functions' theory or just

$$\int_{-90^\circ}^{90^\circ} \cos(2\varphi)\, d\varphi = 0 \tag{26}$$

b) Based on (23), one can introduce a climate-dependent formulation of the planetary albedo α on the global temperature:

$$\alpha(T) = \alpha_0 - \alpha_1 \cdot T_0 \tag{27}$$

Solve the Energy balance model for the case $\alpha(T_0)$ as in (27).

c) Show that the stability of the solution depends on $B - \alpha_1 Q_0$!

d) Explain the ice-albedo effect through this solution!

10. **Questions about advection** *(3 points)*

    I. A ship is steaming northward at a rate of 10 km/h. The surface pressure increases toward the northwest at a rate of 5 Pa/km. What is the pressure tendency recorded at a nearby island station if the pressure aboard the ship decreases at a rate of 100Pa/3h?

    II. The temperature at a point 50 km north of a station is 3°C cooler than at the station. If the wind is blowing from the northeast at 20m/s and the air is being heated by radiation at a rate of 1°C/h, what is the local temperature change at the station?

    III. The following data were received from 50 km to the east, north, west and south of a station, respectively: 90 degree, 10m/s; 120 degree,4m/s; 90degree,8m/s; 60 degree, 4m/s. Given are the angle and absolute value of the wind speed. Calculate the approximate horizontal divergence at the station.

11. **Estimates of overturning** *(4 points)*

    It is observed that water sinks in to the deep ocean in polar regions of the Atlantic basin at a rate of 15 $\boldsymbol{Sv}$. (Atlantic basin: $\mathbf{80,000,000}\,\boldsymbol{km^2 area} \times \mathbf{4}\,\boldsymbol{km}$ depth.)

    I. How long would it take to 'fill up' the Atlantic basin?

    II. Supposing that the local sinking is balanced by large-scale upwelling, estimate the strength of this upwelling. Hint: Upwelling $= \boldsymbol{area} \times \boldsymbol{w}$. Express your answer in $\boldsymbol{m\ y^{-1}}$.

    III. Compare this number with that of the Ekman pumping ! The order of magnitude of the Ekman vertical velocity $\boldsymbol{w_E}$ can be estimated as from a typical wind stress variation of $\mathbf{0.2}\boldsymbol{Nm^{-2}}$ per $\mathbf{2000}$ km in y-direction:

$$\boldsymbol{w_E} \simeq -\frac{\boldsymbol{\Delta\tau_x}}{\boldsymbol{\rho\ f_0 \Delta y}} \tag{28}$$

12. **Questions about tools and statistics** (*16 points*, **for each Q** *2 points*).

Q11: Calculate ∇f, and the divergence of ∇f for the function

$$f(x, y, z) = x^5 + 3x - 4xz + z^4 + \cos(3y) \tag{29}$$

Q12: a) Please write down the Euler forward numerical scheme for $\frac{d}{dt}x = f(x)$!

b) Consider also the special case $f(x) = rx^2 - x^3$.

Q13: What is the necessary condition for stability in a linear system $\frac{d}{dt}x = Ax$ with real vector x and $n \times n$ matrix A?

Q14: Consider Q13 for the case of a non-linear system $\frac{d}{dt}x = f(x) = ax - bx^3$.

Q15: a) What is the Fourier transform of a function $x(t)$?

b) What is the Fourier transform of the $\delta(t)$-function?

Q16: a) What is the definition of auto-correlation and auto-covariance?

b) How is the Fourier transformation of the auto-covariance called?

Q17: The Laplace transform is given by

$$\mathcal{L}\left\{x(t)\right\} = L(s) = \int_0^\infty e^{-st}x(t)dt \tag{30}$$

Show that integration by parts leads to

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sL(s) - x(0) \tag{31}$$

Q18: Show that

$$\mathcal{L}\left\{\exp(-at)\right\} = \frac{1}{s+a} \tag{32}$$

13. **Bifurcations** *(8 points)*

Consider the dynamical system

$$\frac{dx}{dt} = b + x^2 \tag{33}$$

a) Analyze the stability/instability of the equilibria through linearization. The control parameter is b.

b) Explain the graphical method to obtain stability or instability (Fig. 2) !

like ... filled circles with positive slope are unstable ...

c) Calculate the potential and show the stability with a diagram for $b = -1$!

d) Draw the bifurcations as in Fig. 2 for $\frac{dx}{dt} = bx(1-x)$

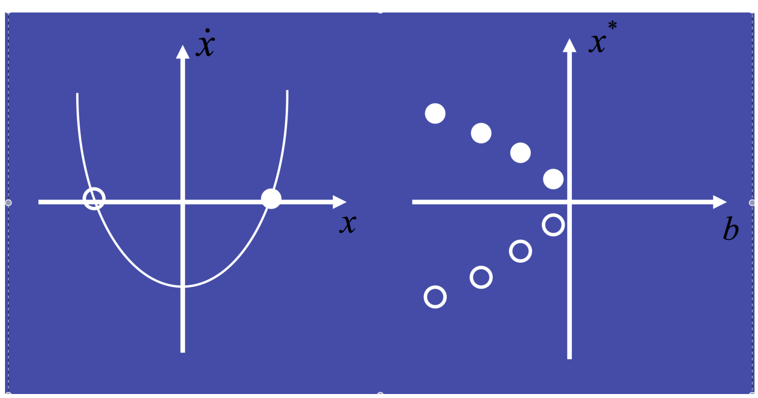

Figure 2: Saddle-node bifurcation diagram using the graphical method.

14. **Stochastic climate model** *(6 points)*

Imagine that the temperature of the ocean mixed layer of depth h is governed by

$$\frac{dT}{dt} = -\lambda T + \frac{Q_{net}}{\gamma_O}\,, \tag{34}$$

where coefficient γ_O is given by the heat capacity $c_p \rho h$, and λ is the typical damping rate of a temperature anomaly. The air-sea fluxes due to weather systems are represented by a white-noise process $Q_{net} = \hat{Q}_\omega e^{i\omega t}$ where $\hat{Q}_\omega$ is the amplitude of the random forcing at frequency ω. $\hat{Q}^*$ is the complex conjugate.

a) Solve Eq. 34 for the temperature response $T = \hat{T}_\omega e^{i\omega t}$ and hence show that:

$$\hat{T}_\omega = \frac{\hat{Q}_\omega}{\gamma_O\,(\lambda + i\omega)} \tag{35}$$

b) Show that it has a spectral density $\hat{T}_\omega\,\hat{T}^*_\omega$ is given by:

$$\hat{T}\,\hat{T}^* = \frac{\hat{Q}\,\hat{Q}^*}{\gamma_O^2\,(\lambda^2 + \omega^2)} \tag{36}$$

and the spectrum

$$S(\omega) =< \hat{T}\,\hat{T}^* >= \frac{1}{\gamma_O^2\,(\lambda^2 + \omega^2)}. \tag{37}$$

The brackets $< \cdots >$ denote the ensemble mean. $< \hat{Q}\,\hat{Q}^* >= 1$

c) Make a sketch of the spectrum using a log-log plot and show that fluctuations with a frequency greater than λ are damped.

15. **Short programming questions.** *(6 points)*

    Write down the output for the following R-commands:

    a) `a<-c(0,-5,3,24); mean(a)`

    b) `max(a)-min(a)`

    c) `paste("The mean value of a is",mean(a),"for sure",sep="_")`

    d) `a*2+c(1,1,-1,0)`

    e) `my.fun<-function(n){return(n*n+1)}`

       `my.fun(10)-my.fun(1)`

    f) Plot the potential (related to 11d)

```
y=-100:100
x=y/50
r=1
z=-r * x^2/2 + r  * x^3/3
plot(x,z,type='l')
```

CDO-related part of the exam: Assume that you have been asked to do some research on climate model output that has been provided to you in a file named 'output.nc'. This file contains various climate-related quantities, including variables 'siced', 'snacl', 'u', 'precip', and 'tsurf'. We assume that all data present in the file has global distribution (0-360 E, -90-90 N), the time resolution is one time step per month, and each time step represents a monthly mean. For each of your answers, give the full CDO command that you would employ when performing this task at your computer. Apply the correct CDO syntax. Since there is more than one variable present in the file, for some of the problems it may be necessary to select a specific variable in your CDO command to derive the correct result. Single quotes given in the problems highlight the exact variable name or file name and should be ommitted in your commands. Allowed supporting material to be used by you:

```
#derive information on the time axis of a file
cdo showdate input.nc

#derive information on the spatial organisation of data (i.e. on the grid) of a file
cdo griddes input.nc

#derive information on the vertical organisation of data (i.e. the levels) of a file
cdo showlevel input.nc

#derive information on the climate data (variables) in a file
cdo pardes input.nc

#extract a variable named "varname" from file input.nc
cdo selvar,varname input.nc output.nc

#extract the first month of all years in file input.nc
cdo selmon,1 input.nc output.nc

#calculate a time average over a time series input.nc
cdo timmean input.nc output.nc

#generate a seasonal mean from input.nc
cdo seasmean input.nc output.nc

#generate an annual mean from input.nc
cdo yearmean input.nc output.nc

#calculate an average annual cycle from file input.nc
cdo ymonmean input.nc output.nc

#select a region from input.nc, spreading from longitude "a" to "b", and from latitude "c" to "d"
cdo sellonlatbox,a,b,c,d input.nc output.nc

#calculate a spatial average of field input.nc
cdo fldmean input.nc output.nc

#write the output of a cdo operator "a" to the screen (omits creation of an output file)
cdo output -a input.nc

#calculate the difference between two NetCDF files input1.nc and input2.nc
cdo sub input1.nc input2.nc output.nc

#multiply two fields input1.nc and input2.nc
cdo mul input1.nc input2.nc output.nc

#add a skalar constant value "a" to field input.nc
cdo addc,a input.nc output.nc

#select only regions of input2.nc, for which mask input1.nc is true (i.e. 1), represents an if-then programming
cdo ifthen input1.nc input2.nc output.nc

#use input2.nc, where mask input1.nc is true - otherwise use input3.nc, represents an if-then-else programming
cdo ifthenelse input1.nc input2.nc input3.nc output.nc

#reduce a data range (a,b) in input.nc to the constant value "c"
cdo setrtoc,a,b,c input.nc output.nc

#replace a data range  (a,b) in input.nc by the missing value ("NaN")
cdo setrtomiss,a,b input.nc output.nc

#calculate trend of time series input.nc; offset "a" and slope "b" of the regression line; stored in a.nc, b.nc
cdo trend input.nc a.nc b.nc

#calculate the horizontal area covered by each grid cell of input.nc, store the result in file output.nc
cdo gridarea input.nc output.nc

#interpolate a spatial data input.nc using nearest neighbor to a specific geolocation of longitude X and latitude Y,
    store the result in file output.nc (X and Y being geolocations in degrees)
cdo remapnn,lon=X/lat=Y input.nc output.nc
```

16. **cdo question 1** *(6 points)*

    Derive information on the climate model output in file ’output.nc’ using the climate data operators (CDO). Explain how you would identify the following listed items of information:
    a) the physical quantity stored in the variable ’snacl’
    b) the various vertical levels present for variable ’u’ (hint: the result of your command should be a list of all vertical levels on which ’u’-data is available, so do not use an operator that provides only the number of levels)
    c) information on the spatial resolution (number of grid cells in lon/lat-direction, longitude and latitude values, ...)

17. **cdo question 2** *(6 points)*

    Compute spatial and temporal averages of climate model output in file ’output.nc’ using the climate data operators (CDO). Explain how you would compute the following characteristics of climate data:
    a) the average of variable ’precip’ in the Northern Hemisphere (longitudes 0-360E, latitudes 0-90N)
    b) the global average and time average of variable ’tsurf’
    c) the time average and spatial average in the Northern Hemisphere (longitudes 0-360E, latitudes 0-90N) of variable ’tsurf’

18. **cdo question 3** *(6 points)*

    Devise CDO commands that compute characteristics of climate model output in file ’output.nc’ in order to answer some scientific questions. For each of the problems, note down how you would further analyse the output of the CDO command devised by you in order to answer the question. The problems to solve are:
    a) How much colder is the month January than the month July in the Northern Hemisphere (longitudes 0-360E, latitudes 0-90N) based on the data in variable ’temp2’?
    b) What is the monthly average temperature at the location of Bremen (geographical coordi-

nates 53.0793 N, 8.8017 E) based on the data in variable 'temp2'? Note: In case you need to employ remapping to interpolate the global data set to the geolocation of Bremen, apply nearest neighbour remapping (CDO operator remapnn). Further note that the result of your CDO command will contain multiple time steps!

c) Which month is coldest at the location of Bremen (geographical coordinates 53.0793 N, 8.8017 E) based on the data in variable 'tsurf'? Note: In case you need to employ remapping to interpolate the global data set to the geolocation of Bremen, apply nearest neighbour remapping (CDO operator remapnn).

19. **Questions about fluid mechanics** (*6 points*, **for each Q** *2 points*).

Q18: Name three different dimensionless parameters which can characterize the flow.

Q19: a) Please state: The dimensionless Reynolds number is $\boldsymbol{Re = U/(L\nu)}$ or $\boldsymbol{Re = UL/\nu}$ or $\boldsymbol{Re = U^2L/\nu}$? $\boldsymbol{\nu}$ denotes the kinematic viscosity, L a length-scale L determined by the geometry of the flow, and U a characteristic velocity.

b) In which context does the Reynolds number play a role?

Q20: Describe in words the Rayleigh-Bénard instability. The basic state possesses a steady-state solution in which there is no motion, and the temperature varies linearly with depth:

$$\boldsymbol{u = w = 0} \tag{38}$$

$$\boldsymbol{T_{eq} = T_0 + \left(1 - \frac{z}{H}\right) \Delta T} \tag{39}$$

When this solution becomes unstable, ... (please continue)

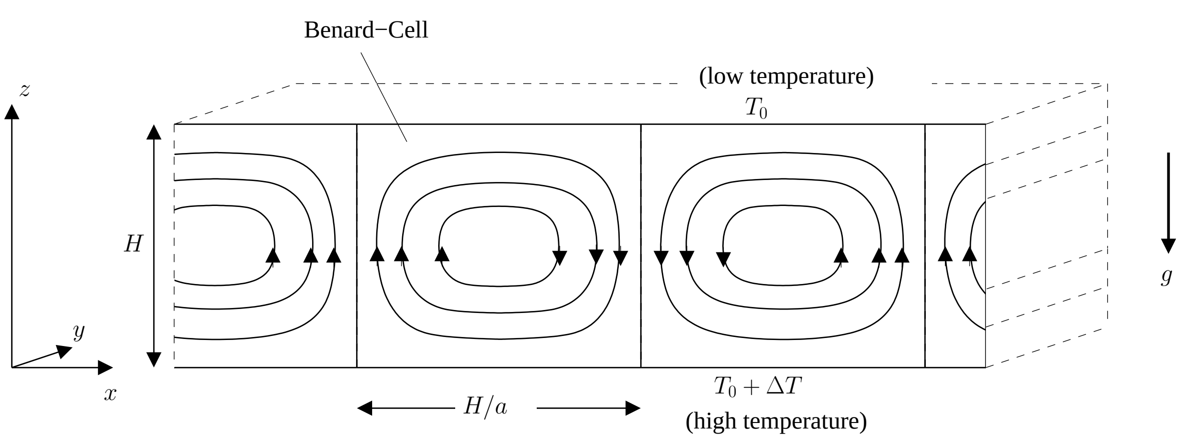

Figure 3: Geometry of the Rayleigh-Bénard system.

20. **Concept of dynamic similarity** *(7 points)*

For the case of an incompressible flow, assuming the temperature effects are negligible and external forces are neglected, the Navier-Stokes equations consist of conservation of mass

$$\nabla \cdot \mathbf{u} = 0 \tag{40}$$

and conservation of momentum

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \mathbf{u} \tag{41}$$

where $\mathbf{u}$ is the velocity vector and p is the pressure, ν denotes the kinematic viscosity.

a) Show: The equations (40,41) can be made dimensionless by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. For example: $u = U \cdot u_d$.

Note: the units of $[\rho_0] = kg/m^3$, $[p] = kg/(ms^2)$, and $[p]/[\rho_0] = m^2/s^2$. Therefore the pressure gradient term in (41) has the scaling U^2/L.

b) Show: The scalings vanish completely in front of the terms except for the $\nabla^2 \mathbf{u_d}$-term! The dimensionless parameter is the Reynolds number and the only parameter left!

Remark: For large Reynolds numbers, the flow is turbulent. In most practical flows Re *is rather large* $(10^4 - 10^8)$, *large enough for the flow to be turbulent.*

Solution for

17.

a) enter the command: **cdo** output -fldmean -sellonlatbox,0,360,0,90 -selvar,precip output.nc
the hemispheric average will be written out for each of the 12 time steps

b) enter the command: **cdo** output -fldmean -timmean -selvar,tsurf output.nc
the global and time average will be written out analog to the following output:
287.583

c) enter the command: **cdo** output -timmean -fldmean -sellonlatbox,0,360,0,90 -selvar,tsurf output.nc
the hemispheric time average will be written out analog to the following output:
288.405

18.

a) enter the command: **cdo** output -fldmean -sellonlatbox,0,360,0,90 -selvar,temp2 output.nc
a monthly climatology will be written out from which the temperature difference between January and July can be computed by hand, e.g.:

```
280.697
281.270
283.700
287.003
290.257
292.723
294.116
293.857
291.883
288.688
285.066
282.221
```

=> January is colder than July by: 294.116 K - 280.697 K = 13.419 K

---

an even better answer would be (such answers could be awarded extra points if this is feasible):
enter the command: **cdo** output -sub -selmon,7 -fldmean -sellonlatbox,0,360,0,90 -selvar,temp2 output.nc \
-selmon,1 -fldmean -sellonlatbox,0,360,0,90 -selvar,temp2 output.nc
in this case the difference is directly computed by CDO, the result 13.4187 K is slightly different due to higher internal precision of CDO

b) enter the command: **cdo** output -remapnn,lon=8.8017/lat=53.0793 -selvar,temp2 output.nc
the monthly average temperature temp2, interpolated to the geolocation of Bremen, will be written out analog to:

```
271.787
272.398
275.225
279.338
283.944
286.991
288.495
287.624
284.712
280.792
276.465
273.783
```

c) enter the command: **cdo** output -remapnn,lon=8.8017/lat=53.0793 -selvar,tsurf output.nc
the monthly climatology of temperature tsurf, interpolated to the geolocation of Bremen, will be written out analog to:

```
271.689
272.348
275.277
279.499
284.253
287.271
288.671
287.760
284.761
280.768
276.365
273.663
```

from this output it can be inferred that the first month (January) is the coldest, with a temperature of 271.689 K