Lecturer: Prof. Dr. G. Lohmann/Dr. M. Ionita Tutors: Smit Doshi, Qiyun Ma Due date: 9.5.2022 2.5.2022

1. Scaling of the dynamical equations (2 points)

We work in the rotating frame of reference of the Earth. The equation can be scaled by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. We can estimate the relative contributions in units of m/s^2 in the horizontal momentum equations:

$$\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{U/T \sim 10^{-8}} + \underbrace{\mathbf{v} \cdot \nabla \mathbf{v}}_{U^2/L \sim 10^{-8}} = \underbrace{-\frac{1}{\rho} \nabla p}_{\delta \mathbf{P}/(\rho \mathbf{L}) \sim \mathbf{10^{-5}}} + \underbrace{2\Omega \times \mathbf{v}}_{\mathbf{f_0} \mathbf{U} \sim \mathbf{10^{-5}}} + \underbrace{fric}_{\nu U/H^2 \sim 10^{-13}} \tag{1}$$

where fric denotes the contributions of friction due to eddy stress divergence (usually $\sim \nu \nabla^2 \mathbf{v}$). Typical values are given in Table 1. The values have been taken for the ocean.

- a) Please repeat the estimate for the atmosphere using Table 1.
- b) The Rossby number Ro is the ratio of inertial (the left hand side in (1)) to Coriolis (second term on the right hand side in (1)) terms

$$Ro = \frac{(U^2/L)}{(fU)} = \frac{U}{fL} \quad . \tag{2}$$

Ro is small when the flow is in a so-called geostrophic balance. Please calculate Ro for the atmosphere and ocean using Table 1.

	Quantity	Atmosphere	Ocean
horizontal velocity	U	$10 \ ms^{-1}$	$10^{-1} ms^{-1}$
horizontal length	L	$10^{6} m$	$10^6 m$
horizonal Pressure changes	δP (horizontal)	$10^{3} Pa$	$10^4 Pa$
time scale	T	$10^{5} s$	$10^{7} s$
Coriolis parameter at 45°N	$f_0 = 2\Omega\sin\varphi_0$	$10^{-4} s^{-1}$	$10^{-4} s^{-1}$
density	ρ	$1 kgm^{-3}$	$\begin{array}{ c c c c }\hline 10^3 kgm^{-3} \\ 10^{-6} kgm^{-3} \\ \end{array}$
viscosity (turbulent)	ν	$10^{-5} kgm^{-3}$	$10^{-6} kgm^{-3}$

Table 1: Table shows the typical scales in the atmosphere and ocean system.

Dynamics 2
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2. Concept of dynamic similarity (3 points)

For the case of an incompressible flow, assuming the temperature effects are negligible and external forces are neglected, the Navier-Stokes equations consist of conservation of mass

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

Exercise 1, Summer semester 2022

and conservation of momentum

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{u}$$
 (4)

where **u** is the velocity vector and p is the pressure, ν denotes the kinematic viscosity.

a) Show: The equations (3,4) can be made dimensionless by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. For example: $u = U \cdot u_d$.

Note: the units of $[\rho_0] = kg/m^3$, $[p] = kg/(ms^2)$, and $[p]/[\rho_0] = m^2/s^2$. Therefore the pressure gradient term in (4) has the scaling U^2/L .

b) Show: The scalings vanish completely in front of the terms except for the $\nabla^2 \mathbf{u_d}$ -term! The dimensionless parameter is the Reynolds number and the only parameter left!

Remark: For large Reynolds numbers, the flow is turbulent. In most practical flows Re is rather large $(10^4 - 10^8)$, large enough for the flow to be turbulent.

3. Advection (3 points)

A ship is steaming northward at a rate of 10 km/h. The surface pressure increases toward the northwest at a rate of 5 Pa/km. What is the pressure tendency recorded at a nearby island station if the pressure aboard the ship decreases at a rate of 100Pa/3h?

4. **Download and install the R version** for your operating system (for many linux distributions R is also available in the package management system). Furthermore, look at the web page for R studio http://www.rstudio.com/, R studio is a free and open source user interface for R. One particular package is Shiny. This makes it super simple for R users like you to turn analyses into interactive web applications that anyone can use. The latest version of R for Linux, OS X and Windows is freely available on the CRAN webpage: http://cran.r-project.org (Fig. 1).

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Figure 1: R is available for download from the CRAN webpage: http://cran.r-project.org.

5. Short programming questions. (2 points)

Write down the output for the following R-commands:

a) a<-c(0,-5,4,20); mean(a)
b) max(a)-min(a)
c) a*2+c(3,1,-1,0)
f) Plot the potential
 y=-100:100
 x=y/50
 r=1
 z=-r * x^2/2 + r * x^3/3
 plot(x,z,type='l')
 and the derivative of z(x)</pre>

Notes on submission form of the exercises: Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Smit Doshi (Smit.Doshi@awi.de), Dr. Qiyun Ma (Qiyun.Ma@awi.de).