Dynamics II, Summer semester 2025 | Exercise 1
Lecturer: Prof. Dr. G. Lohmann, Dr. M. Ionita | 7.4.2024
Tutors: Alessandro Gagliardi, Georg Hüttner | Due date: 14.4.2024

1. **Scaling of the dynamical equations** *(2 points)*

We work in the rotating frame of reference of the Earth. The equation can be scaled by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. We can estimate the relative contributions in units of m/s^2 in the horizontal momentum equations:

$$\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{U/T\sim 10^{-8}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{U^2/L\sim 10^{-8}} = \underbrace{-\frac{1}{\rho}\nabla p}_{\boldsymbol{\delta P/(\rho L)\sim 10^{-5}}} + \underbrace{2\boldsymbol{\Omega}\times\mathbf{v}}_{\mathbf{f_0 U\sim 10^{-5}}} + \underbrace{fric}_{\nu U/H^2\sim 10^{-13}} \tag{1}$$

where fric denotes the contributions of friction due to eddy stress divergence (usually $\sim \nu\nabla^2\mathbf{v}$). Typical values are given in Table 1. The values have been taken for the ocean.

a) Please repeat the estimate for the atmosphere using Table 1.

b) The Rossby number Ro is the ratio of inertial (the left hand side in (1)) to Coriolis (second term on the right hand side in (1)) terms

$$Ro = \frac{(U^2/L)}{(fU)} = \frac{U}{fL} \quad . \tag{2}$$

Ro is small when the flow is in a so-called geostrophic balance. Please calculate Ro for the atmosphere and ocean using Table 1.

	Quantity	Atmosphere	Ocean
horizontal velocity	U	$10\ ms^{-1}$	$10^{-1}\, ms^{-1}$
horizontal length	L	$10^6\, m$	$10^6\, m$
horizonal Pressure changes	δP (horizontal)	$10^3\, Pa$	$10^4\, Pa$
time scale	T	$10^5\, s$	$10^7\, s$
Coriolis parameter at 45°N	$f_0 = 2\Omega\sin\varphi_0$	$10^{-4}\, s^{-1}$	$10^{-4}\, s^{-1}$
density	ρ	$1\ kgm^{-3}$	$10^3\, kgm^{-3}$
viscosity (turbulent)	ν	$10^{-5}\, kgm^{-3}$	$10^{-6}\, kgm^{-3}$

Table 1: Table shows the typical scales in the atmosphere and ocean system.

2. **Advection** *(3 points)*

   A ship is steaming northward at a rate of 10 km/h. The surface pressure increases toward the northwest at a rate of 5 Pa/km. What is the pressure tendency recorded at a nearby island station if the pressure aboard the ship decreases at a rate of 100Pa/3h?

3. **Questions about atmosphere-ocean dynamics** (*5 points*, **for each** *1 point*)**.**

   a) Please clarify: On the Northern Hemisphere, particles tend to go to the right or left relative to the direction of motion due to the Coriolis force?

   b) How is the Coriolis parameter f defined ?

   c) What is the hydrostatic approximation in the momentum equations?

   d) What are the two dominant terms in the horizontal momentum balance for the large-scale dynamics at mid-latitudes for the atmosphere and ocean flow ?

   e) Draw the direction of large-scale motions in the atmosphere in Fig. 1 using the geostrophic balance.

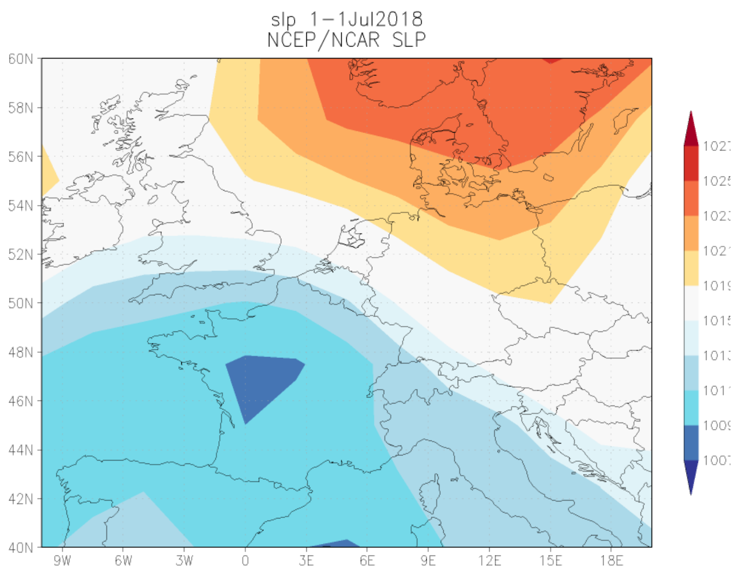

Figure 1: Sea level pressure (hPa) field for July 1, 2018. Source: NCEP/NCAR reanalysis.

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Alessandro Gagliardi (Alessandro.Gagliardi@awi.de), Georg Huettner (Georg.Huettner@awi.de).*