

1. **Wind-driven ocean circulation** (5 points)

the Sverdrup transport V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x} \tau_y - \frac{\partial}{\partial y} \tau_x \quad (1)$$

where τ_x and τ_y are the components of the wind stress.

The Ekman transports V_E, U_E describe the dynamics in the upper mixed layer:

$$fV_E = -\tau_x/\rho_0 \quad , \quad fU_E = \tau_y/\rho_0 \quad (2)$$

where $U_E = \int_{-E}^0 u dz$ and $V_E = \int_{-E}^0 v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface. Denote w_E as the Ekman vertical velocity at the bottom of the Ekman layer. Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

$$-\int_{-E}^0 \frac{\partial w}{\partial z} dz = w_E = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x} \left(\frac{\tau_y}{\rho_0 f} \right) - \frac{\partial}{\partial y} \left(\frac{\tau_x}{\rho_0 f} \right) \quad (3)$$

a) Assume that the windstress is only zonal with

$$\tau_x = -\tau_0 \cos(\pi y/B) \quad (4)$$

for an ocean basin $0 < x < L$, $0 < y < B$. Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case. Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

b) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4} \text{ s}^{-1}$ and $\beta = 1.8 \cdot 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ and $B = 5000 \text{ km}$, $\tau_0/\rho_0 = 10^{-4} \text{ m}^2 \text{ s}^{-2}$.

c) Using the values in b), calculate the maximum of w_E for constant f .

	Quantity	Ocean
horizontal velocity	U	$1.6 \cdot 10^{-2} m s^{-1}$
horizontal length	L	$10^6 m$
vertical length	E	$10^2 m$
wind stress	τ_0	$1.5 \cdot 10^{-1} Pa$
Coriolis parameter at 45°N	$f_0 = 2\Omega \sin \varphi_0$	$10^{-4} s^{-1}$
density	ρ_0	$10^3 kg m^{-3}$
viscosity (turbulent)	A_H	$10^2 - 10^4 m^2 s^{-1}$

Table 1: Table shows the typical scales in the ocean system for the exercise.

2. **Non-dimensional vorticity dynamics including wind stress** (4 points)

a) Show that (1) is a special case of the vorticity equation

$$\frac{D}{Dt} (\zeta + f) = A_H \nabla^2 \zeta + \frac{1}{\rho E} \left(\frac{\partial}{\partial x} \tau_y - \frac{\partial}{\partial y} \tau_x \right) . \quad (5)$$

b) Derive the the non-dimensional version of (5). Include the Reynolds number $Re = UL/A_H$, Rossby number $Ro = U/(f_0L)$, and the wind stress strength number $\alpha = \tau_0 L/(\rho_0 E U^2)$.

c) Estimate the order of magnitude of the characteristic numbers for the ocean ! Use Table 1.

3. **Questions about the course** (3 points)

a) Explain the Taylor-Proudman Theorem! (remember $f = f_0$, barotropic circulation) Why does the flow not go over the obstacle?

Laboratory experiments showing the formation of a Taylor column, go to 2:50,

b) Please write down the barotropic potential vorticity equation for large-scale motion!

c) What are the two dominant terms in the horizontal momentum balance for the large-scale dynamics at mid-latitudes?

d) What are the names of the 3 meridional cells in the atmosphere? Are these cells geostrophically driven or not?

Notes on submission form of the exercises: Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Smit Doshi (Smit.Doshi@awi.de), Dr. Qiyun Ma (Qiyun.Ma@awi.de).