1. **Lorenz equations:** *(5 points)*

   Consider the Lorenz equations which were derived from the Rayleigh-Bénard system in the lecture:

$$\dot{x} = \sigma(y - x) \tag{1}$$
$$\dot{y} = rx - xz - y \tag{2}$$
$$\dot{z} = xy - bz \tag{3}$$

   with $\sigma, r, b > 0$. σ is the Prandtl number. Furthermore, Rayleigh number $R_a \sim \Delta T$, critical Rayleigh number R_c, and $r = R_a/R_c$.

   (a) Evaluate the equilibrium points!
   (b) Determine the stability of the $(0, 0, 0)-$equilibrium through linearization! Control parameter is r.
   (c) Show the symmetry: The Lorenz equation has the following symmetry $(x, y, z) \rightarrow (-x, -y, z)$ independent on the parameters σ, r, b.

2. **Lorenz equations on the computer:** *(2 points)*

   Solve the Lorenz equations numerically using the parameters $\sigma, r, b = 10, 28, 8/3$. As initial conditions use $(x_0, y_0, z_0) = (1, 3, 5)$. Provide a short description.

3. **Ocean heat transport** *(4 points)*

   a) One can introduce a streamfunction

$$\Phi(y, z) : v = -\partial_z \Phi; w = \partial_y \Phi$$

   where v is the zonally integrated transport. One can formulate the volume transport with a simple ansatz satisfying that the normal velocity at the boundary vanishes

$$\Phi(y, z, t) = \Phi_{max}(t) \sin\left(\frac{\pi y}{L}\right) \times \sin\left(\frac{\pi z}{H}\right) \quad .$$

   Estimate the ocean heat transport

$$H = c_p \rho_0 \int_{bottom}^{top} vT dz \tag{4}$$
$$= -c_p \rho_0 \int_{bottom}^{top} \frac{\partial \Phi}{\partial z} \quad T dz \tag{5}$$
$$= c_p \rho_0 \int_{bottom}^{top} \Phi \quad \frac{\partial T}{\partial z} dz \tag{6}$$
$$\approx c_p \rho_0 \Phi_{max} \int_{T(bottom)}^{T(top)} \quad dT \tag{7}$$

Therefore, the heat transport can be estimated in terms of the mass transport in the temperature layers:

$$H \approx c_p \rho_0 \underbrace{(T(top) - T(bottom))}_{15K} \qquad \underbrace{\Phi_{max}}_{15 \cdot 10^6 m^3/s} \tag{8}$$

Which is about how many $PW (PW = 10^{15} W)$?
Compare this number with the atmospheric heat transport!

b) It is observed that water sinks in to the deep ocean in polar regions of the Atlantic basin at a rate of 15 Sv. (Atlantic basin: 80,000,000 km^2 area * 4 km depth.)

Q1: How long would it take to 'fill up' the Atlantic basin?

Q2: Supposing that the local sinking is balanced by large-scale upwelling, estimate the strength of this upwelling.

4. **Ocean thermohaline circulation** *(3 points)*

Consider a geostrophic flow (u, v)

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \tag{9}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \quad . \tag{10}$$

The meridional overturning stream function $\Phi(y, z)$ is defined via

$$\Phi(y, z) = \int_0^z \frac{\partial \Phi}{\partial \tilde{z}}\, d\tilde{z} \tag{11}$$

$$\frac{\partial \Phi}{\partial \tilde{z}} = \int_{x_e}^{x_w} v(x, y, \tilde{z})\, dx \quad \text{(zonally integrated transport)}, \tag{12}$$

where x_e and x_w are the eastward and westward boundaries in the ocean basin (think e.g. of the Atlantic Ocean). Units of Φ are $m^3 s^{-1}$. At the surface $\Phi(y, 0) = 0$.

Calculate $\Phi(y, z)$ as a function of density ρ at the basin boundaries!

Use

$$v(x, y, z) = \frac{1}{\rho_0 f}\frac{\partial p}{\partial x}$$

and the hydrostatic approximation

$$\frac{\partial p}{\partial z} = -g\rho$$

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00 pm) to Alessandro Gagliardi (Alessandro.Gagliardi@awi.de), Georg Huettner (Georg.Huettner@awi.de).*