1. **Analytical EBM** *(3 points)*

   The temperature is described as $T(y)$ and the heat transport (sensible, latent and ocean) is modelled as diffusion:

   $$C_p\,\partial_t T \;+\; k\,\partial_y^2 T \;=\; (1-\alpha) Q_S^{top} - (A + B\;T) \tag{1}$$

   Show the solution if the planetary albedo α is chosen as a constant parameter. Use the ansatz with a global component and a latitude component

   $$T(y,t) \;=\; T_0(t) + T_1(t)\cdot\cos\left(\frac{2\,y}{R}\right) \tag{2}$$

   $$Q_S^{top} \;=\; Q_0 + Q_1\cdot\cos\left(\frac{2\,y}{R}\right) \tag{3}$$

   with $y = R\varphi$, R is the Earth radius, φ the latitude.

   Separate the dynamics for T_0 and T_1 and solve the differential equations!

   Hint: This is possible because the base functions 1 and $\cos(2\varphi)$ are orthogonal

   $$\int_{-90^\circ}^{90^\circ} 1\cdot\cos(2\varphi)\,d\varphi = \int_{-180^\circ}^{180^\circ} \cos(\varphi)\,d\varphi = 0 \tag{4}$$

2. **Questions about fluid mechanics (***3 points***, for each Q** *1 point***).**

   Q1: Name three different dimensionless parameters which can characterize the flow.

   Q2: Please state: The dimensionless Reynolds number is $Re = U/(L\nu)$ or $Re = UL/\nu$ or $Re = U^2L/\nu$? ν denotes the kinematic viscosity, L a length-scale L determined by the geometry of the flow, and U a characteristic velocity.

   Q3: Describe in words the Rayleigh-Bénard instability. The basic state possesses a steady-state solution in which there is no motion, and the temperature varies linearly with depth:

   $$u = w = 0 \tag{5}$$

   $$T_{eq} = T_0 + \left(1 - \frac{z}{H}\right)\Delta T \tag{6}$$

   When this solution becomes unstable, ... (please continue)

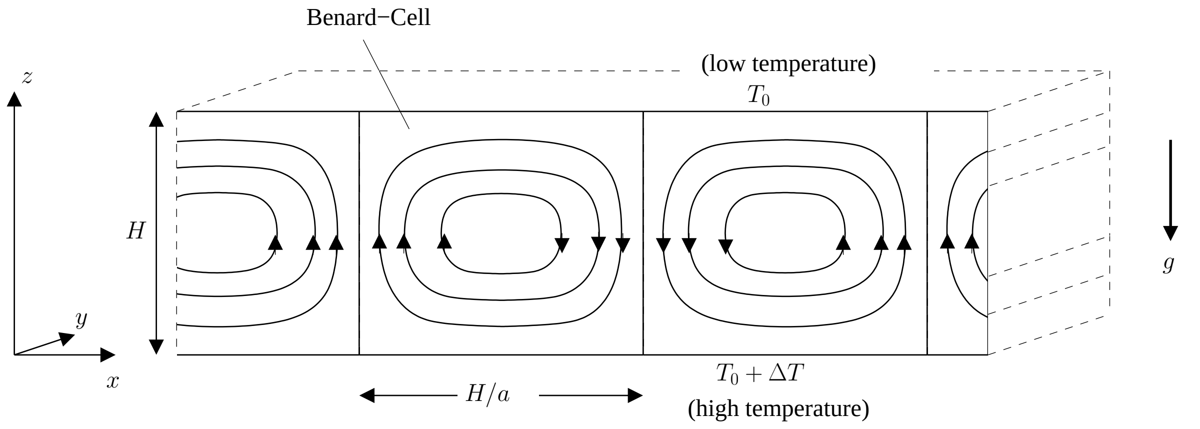

Figure 1: Geometry of the Rayleigh-Bénard system.

3. **Lorenz equations:** *(4 points)*

Consider the Lorenz equations which were derived from the Rayleigh-Bénard system (Fig. 1) in the lecture:

$$\dot{x} = \sigma(y - x) \tag{7}$$
$$\dot{y} = rx - xz - y \tag{8}$$
$$\dot{z} = xy - bz \tag{9}$$

with $\sigma, r, b > 0$. σ is the Prandtl number. Furthermore, Rayleigh number $R_a \sim \Delta T$, critical Rayleigh number R_c, and $r = R_a/R_c$.

(a) Evaluate the equilibrium points.
(b) Determine the stability of the $(0, 0, 0)-$equilibrium through linearization! Control parameter is r.
(c) Show the symmetry: The Lorenz equation has the following symmetry $(x, y, z) \rightarrow (-x, -y, z)$ independent on the parameters σ, r, b.

4. **Circulation and temperature in May 2017 and 2018** *(3 points)*

Consider the temperatures on May 8 in the years 2017 and 2018 in Fig. 2. The temperature differences over Central and Northern Europe are stricing. Explain the temperature differences over this area by the large-scale atmospheric circulation. The associated circulation can be derived from the Sea Level Pressure (Pa) patterns in Fig. 3 (geostrophic balance). Explain your observation in words (not more than 4 sentences).

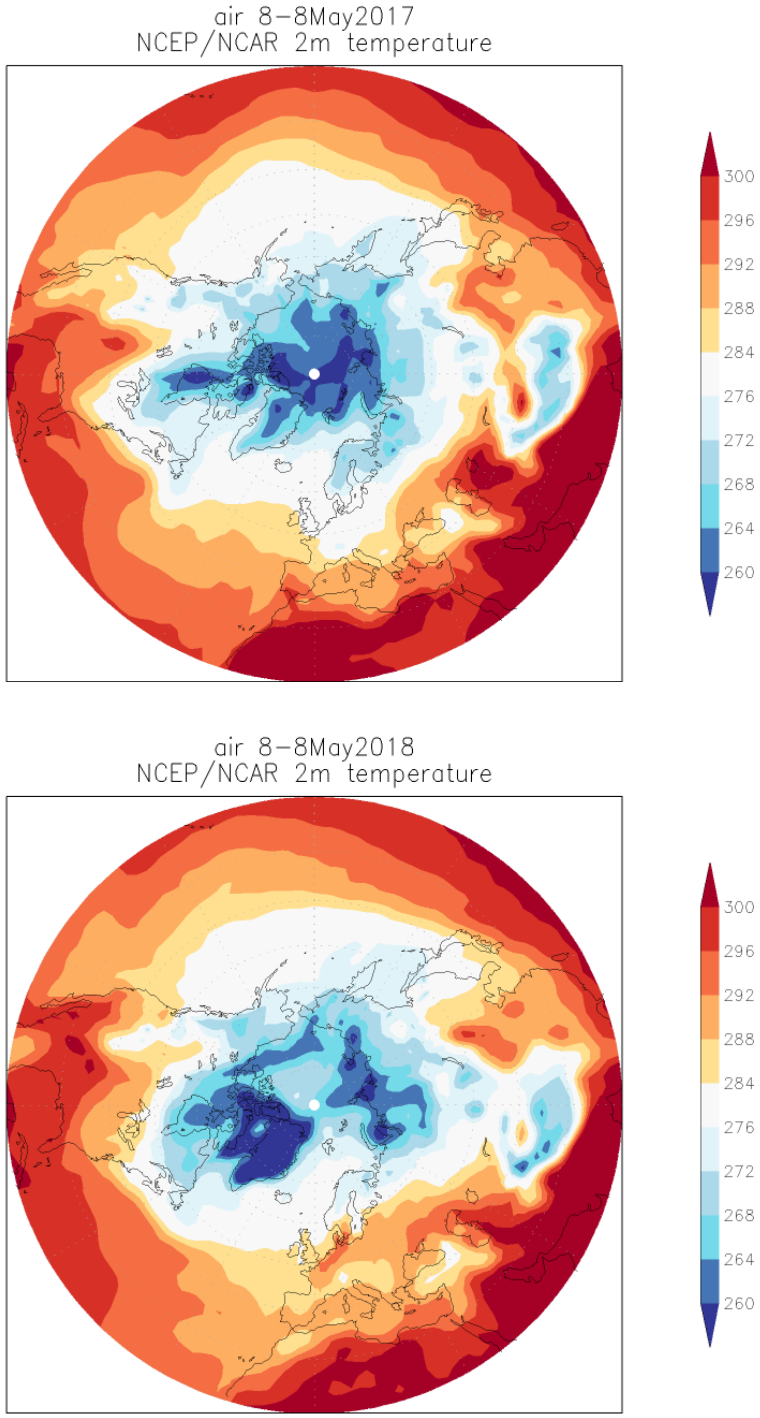

Figure 2: Surface Air Temperature (K) for May 8 in the years 2017 (upper) and 2018 (lower panel). Data are from the NCEP/NCAR reanalysis project (Kalnay et al., Bull. Amer. Meteor. Soc., 77, 437-470, 1996).

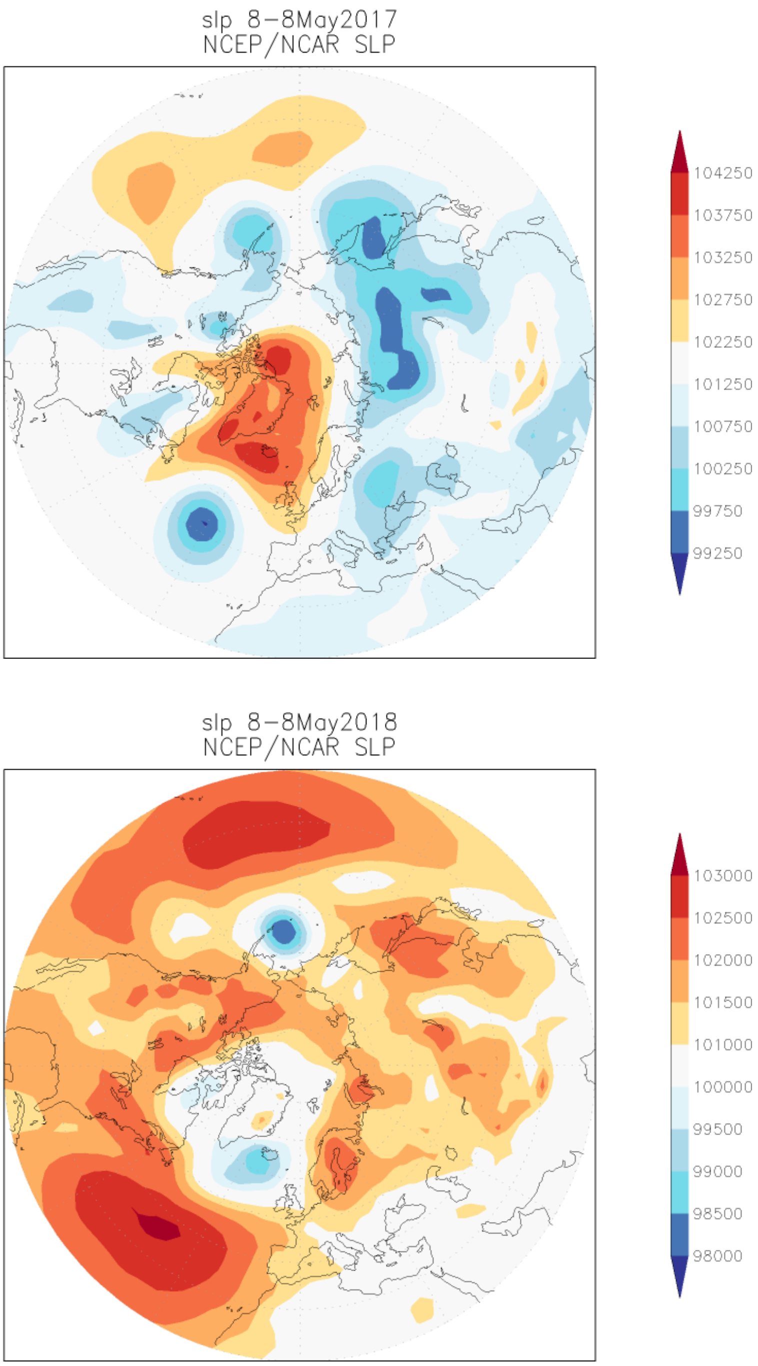

Figure 3: As in Fig. 2, but for Sea Level Pressure (Pa). The circulation in 2017 is characterized by a high pressure over Greenland, Iceland, and the Nordic Sea, and by surrounded low pressure systems.