Dynamics 2 | Exercise 5, Summer semester 2019
Lecturer: Prof. Dr. G. Lohmann | Tutor: Yuchen Sun
Due date: 20.5.2019 | 13.5.2019

1. **Wind-driven ocean circulation** *(5 points)*

   the Sverdrup transport V for the depth-integrated flow is calculated by

   $$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \tag{1}$$

   where τ_x and τ_y are the components of the wind stress.

   The Ekman transports V_E, U_E describe the dynamics in the upper mixed layer:

   $$f V_E = -\tau_x/\rho_0 \qquad , \qquad f U_E = \tau_y/\rho_0 \tag{2}$$

   where $U_E = \int_{-E}^{0} u dz$ and $V_E = \int_{-E}^{0} v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface. Denote w_E as the Ekman vertical velocity at the bottom of the Ekman layer. Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

   $$-\int_{-E}^{0} \frac{\partial w}{\partial z} dz = w_E = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0\, f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0\, f}\right) \quad . \tag{3}$$

   a) Assume that the windstress is only zonal with

   $$\tau_x = -\tau_0 \cos(\pi y/B) \tag{4}$$

   for an ocean basin $0 < x < L, \quad 0 < y < B$. Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case. Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

   b) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}\ \mathrm{s}^{-1}$ and $\beta = 1.8 \cdot 10^{-11}\ \mathrm{m}^{-1}\mathrm{s}^{-1}$ and $B = 5000\ \mathrm{km}$, $\tau_0/\rho_0 = 10^{-4}\ \mathrm{m}^2\mathrm{s}^{-2}$.

   c) Using the values in b), calculate the maximum of w_E for constant f.

	Quantity	Ocean
horizontal velocity	U	$1.6 \cdot 10^{-2}\, ms^{-1}$
horizontal length	L	$10^6\, m$
vertical length	E	$10^2\, m$
wind stress	τ_0	$1.5 \cdot 10^{-1}\, Pa$
Coriolis parameter at 45°N	$f_0 = 2\Omega \sin \varphi_0$	$10^{-4}\, s^{-1}$
density	ρ_0	$10^3\, kgm^{-3}$
viscosity (turbulent)	A_H	$10^2 - 10^4\, m^2 s^{-1}$

Table 1: Table shows the typical scales in the ocean system for the exercise.

2. **Non-dimensional vorticity dynamics including wind stress** *(4 points)*
   a) Show that (1) is a special case of the vorticity equation

$$\frac{D}{Dt}(\zeta + f) = A_H \nabla^2 \zeta + \frac{1}{\rho E}\left(\frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x\right) \quad . \tag{5}$$

   b) Derive the the non-dimensional version of (5). Include the Reynolds number $Re = UL/A_H$, Rossby number $Ro = U/(f_0 L)$, and the wind stress strength number $\alpha = \tau_0 L/(\rho_0 E U^2)$.

   c) Estimate the order of magnitude of the characteristic numbers for the ocean ! Use Table 1.

3. **Questions about the course** *(3 points)*
   a) Explain the Taylor-Proudman Theorem! (remember $f = f_0$, barotropic circulation) Why does the flow not go over the obstacle?
   Laboratory experiments showing the formation of a Taylor column, go to 2:50,

   b) Please write down the barotropic potential vorticity equation for large-scale motion!

   c) What are the two dominant terms in the horizontal momentum balance for the large-scale dynamics at mid-latitudes?

   d) What are the names of the 3 meridional cells in the atmosphere?
   Are these cells geostrophically driven or not?

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date to Yuchen Sun (yuchen.sun@awi.de).*