1. **Concept of dynamic similarity** *(3 points)*

For the case of an incompressible flow, assuming the temperature effects are negligible and external forces are neglected, the Navier-Stokes equations consist of conservation of mass

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and conservation of momentum

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \mathbf{u} \tag{2}$$

where $\mathbf{u}$ is the velocity vector and p is the pressure, ν denotes the kinematic viscosity.

a) Show: The equations (1,2) can be made dimensionless by a length-scale L, determined by the geometry of the flow, and by a characteristic velocity U. For example: $u = U \cdot u_d$.

Note: the units of $[\rho_0] = kg/m^3$, $[p] = kg/(ms^2)$, and $[p]/[\rho_0] = m^2/s^2$. Therefore the pressure gradient term in (2) has the scaling U^2/L.

b) Show: The scalings vanish completely in front of the terms except for the $\nabla^2 \mathbf{u_d}$-term! The dimensionless parameter is the Reynolds number and the only parameter left!

Remark: For large Reynolds numbers, the flow is turbulent. In most practical flows Re is rather large $(10^4 - 10^8)$, *large enough for the flow to be turbulent.*

2. **Angular momentum and Hadley cell** *(5 points)*

Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0, \tag{3}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\,(u + \Omega r) = \Omega R^2 \cos^2 \varphi + uR\cos\varphi \quad . \tag{4}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

a) Show, for inviscid zonally symmetric flow, that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

in (x, y, z) coordinates, where $y = R\varphi$.

b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

c) If the Hadley circulation is symmetric about the equator, and its edge is at 20° latitude, determine the strength of the subtropical jet. Use (3, 4).

3. **Conservation of potential vorticity:** *(3 points)*
An air column at 53°N with $\zeta = 0$ initially streches from the surface to a fixed tropopause at 10 km height. If the air column moves until it is over a mountain barrier of 2 km height at 30°N, what is its absolute vorticity and relative vorticity as it passes the mountain top?

Assume: $\sin 53° = 0.8; \sin 30° = 0.5$
The angular velocity of the Earth $\Omega = 2\pi/(1 \text{ day})$.
Potential vorticity: $(\zeta + f)/h$

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Fernanda Matos (Fernanda.Matos@awi.de), Ahmadreza Masoum (Ahmadreza.Masoum@awi.de).*