1. **Wind-driven ocean circulation** *(6 points)*

The **Sverdrup transport** V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \tag{1}$$

where τ_x and τ_y are the components of the wind stress.

The **Ekman transports** V_E, U_E describe the dynamics in the upper mixed layer:

$$fV_E = -\tau_x/\rho_0 \qquad , \qquad fU_E = \tau_y/\rho_0 \tag{2}$$

where $U_E = \int_{-E}^{0} u dz$ and $V_E = \int_{-E}^{0} v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface.

Ekman vertical velocity w_E: Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

$$w_E = -\int_{-E}^{0} \frac{\partial w}{\partial z} dz = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0 f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0 f}\right) \quad . \tag{3}$$

a) Assume that the windstress is only zonal with

$$\tau_x = -\tau_0 \cos(\pi y/B) \qquad \text{for an ocean basin with } \; 0 < x < L, \; 0 < y < B. \tag{4}$$

Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case.

b) Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

c) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B =$ 5000 km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

d) Using the values in b), calculate the maximum of w_E for constant f.

2. **Rossby wave formula (long waves in the westerlies)** *(4 points)*

a) Assume a mean flow with constant zonal velocity $u = U = const > 0$ and a varying north-south component $v = v(x, t)$ which gives the total motion a wave-like form. Furthermore, h =const.

Write down the vorticity equation for this specific flow! Remember that the vorticity equation is

$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) = 0 \tag{5}$$

b) Use a) and the ansatz

$$v(x,t) = A\cos[(kx - \omega t)] \tag{6}$$

to determine the disperion relation $\omega(k)$, the group velocity $\frac{\partial\omega}{\partial k}$, and the phase velocity $c = \omega/k$.

3. **Conservation of potential vorticity:** *(2 points)*

An air column at 53°N with $\zeta = 0$ initially streches from the surface to a fixed tropopause at 10 km height. If the air column moves until it is over a mountain barrier of 2 km height at 30°N, what is its absolute vorticity and relative vorticity as it passes the mountain top?

Assume: $\sin 53° = 0.8; \sin 30° = 0.5$
The angular velocity of the Earth $\Omega = 2\pi/(1 \text{ day})$.
Potential vorticity: $(\zeta + f)/h$

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00 pm) to Alessandro Gagliardi (Alessandro.Gagliardi@awi.de), Georg Huettner (Georg.Huettner@awi.de).*