Dynamics 2 | Exercise 6, Summer semester 2019
Lecturer: Prof. Dr. G. Lohmann | Tutor: Yuchen Sun
Due date: 27.05.2019 | 20.05.2019

1. **Rossby, gravity, and Kelvin waves** *(5 points)*

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \quad (1)$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \quad (2)$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad (3)$$

with H=const. as mean depth and η as surface anomaly.

a) With the elimination of the fast gravity waves in equation (3)

$$\frac{\partial \eta}{\partial t} = 0$$

derive the dispersion relation for divergence-free Rossby waves! Ansatz: Introduce a streamfunction for u,v: $\Psi \sim \exp(ikx + ily - i\omega t)$

b) With the assumption of $f = f_0 = 0$ derive the dispersion relation for gravity waves! The restoring force is related to gravity. Ansatz: take one of the equations (1,2,3) and derive the solution.

c) Kelvin waves:
What is the dispersion relation for Kelvin waves?
Make a sketch of the coastally trapped Kelvin wave on the Northern Hemisphere ocean basin.
Make a sketch of the equatorial trapped Kelvin waves.

d) Explain the difference between dispersive and non-dispersive waves!
You could use the $\omega(k)$ formula for Rossby and Kelvin waves.

2. **Rossby wave formula (long waves in the westerlies)** *(5 points)*
Consider the vorticity equation

$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) = 0 \tag{4}$$

a) Assume a mean flow with constant zonal velocity $u = U = const > 0$ and a varying north-south component $v = v(x, t)$ which gives the total motion a wave-like form. Furthermore, h =const. Write down the vorticity equation for this specific flow!

b) Use a) and the ansatz

$$v(x, t) = A\cos[(kx - \omega t)] \tag{5}$$

to determine the disperion relation $\omega(k)$, group velocity $\frac{\partial\omega}{\partial k}$, and the phase velocity $c = \omega/k$.

c) Derive the wavelength $L = 2\pi/k$ of the stationary wave given by $c = 0$.

3. **General Questions** *(3 points)*
a) Make a sketch of the Foucault pendulum (see script). Explain the horizontal dynamics of the Foucault pendulum

$$\ddot{x} = 2\Omega\sin\varphi\dot{y} - \frac{g}{L}x \tag{6}$$

$$\ddot{y} = -2\Omega\sin\varphi\dot{x} - \frac{g}{L}y \tag{7}$$

b) What is the hydrostatic approximation in the momentum equations?

c) Draw a schematic figure of the Atlantic Ocean meridional overturning! Include the directions N,S,E,W or depth in your sketch.

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date to Yuchen Sun (yuchen.sun@awi.de).*