1. **Wind-driven ocean circulation** *(6 points)*

The **Sverdrup transport** V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \tag{1}$$

where τ_x and τ_y are the components of the wind stress.

The **Ekman transports** V_E, U_E describe the dynamics in the upper mixed layer:

$$f V_E = -\tau_x/\rho_0 \qquad , \qquad f U_E = \tau_y/\rho_0 \tag{2}$$

where $U_E = \int_{-E}^{0} u dz$ and $V_E = \int_{-E}^{0} v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface.

Ekman vertical velocity w_E: Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

$$w_E = -\int_{-E}^{0} \frac{\partial w}{\partial z} dz = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0 f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0 f}\right) \quad . \tag{3}$$

a) Assume that the windstress is only zonal with

$$\tau_x = -\tau_0 \cos(\pi y/B) \qquad \text{for an ocean basin with } \; 0 < x < L, \; 0 < y < B. \tag{4}$$

Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case.

b) Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

c) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B = 5000$ km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

d) Using the values in b), calculate the maximum of w_E for constant f.

2. **Angular momentum and Hadley cell** *(6 points)*

Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0, \tag{5}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\,(u + \Omega r) = \Omega R^2 \cos^2\varphi + uR\cos\varphi \quad . \tag{6}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

a) Show, for inviscid zonally symmetric flow, that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{7}$$

in (x, y, z) coordinates, where $y = R\varphi$.

b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

c) If the Hadley circulation is symmetric about the equator, and its edge is at 20° latitude, determine the strength of the subtropical jet. Use (5, 6).

d) Is the Hadley Cell geostrophically driven or not?

<u>Notes on submission form of the exercises:</u> *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00 pm) to Alessandro Gagliardi (Alessandro.Gagliardi@awi.de), Georg Huettner (Georg.Huettner@awi.de).*