Dynamics 2 | Exercise 7, Summer semester 2019
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Due date: 7.06.2019 | 27.05.2019

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1. **Laplace transform** *(1 point)*

is given by $$\mathcal{L}\left\{x(t)\right\} = L(s) = \int_0^\infty e^{-st}x(t)dt \tag{1}$$

a) Show that

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\} = sL(s) - x(0) \tag{2}$$

through integration by parts.
b) Show furthermore that

$$\mathcal{L}\left\{\exp(-at)\right\} = \frac{1}{s+a} \tag{3}$$

2. **Angular momentum and Hadley cell** *(4 points)*

Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0, \tag{4}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\left(u + \Omega r\right) = \Omega R^2 \cos^2\varphi + uR\cos\varphi \quad . \tag{5}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

a) Show, for inviscid zonally symmetric flow, that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = 0 \tag{6}$$

in (x, y, z) coordinates, where $y = R\varphi$. We assume that $-\frac{1}{\rho}\frac{\partial p}{\partial x} = 0$

b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

c) If the Hadley circulation is symmetric about the equator, and its edge is at 20° latitude, determine the strength of the subtropical jet. Use (4, 5).

3. **Climate Data Operators zonal mean** *(4 points)*

   At which latitude is the zonal average of annual average surface temperature (tsurf) largest (smallest) for the climate states
   INIOM_PD_echam5_main_mm_3901-4000_climatological_mean.nc
   and LGM-W_echam5_6100-6200_climatological_mean.nc ?

   Solve this problem in two ways:
   a) design and execute a CDO command that produces this information directly
   b) use CDO only as a means to create zonal average annual average surface temperature; use the resulting data for generating a graphical solution to the problem.

   Details of this approach:
   -use CDO to first compute the zonal average of the annual average; instead of printing the result to screen, store the result in a new file
   -produce a graphical representation of the data (zonal temperature on y-axis, latitude on x-axis) with the software of your choice (e.g. ncview, panoply)
   -you may also use plotting software that is not able to process NetCDF data directly; if you want to do that, then:
   *extract the results of the computation of zonal mean annual average temperature from the NetCDF file via the CDO output-operator (that can dump content of an existing NetCDF file to the screen)
   *plot the respective temperature array with the software of your choice
   *as the CDO output-operator does not provide information on the latitudes, you will have to derive that information with an additional call of "CDO griddes" or similar.
   Hand in results for both a) and b) as solution to this exercise.

4. **Climate Data Operators max** *(2 points)*

   What is the coldest (warmest) surface temperature (tsurf) found during any month of a year at any location of the Earth for climate states (that is the one smallest (largest) temperature present in the files)
   INIOM_PD_echam5_main_mm_3901-4000_climatological_mean.nc
   and LGM-W_echam5_6100-6200_climatological_mean.nc ?

   During which month do extreme temperatures occur for the two climate states?

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date to to Yuchen Sun (yuchen.sun@awi.de).*