

1. **Rossby, gravity, and Kelvin waves** (5 points)

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g \frac{\partial \eta}{\partial x} \quad (1)$$

$$\frac{\partial v}{\partial t} + fu = -g \frac{\partial \eta}{\partial y} \quad (2)$$

$$\frac{\partial \eta}{\partial t} + H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (3)$$

with $H = \text{const.}$ as mean depth and η as surface anomaly.

a) With the elimination of the fast gravity waves in equation (3)

$$\frac{\partial \eta}{\partial t} = 0$$

derive the dispersion relation for divergence-free Rossby waves! Ansatz: Introduce a streamfunction for u, v : $\Psi \sim \exp(ikx + ily - i\omega t)$

b) With the assumption of $f = f_0 = 0$ derive the dispersion relation for gravity waves! The restoring force is related to gravity. Ansatz: take one of the equations (1,2,3) and derive the solution.

c) Kelvin waves:

What is the dispersion relation for Kelvin waves?

Make a sketch of the coastally trapped Kelvin wave on the Northern Hemisphere ocean basin.

Make a sketch of the equatorial trapped Kelvin waves.

d) Explain the difference between dispersive and non-dispersive waves!

You could use the $\omega(k)$ formula for Rossby and Kelvin waves.

2. **Rossby wave formula (long waves in the westerlies)** (5 points)

Consider the vorticity equation

$$\frac{D}{Dt} \left(\frac{\zeta + f}{h} \right) = 0 \quad (4)$$

a) Assume a mean flow with constant zonal velocity $u = U = \text{const} > 0$ and a varying north-south component $v = v(x, t)$ which gives the total motion a wave-like form. Furthermore, $h = \text{const}$. Write down the vorticity equation for this specific flow!

b) Use a) and the ansatz

$$v(x, t) = A \cos[(kx - \omega t)] \quad (5)$$

to determine the dispersion relation $\omega(k)$, group velocity $\frac{\partial \omega}{\partial k}$, and the phase velocity $c = \omega/k$.

c) Derive the wavelength $L = 2\pi/k$ of the stationary wave given by $c = 0$.

3. **General Questions** (3 points)

a) Make a sketch of the Foucault pendulum (see script). Explain the horizontal dynamics of the Foucault pendulum

$$\ddot{x} = 2\Omega \sin \varphi \dot{y} - \frac{g}{L} x \quad (6)$$

$$\ddot{y} = -2\Omega \sin \varphi \dot{x} - \frac{g}{L} y \quad (7)$$

b) What is the hydrostatic approximation in the momentum equations?

c) Draw a schematic figure of the Atlantic Ocean meridional overturning! Include the directions N,S,E,W or depth in your sketch.

Notes on submission form of the exercises: Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Smit Doshi (Smit.Doshi@awi.de), Dr. Qiyun Ma (Qiyun.Ma@awi.de).