

1. **The stochastic climate model** (6 points)

Figure 1 shows the mixed layer dynamics in the ocean. Imagine that the temperature of the ocean mixed layer of depth h is governed by

$$c_p \rho h \frac{dT}{dt} = -\lambda' T + Q'_{net}(t), \quad (1)$$

where $c_p \rho h$ is the heat capacity of the mixed layer with depth h . λ' is the damping rate of a temperature anomaly with a typical value of $2 \text{ W m}^{-2} \text{ K}^{-1}$. For simplicity define

$$\lambda = \lambda' / (c_p \rho h) \quad (2)$$

$$Q_{net}(t) = Q'_{net}(t) / (c_p \rho h) \quad (3)$$

The air-sea fluxes due to weather systems are represented by a white-noise process $Q_{net} = \hat{Q}_\omega e^{i\omega t}$ where $\hat{Q}_\omega$ is the amplitude of the random forcing at frequency ω and $\hat{Q}^*$ is the complex conjugate. Remember that Q_{net} can be described through its distribution and its correlation properties: a Gaussian distribution of zero average $\langle Q_{net} \rangle = 0$ and δ -correlated in time $Cov_Q(\tau) = \langle Q_{net}(t) Q_{net}(t+\tau) \rangle = q \delta(\tau)$. The brackets indicate an average over realizations of the random force (ensemble mean).

a) Solve (1) for the temperature response $T = \hat{T}_\omega e^{i\omega t}$ and hence show that:

$$\hat{T}_\omega = \frac{\hat{Q}_\omega}{(\lambda + i\omega)} \quad (4)$$

b) Show that it has a spectral density $\hat{T}_\omega \hat{T}_\omega^*$ is given by:

$$\hat{T} \hat{T}^* = \frac{\hat{Q} \hat{Q}^*}{(\lambda^2 + \omega^2)} \quad (5)$$

and the spectrum

$$S(\omega) = \langle \hat{T} \hat{T}^* \rangle = \frac{q}{(\lambda^2 + \omega^2)}. \quad (6)$$

Use: The Fourier transform of the auto-correlation function $Cov_Q(\tau)$ is the spectrum

$$S_Q(\omega) = \langle \hat{Q} \hat{Q}^* \rangle = \int_{\mathbb{R}} Cov_Q(\tau) e^{-i\omega\tau} d\tau = \int_{\mathbb{R}} q \cdot \delta(\tau) e^{-i\omega\tau} d\tau = q \quad (7)$$

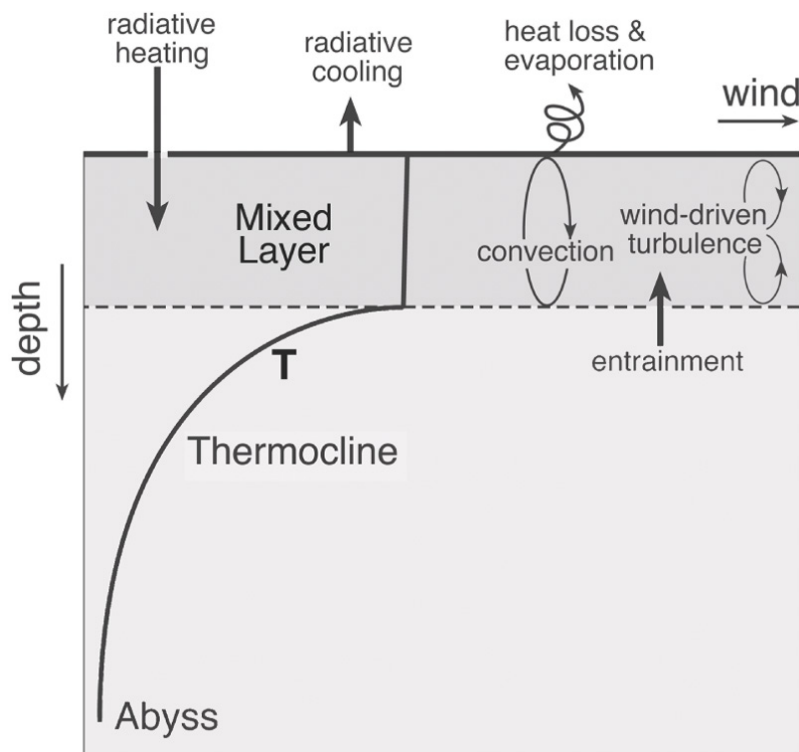


Figure 1: Schematic picture of mixed layer in the ocean.

- c) Make a sketch of the spectrum using a log-log plot and show that fluctuations with a frequency greater than λ are damped. Compare the spectrum of T with the spectrum of Q.

Why is the spectrum of Q called white ? Remember that

$$\int_R \exp(-i\omega t) \delta(t) dt = 1 \quad (8)$$

- d) Compare the spectrum of T between high and low latitudes using the distribution of the seasonal mixed layer depth variations (Fig. 1) due to the different damping rates which are related to the mixed layer depth h. Argue qualitatively with the above equations (1,2). Remember that $c_p \rho h$ is the heat capacity of the mixed layer ($c_p = 4.2 \cdot 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$; $\rho = 1023 \text{ kg m}^{-3}$).

- e) Consider now the analogous deterministic equation

$$c_p \rho h \frac{dT}{dt} = -\lambda' T + F'(t), \quad (9)$$

where $F'(t)$ is the forcing due to increase in greenhouse gas concentrations. What is the change in equilibrium temperature when the CO_2 content in the atmosphere is doubled which is equivalent to a forcing of $F'(t) = +4 \text{ W/m}^2$?

Note that $F'(t)/\lambda'$ is also called the sensitivity of oceanic mixed layer temperature to doubling of CO_2 .

2. Questions about atmosphere-ocean dynamics (4 points, for each 1 point).

- a) Please write down the barotropic potential vorticity equation for large-scale motion! Which quantity is conserved along the streamlines?
- b) What are the two dominant terms in the horizontal momentum balance for the large-scale dynamics at mid-latitudes for the atmosphere and ocean flow ?
... and with wind stress forcing?
- c) Explain the Taylor-Proudman theorem! Assume constant density ρ_0 on a plane with constant rotation $f = f_0 \neq 0$.
- d) Draw a schematic figure of the Atlantic Ocean meridional overturning! Include the directions (N,S), (E,W), depth in your sketch.

Notes on submission form of the exercises: Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Aparna Prasannakumar (apa_pra@uni-bremen.de) and Yifan Ma (yifan.ma@awi.de).