$$\Phi_{xy}(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}(\Gamma_{xy}(\omega))}{\mathrm{Re}(\Gamma_{xy}(\omega))}\right).$$

A cross-spectrum for two similar processes, but with one shifted in time with respect to the other: $x(t)$ and $x(t+\tau)$, gives the same power spectrum as for the same analysis applied to two identical time series, $x(t)$, but instead of a phase difference of zero, the phase is linear in frequency with a slope proportional to the phase shift: $\Phi_{xy}(\omega) = 2\pi\tau\omega$.

The coherence spectrum is analogous to the conventional correlation coefficient and is defined as:

$$\kappa_{xy}(\omega) = \frac{A_{xy}(\omega)^2}{\Gamma_{xx}(\omega)\Gamma_{yy}(\omega)}.$$

(Von Storch and Zwiers (1999) [?], p. 234-241)

10.4.3 Singular Spectrum Analysis*

Singular spectrum analysis (SSA) [?] (p.451-459) is a method which is similar to the EOF analysis in terms of the mathematical formulation, but yields results which may be categorised under the spectral method groups. The method is suitable for extracting information from short and noisy time series. SSA unravels the information embedded in the delay-coordinate phase space by decomposing the sequence into elementary patterns of behaviour in time and spectral domains, by using "data-adaptive filters" that help separating the time series into statistically independent components, which can be classified as trends, deterministic oscillation, or noise.

First, a window size (M), which sets the maximum lag, is chosen. Then, a $M \times M$ covariance matrix $\mathbf{C}_X$ is computed from the M sequences of the $N \times 1$ data series $\mathbf{X}$, each corresponding to a 1-lag shift:

$$\tilde{\mathbf{X}}_k(t) = [\mathbf{X}(t+k), \mathbf{X}(t+k+1), ..., \mathbf{X}(N-M+k)],$$

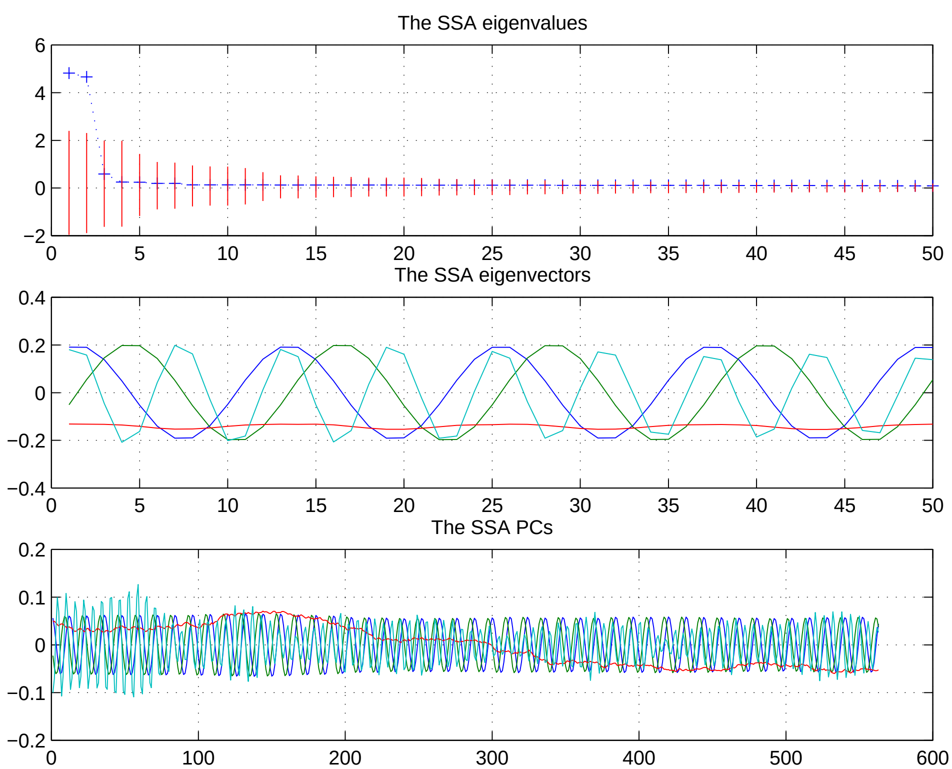

Figure 10.6: *SSA results for the leading PC from the NCEP SLP. [stats_uib_10_1.m]*

where $k = 1, ..., M$. The eigenvalue problem

$$\mathbf{C}_x \rho_k = \lambda_k \rho_k,$$

or

$$\mathbf{E}_X^T \mathbf{C}_X \mathbf{E}_X = \Lambda_X,$$

is solved (using SVD), giving the eigenvectors $\mathbf{E}_x$ corresponding to the eigenvalues ρ_k (given in decreasing order). The S/N separation is obtained by plotting the Eigenvalue spectrum.

The SSA does not yield spectral estimates directly, but may be used to reconstruct a cleaner signal that may be spectrally analysed.

Multi-channel SSA (M-SSA) [**?**] (p.469-470) performs the SSA on a vector or map of data, such as in gridded form.

10.4.4 Wavelet analysis*

Wavelet analysis can give information about the time evolution of the spectral properties of a quantity. A good introduction is given by Torrence and Compo (1998) [**?**] (See URL: http://paos.colorado.edu/research/wavelets/).

Wavelet analysis has traditionally been a tool for analysing sound and music, but is now becoming increasingly more common in climate science.