1 Description of the 7-Box-Model

1.1 Model overview

Here we introduce a interhemispheric boxmodel of the thermohaline circulation. Like in the model of Rooth (1982) the atlantic ocean is described over both hemispheres. The ocean model follows the one of Rahmstorf (1996) while the adding of equations for the atmosphere and the coupling of the latter with the ocean through heat and freshwater fluxes leads to strong similarities with the model version of Knorr (2005).
The box model contains out of four oceanic and three atmospheric boxes, as indicated in figure 1.1. The ocean boxes represent the atlantic ocean from $80°N$ to $60°S$. The northern ocean box ranges from $80°N$ to $45°N$, where the tropical and the underneath deep box are attached. These reach to the southern box, which is located from $30°S$ to $60°S$. The boundaries of the atmospheric boxes differ from that only by the extension of the high latitude boxes to the north and the south pole, respectively. The width of the atlantic ocean is uniformly set to 80 degrees of longitude and the bottom boundary is approximated with the average ocean depth of 4.000m. The depth of the tropical box is set to 600m, which leads to a vertical extent of 3.400m for the deep box.

The indices of the temperatures T, the salinities S, the surface heat fluxes H, the atmospheric heat fluxes F, the radiation terms R as well as later on the volumes bear on the different boxes (N for the northern, M for the tropical, D for the deep and S for the southern box). Temperatures are given in $°C$, salinities in pSU and the overturning flow in Sv (1 Sverdrup=$10^6 \frac{m^3}{s}$). The discrete boxes are utterly homogenious, which implies that the temperatures and the salinities everywhere within one box are alike.

1.2 Model equations

The prognostic equations for the temperatures of the ocean boxes contain out of two parts. The first part is proportional to the overturning flow Φ and represents the advective coupling between the boxes. The second part, which is dependent on the surface heat flux H, stands for the coupling between the ocean and the atmosphere. The latter part does not take account in the deep box, as it is not connected to the atmosphere. The four differential equations for the ocean temperatures

$$\frac{d}{dt}T_N = -\left(T_N - T_M\right)\frac{\Phi}{V_N} + \frac{H_N}{\rho_0 c_p dz_2}, \tag{1.1}$$

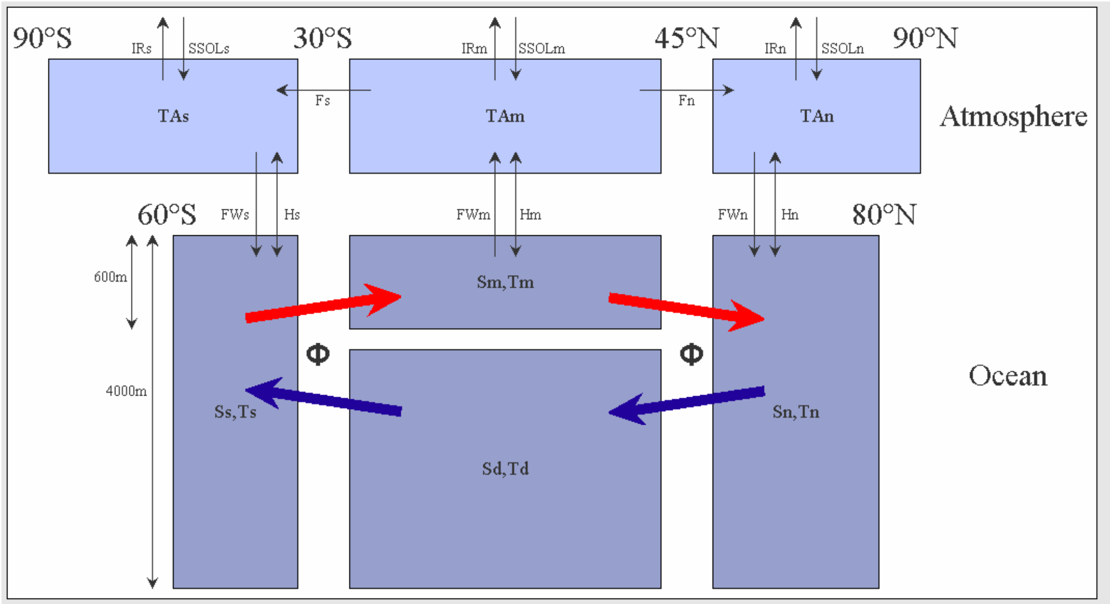

Figure 1.1: Schematic illustration of the 7-Box-Model

$$\frac{d}{dt}T_M = -\left(T_M - T_S\right)\frac{\Phi}{V_M} + \frac{H_M}{\rho_0 c_p dz_1}, \tag{1.2}$$

$$\frac{d}{dt}T_S = -\left(T_S - T_D\right)\frac{\Phi}{V_S} + \frac{H_S}{\rho_0 c_p dz_2} and \tag{1.3}$$

$$\frac{d}{dt}T_D = -\left(T_D - T_N\right)\frac{\Phi}{V_D} \tag{1.4}$$

are similar to those in Knorr (2005) or Prange et al. (1997).
ρ_0 denotes a reference density for saltwater of $1025\frac{kg}{m^3}$ and c_p the specific heat capacity $4200\frac{J}{kgK}$ of water. The factors dz_i indicate the depths of the discrete ocean boxes, respectivly ($dz_1 = 4.000m, dz_2 = 600m$). These and the width of 80 degrees in longitude lead to

$$V = 2\pi r^2 dz_i \left(\varphi_S - \varphi_N\right)\frac{80}{360} \tag{1.5}$$

for the calculation of the volumes of the ocean boxes, where φ_i represent the limiting latitudes of the particular box and r=6371km the earths radius. By this means the volumes of the ocean boxes are given in table 1.1.
The overturning flow or rather the volume flux Φ of the THC is defined by the density gradients of the oceans boxes, as already in Stommel (1961). Like in Rahmstorf (1996) the northern and the southern box will be taken into account for this, which leads to

V_S	V_M	V_D	V_N
$8,298 \cdot 10^{16} m^3$	$4,105 \cdot 10^{16} m^3$	$2,326 \cdot 10^{17} m^3$	$6,295 \cdot 10^{16} m^3$

Table 1.1: Volumes of the ocean boxes

the equation

$$\Phi = c\left[-\alpha\left(T_N - T_S\right) + \beta\left(S_N - S_S\right)\right] \tag{1.6}$$

from Knorr (2005) for the calculation of the overturning flow. $\alpha = 1,5 \cdot 10^{-4}\frac{1}{K}$ and $\beta = 8,0 \cdot 10^{-4}\frac{1}{pSU}$ represent the thermal and the haline expansion coefficients (from Lohmann et al. (1996)). c is a freely tunable parameter which here is set to $1,5264 \cdot 10^{10}\frac{m^3}{s}$ to produce a value for Φ, which is consistent with present-day climate conditions. The surface heat fluxes follow the equations

$$H_i = Q_{1_i} - Q_2\left(T_i - T_{A_i}\right) \tag{1.7}$$

from Haney (1971). Q_{1_i} and Q_2 are tuning parameters. While $Q_2 = 50\frac{W}{m^2K}$ can be seen as a global constant, Q_{1_i} varies with each box ($Q_{1_N} = 20\frac{W}{m^2}, Q_{1_M} = 70\frac{W}{m^2}, Q_{1_S} = 100\frac{W}{m^2}$). More details about these tuning parameters will be provided in chapter 1.3.

Analogue to the equations 1.1 to 1.4 the prognostic differential equations for the salinities consist out of two components. One of those is again the advective part, caused by the interconnection between the boxes and the other one is the influence of the freshwater fluxes between the ocean and the atmosphere. The latter is again only for the boxes near the surface, thus the equations are

$$\frac{d}{dt}S_N = -\left(S_N - S_M\right)\frac{\Phi}{V_N} - S_{ref}\frac{FW_N}{V_N}, \tag{1.8}$$

$$\frac{d}{dt}S_M = -\left(S_M - S_S\right)\frac{\Phi}{V_M} + S_{ref}\frac{FW_M}{V_M}, \tag{1.9}$$

$$\frac{d}{dt}S_S = -\left(S_S - S_D\right)\frac{\Phi}{V_S} - S_{ref}\frac{FW_S}{V_S}, \tag{1.10}$$

$$\frac{d}{dt}S_D = -\left(S_D - S_N\right)\frac{\Phi}{V_D}. \tag{1.11}$$

The reference salinity $S_{ref} = 39,4pSU$ is a characteristic average value for the entire atlantic ocean which is included in the model as a fixed upper boundary ("rigid-lid") condition.
The freshwater fluxes can be considered as the proportion of precipitation minus evaporation (P-E) and thus have to be related to the volume of the particular box. So $\frac{FW_i}{V_i}$ corresponds to the relation $\frac{(P-E)_i}{dz_i}$ (Lohmann et al. (1996) or Prange et al. (1997)) and

flows, as to be seen in image 1.1, from the tropical ocean box into the atmosphere and from there into the high latitude ocean boxes. Hence the diagnostic equations for the freshwater fluxes FW_i dependent on the latent heat fluxes F_{l_i} are

$$FW_N = 2,5 \cdot \frac{2\pi r \cdot cos\,(45)}{L_v \rho_w} F_{ln} \cdot \frac{80}{360}, \tag{1.12}$$

$$FW_S = \frac{2\pi r \cdot cos\,(30)}{L_v \rho_w} F_{ls} \cdot \frac{80}{360}, \tag{1.13}$$

$$FW_M = FW_N + FW_S. \tag{1.14}$$

Here L_v represents the specific heat of vaporisation of water $(2,5 \cdot 10^6 \frac{J}{m^3})$ and ρ_w the density of freshwater $(1000 \frac{kg}{m^3})$. The factor $\frac{80}{360}$ scales the width of the atlantic ocean in relation to the total earths area. Freshwater fluxes north of $80°N$ and south of $60°S$ are neglected. Due to the much larger caption area in the north atlantic, the freshwater flux in the northern ocean box is multiplied by 2.5. Another assumption from Prange et al. (1997) is the proportional dependence of the latent heat fluxes to the meridional moisture gradient $\left(\frac{\partial q}{\partial y}\right)$ in the atmosphere

$$F_l = K_l \left(\frac{\partial q}{\partial y}\right). \tag{1.15}$$

Similar to c in equation 1.6, K_l is a freely tunable parameter, which is set to $7,65 \cdot 10^{17} Wm$ here and will be dicussed in chapter 1.3. The introduction of the specific saturated humidity q_s and the transcribtion of the derivative to a sphere leads to

$$F_l = K_l \frac{r_h}{r} \frac{\partial q_s\,(T_{L_i}, p)}{\partial T_{A_i}} \frac{\partial T_{A_i}}{\partial \varphi}, \tag{1.16}$$

where $r_h = 0,8$ is an approximated constant relative humidity and T_{L_i} the temperature at the latitude where the flux actually takes place. As the specific humidity can be described with $q_s = 0,622 \frac{e_s}{p}$ and also the air pressure is considered as constant $(p = p_0 = 1000hPa)$ in the model, F_l can be formulated as

$$F_l = K_l \frac{r_h}{r} \cdot \frac{0,622}{p_0} \cdot \frac{\partial e_s\,(T_{L_i})}{\partial T_{A_i}} \cdot \frac{\partial T_{A_i}}{\partial \varphi}. \tag{1.17}$$

The saturatd vapour pressure e_s here is calculated in dependence of the temperature with one of the Magnus-formulas

$$e_s\,(T_{L_i}) = 6,122 \cdot exp\left(\frac{17,67 \cdot T_{L_i}}{T_{L_i} + 243,5}\right). \tag{1.18}$$

To determine the temperatures at $30°S$ and $45°N$, a linearly approximated latitudinal profile for the zonal air temperatur is assumed. As the temperatures T_{A_i} are regarded for the exact center of the particular box, they are assessed with the distance from the center to the edge of the box to obtaine T_{L_i} which stands for T(30) (temperature at

$30°S$) and (T45) (Temperature at $45°N$), respectively:

$$T\,(30) = \frac{15}{48} T_{A_M} + \frac{33}{48} T_{A_S} \tag{1.19}$$

$$T\,(45) = \frac{17}{48} T_{A_M} + \frac{31}{48} T_{A_N} \tag{1.20}$$

Similar to Knorr (2005) und Prange et al. (1997) the ocean model is coupled to an atmospheric energy-balance-model (EBM), which allows the temperatures to vary, in comparison to Stommel (1961). This EBM is related to the one of Chen et al. (1995) and contains sensible and latent heat fluxes, outgoing infrared radiation, solar radiation as well as the surface heat fluxes H between the atmosphere and the ocean. These components influence the temperatures of the atmospheric boxes, whose prognostic equations are

$$\beta_N c_p \frac{d}{dt} T_{A_N} = \frac{cos\,(45°)\,(F_{s_N} + F_{l_N})}{2\pi r^2\,(sin\,(80°) - sin\,(45°))} + R_N - f_N H_N, \tag{1.21}$$

$$\beta_M c_p \frac{d}{dt} T_{A_M} = \frac{cos\,(30°)\,(F_{s_S} + F_{l_S}) + cos\,(45°)\,(F_{s_N} + F_{l_N})}{2\pi r^2\,(sin\,(30°) + sin\,(45°))} + R_M - f_M H_M and \tag{1.22}$$

$$\beta_S c_p \frac{d}{dt} T_{A_S} = \frac{cos\,(30°)\,(F_{s_S} + F_{l_S})}{2\pi r^2\,(sin\,(60°) - sin\,(30°))} + R_S - f_S H_S. \tag{1.23}$$

$c_p = 1004 \frac{J}{kgK}$ is the specific heat of air and $\beta_i c_p$ is characterised as the thermal inertia or rather the timescale of atmospherical reactions by Chen et al. (1995). This factor emerges from the connection between the vertical integrated temperature and the surface temperature

$$\frac{1}{g} \int_0^{p_0} c_p \frac{\partial T}{\partial t} dp = \beta_i c_p \frac{\partial T_0}{\partial t}. \tag{1.24}$$

In this equation, where g represents the acceleration of gravity, the constant β_i is composed of the surface temperature of air T_0 and the effective height of the atmosphere. According to Chen et al. (1995), this can be approximated with the value $5300 \frac{kg}{m^2}$ for the whole atmosphere.
The diagnostic equations for the sensible heat fluxes F_{s_i} are described in dependence of the meridional gradient of the surface temperature with

$$F_s = K_s \frac{\partial T_A}{r \partial \varphi} \tag{1.25}$$

after Lohmann et al. (1996). K_s is another freely tunable parameter, which serves to generate realistic values for sensible heat fluxes and again is described more precisely in the following chapter 1.3.
The radiation terms R_i in the equations 1.21 to 1.23 consist of an incoming solar short-

wave (S_i) and an outgoing infrared longwave (I_i) part. The extraterrestrial solar radiation is not absorbed entirely, though and thus has to be balanced with a box-dependent average albedo value α_i. The outgoing infrared radiation I_i is generated with the empirical formula of Budyko (1969), which uses a linear approach from the atmosphere temperatures. This is contrary to the Stefan-Boltzmann-Law, which determines the dependence of the longwave radiation with σT^4, but after North (1975) justifiable for small temperature intervalls. Thus, the equation for the radiation balance on the top of the atmosphere is

$$R_i = S_i - I_i = S_{sol,i}\,(1 - \alpha_i) - (A + BT_{A_i})\,. \tag{1.26}$$

The constants $A = 213,35\frac{W}{m^2}$ and $B = 2,22\frac{W}{m^2K}$ are taken from Chen et al. (1995) and can be found in Knorr (2005) or in Prange et al. (1997), as well. The surface heat fluxes H_i are multiplied by $f_i = \frac{80}{360}$ in the equations 1.21 to 1.23 to account for the part of the surface area of the atlantic on earth.

1.3 Properties and initial model conditions

The very coarse resolution of the model leads to the neglection of several spacial and temporal processes. Synoptic processes like the development and the movement of pressure systems through barocline instabilities and Rossby-waves, that mainly control the transport of sensible and latent heat, or seasonal and spacial differences of the ocean temperature and salinity, that have impacts on the strength of the THC, are not considered. Therefore the freely tunable parameters $(c, Q_{1_i}, Q_2, K_l, K_s)$, mentioned in chapter 1.2, are to be adjusted such as to describe the physical processes correctly, at least. These parameters often are either not explicitly defined or too complex to be integrated into a conceptual model. Still these values are necessary to generate as realistic climate conditions as possible and to average out various not resolvable processes, respectively. This however burrows diverse uncertainties, as several atmospheric and oceanic phenomena like the north atlantic oscillation or the ENSO circulation possess very anomalous fluctuations, which causes difficulties in developing realistic climatological means.
Furthermore convective as well as diffusive processes are neglected in the model. The model equations for the freshwater fluxes in conjunction with the prognostic salinity equations clinch the salinity conservation of the system. Thus the total mass of salt is constant in time. The integration of the system is implemented with a euler foreward scheme. The time step is 1/100 of a year, so about 3.65 days, which fits the Courant-Friedrich-Levy-Criterion (CFL-Criterion) and therefore ensures the stability of the system.

For example seasonal variability which causes fluctuations of the THC or the spacial distribution of salinities and temperatures in the ocean, leading to cross circulations and varying values for the overturning flow at different longitudes, are aspects that are implied in the factor c. Thus the adjustment of the parameter has to be done in order to generate a climatological and spacial mean for the overturning flow.

The parameters K_s and K_l and the associated sensible and latent heat fluxes are very dependent on large scale pressure systems and therefore alter on small time scales. The characteristics and the location of Rossby-waves, front lines or streaming patterns can determine the advection of air masses with various temperature and moisture properties and thus define the sensible and latent heat fluxes. These small scale driving factors are dependent on seasonal variablity or the general weather situation, and of course can not be considered in a box model. Therefore they have to be assimilated somehow within the parameters K_s and K_l.
Q_{1_i} and Q_2 are affected vastly by the atmosphere temperatures on the one hand and the different albedo values on the other. The equation 1.7 has been developed by Haney (1971) specifically for the ocean. It provides zonally averaged magnitudes for the amount of the outgoing infrared radiation and sensible and latent heat per $°C$ of the temperature difference between ocean and atmosphere. Different values of Q_{1_i} for each of the three boxes are applied, as the meridional differences are relatively large. Per contra a global mean for Q_2 is taken, which is defensible because of the much smaller alterations.

The initial values of the model are given in table 1.2. They represent averages for present-day climate conditions. An overturning flow of 10.15Sv is reached that way, which is consistent with the magnitudes of the overturning flow in literature like Rahmstorf (1996). These values are already balanced and can be used for various experiments to gain qualitative and conditionally also quantitative results.

TA_S	TA_M	TA_N	T_S	T_M	T_N
4,57	23,23	1,43	4,70	24,38	3,01
T_D	S_S	S_M	S_N	S_D	Φ
3,02	34,40	35,92	34,91	34,91	10,15

Table 1.2: Initial values for the model (T in $°C$, S in pSU, Φ in Sv).

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