Abrupt Climate Change Modeling

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Article Outline

Glossary

Atmosphere The atmosphere is involved in many processes of abrupt climate change, providing a strong non-linearity in the climate system and propagating the influence of any climate forcing from one part of the globe to another. Atmospheric temperature, composition, humidity, cloudiness, and wind determine the Earth's energy fluxes. Wind affects the ocean's surface circulation and upwelling patterns. Atmospheric moisture transport determines the freshwater balance for the oceans, overall water circulation, and the dynamics of glaciers.

Oceans Because water has enormous heat capacity, oceans typically store 10–100 times more heat than equivalent land surfaces. The oceans exert a profound influence on climate through their ability to transport heat from one location to another. Changes in ocean circulation have been implicated in abrupt climate change of the past. Deglacial meltwater has freshened the North Atlantic and reduced the ability of the water to sink, inducing long-term coolings.

Land surface The reflective capacity of the land can change greatly, with snow or ice sheets reflecting up to 90% of the sunlight while dense forests absorb more than 90%. Changes in surface characteristics can also affect solar heating, cloud formation, rainfall, and surface-water flow to the oceans, thus feeding back strongly on climate.

Cryosphere The portion of the Earth covered with ice and snow, the cryosphere, greatly affects temperature. When sea ice forms, it increases the planetary reflective capacity, thereby enhancing cooling. Sea ice also insulates the atmosphere from the relatively warm ocean, allowing winter air temperatures to steeply decline and reduce the supply of moisture to the atmosphere. Glaciers and snow cover on land can also provide abrupt-change mechanisms. The water frozen in a glacier can melt if warmed sufficiently, leading to possibly rapid discharge, with consequent effects on sea level and ocean circulation. Meanwhile, snow-covered lands of all types maintain cold conditions because of their high reflectivity and because surface temperatures cannot rise above freezing until the snow completely melts.

External factors Phenomena external to the climate system can also be agents of abrupt climate change. For example, the orbital parameters of the Earth vary over time, affecting the latitudinal distribution of solar energy. Furthermore, fluctuations in solar output, prompted by sunspot activity or the effects of solar wind, as well as volcanoes may cause climate fluctuations.

Climate time scales The climate system is a composite system consisting of five major interactive components: the atmosphere, the hydrosphere, including the oceans, the cryosphere, the lithosphere, and the biosphere. All subsystems are open and non-isolated, as the atmosphere, hydrosphere, cryosphere and biosphere act as cascading systems linked by complex feedback processes. Climate refers to the average conditions in the Earth system that generally occur over periods of time, usually several decades or longer. This time scale is longer than the typical response time of the atmosphere. Parts of the other components of the Earth system (ice, ocean, continents) have much slower response times (decadal to millennial).

Climate variables and forcing State variables are temperature, rainfall, wind, ocean currents, and many other variables in the Earth system. In our notation, the variables are described by a finite set of real variables in a vector $x(t) \in \mathbb{R}^n$. The climate system is subject to two main external forcings $F(x, t)$ that condition its behavior, solar radiation and the action of gravity. Since $F(x, t)$ has usually a spatial dependence, F is also a vector $\in \mathbb{R}^n$. Solar radiation must be regarded as the primary forcing mechanism, as it provides almost all the energy that drives the climate system. The whole climate system can be regarded as continuously evolving, as solar radiation changes on diurnal, seasonal and longer time scales, with parts of the system leading or lagging in time. Therefore, the subsystems of the climate system are not always in equilibrium with each other. Indeed, the climate system is a dissipative, highly non-linear system, with many instabilities.

Climate models are based on balances of energy, momentum, and mass, as well as radiation laws. There are several model categories, full circulation models, low-order models, and models of intermediate complexity. Climate models simulate the interactions of the atmosphere, oceans, land surface, and ice. They are used for a variety of purposes from study of the dynamics of the weather and climate system, past climate to projections of future climate.

Global climate models or General circulation models (GCMs) The balances of energy, momentum, and mass are formulated in the framework of fluid dynamics on the rotating Earth. GCMs discretize the equations for fluid motion and energy transfer and integrate these forward in time. They also contain parametrization for processes – such as convection – that occur on scales too small to be resolved directly. The dimension of the state vector is in the order of $n \sim 10^5 - 10^8$ depending on the resolution and complexity of the model.

Model categories In addition to complex numerical climate models, it can be of great utility to reduce the system to low-order, box, and conceptual models. This complementary approach has been successfully applied to a number of questions regarding feedback mechanisms and the basic dynamical behavior, e. g. [48,84]. In some cases, e. g. the stochastic climate model of Hasselmann [32], such models can provide a null hypothesis for the complex system. The transition from highly complex dynamical equations to a low-order description of climate is an important topic of research. In his book "Dynamical Paleoclimatology", Saltzman [77] formulated a dynamical system approach in order to differentiate between fast-response and slow-response variables. As an alternative to this method, one can try to derive phenomenologically based concepts of climate variability, e. g. [21,43]. In between the comprehensive models and conceptual models, a wide class of "models of intermediate complexity" were defined [12].

Earth-system models of intermediate complexity (EMICs) Depending on the nature of questions asked and the pertinent time scales, different types of models are used. There are, on the one extreme, conceptual models, and, on the other extreme, comprehensive models (GCMs) operating at a high spatial and temporal resolution. Models of intermediate complexity bridge the gap [12]. These models are successful in describing the Earth system dynamics including a large number of Earth system components. This approach is especially useful when considering long time scales where the complex models are computationally too expensive, e. g. [47]. Improvements in the development of coupled models of intermediate complexity have led to a situation where modeling a glacial cycle, even with prognostic atmospheric CO_2 is becoming possible.

Climate simulation A climate simulation is the output of a computer program that attempts to simulate the climate evolution under appropriate boundary conditions. Simulations have become a useful part of climate science to gain insight into the sensitivity of the system.

Climate variability pattern Climate variability is defined as changes in integral properties of the climate system. True understanding of climate dynamics and prediction of future changes will come only with an understanding of the Earth system as a whole, and over past and present climate. Such understanding requires identification of the patterns of climate variability and their relationships to known forcing. Examples for climate variability patterns are the North Atlantic Oscillation (NAO) or the El Niño-Southern Oscillation (ENSO).

Abrupt climate change One can define abrupt climate change in the time and frequency domain. (a) Time domain: Abrupt climate change refers to a large shift in climate that persists for years or longer, such as marked changes in average temperature, or altered patterns of storms, floods, or droughts, over a widespread area that takes place so rapidly that the natural system has difficulty adapting to it. In the context of past abrupt climate change, "rapidly" typically means on the order of a decade. (b) Frequency domain: An abrupt change means that the characteristic periodicity changes. Also the phase relation between certain climate variables may change in a relatively short time. For both types of changes examples will be provided.

Regime shifts are defined as rapid transitions from one state to another. In the marine environment, regimes may last for several decades, and shifts often appear to be associated with changes in the climate system. If the shifts occur regularly, they are often referred to as an oscillation (e. g., Atlantic Multi-decadal Oscillation, Pacific Decadal Oscillation). Similarly, one can define a regime shift in the frequency domain.

Anthropogenic climate change Beginning with the industrial revolution in the 1850s and accelerating ever since, the human consumption of fossil fuels has elevated CO_2 levels from a concentration of ~ 280 ppm to more than 380 ppm today. These increases are projected to reach more than 560 ppm before the end of the 21st century. As an example, a concomitant shift of ocean circulation would have serious consequences for both agriculture and fishing.

Multiple equilibria Fossil evidence and computer models demonstrate that the Earth's complex and dynamic climate system has more than one mode of operation. Each mode produces different climate patterns. The evidence of models and data analysis shows that the Earth's climate system has sensitive thresholds. Pushed past a threshold, the system can jump from one stable operating mode to a completely different one.

Long-term climate statistics Starting with a given initial state, the solutions $x(t)$ of the equations that govern the dynamics of a non-linear system, such as the atmosphere, result in a set of long-term statistics. If all initial states ultimately lead to the same set of statistical properties, the system is ergodic or transitive. If, instead, there are two or more different sets of statistical properties, where some initial states lead to one set, while the other initial states lead to another, the system is called intransitive (one may call the different states regimes). If there are different sets of statistics that a system may assume in its evolution from different initial states through a long, but finite, period of time, the system is called almost intransitive [50,51,53]. In the transitive case, the equilibrium climate statistics are both stable and unique. Long-term climate statistics will give a good description of the climate. In the almost intransitive case, the system in the course of its evolution will show finite periods during which distinctly different climatic regimes prevail. The almost intransitive case arises because of internal feedbacks, or instabilities involving the different components of the climatic system. The climatic record can show rapid step-like shifts in climate variability that occur over decades or less, including climatic extremes (e. g. drought) that persist for decades.

Feedbacks A perturbation in a system with a negative feedback mechanism will be reduced whereas in a system with positive feedback mechanisms, the perturbation will grow. Quite often, the system dynamics can be reduced to a low-order description. Then, the growth or decay of perturbations can be classified by the systems' eigenvalues or the pseudospectrum. Consider the stochastic dynamical system

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = f(x) + g(x)\xi + F(x,t) \,, \tag{1}$$

where ξ is a stochastic process. The functions f, g describe the climate dynamics, in this case without explicit time dependence. The external forcing $F(x,t)$ is generally time-, variable-, and space-dependent. In his theoretical approach, Hasselmann [32] formulated a linear stochastic climate model

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = Ax + \sigma\xi + F(t) \,, \tag{2}$$

with system matrix $A \in \mathbb{R}^{n\times n}$, constant noise term σ, and stochastic process ξ. Interestingly, many features of the climate system can be well described by (2), which is analogous to the Ornstein–Uhlenbeck process in statistical physics [89]. In the climate system, linear and non-linear feedbacks are essential for abrupt climate changes.

Paleoclimate Abrupt climate change is evident in model results and in instrumental records of the climate system. Much interest in the subject is motivated by the evidence in archives of extreme changes. Proxy records of paleoclimate are central to the subject of abrupt climate change. Available paleoclimate records provide information on many environmental variables, such as temperature, moisture, wind, currents, and isotopic compositions.

Thermohaline circulation stems from the Greek words "thermos" (heat) and "halos" (salt). The ocean is driven to a large extent by surface heat and freshwater fluxes. As the ocean is non-linear, it cannot be strictly separated from the wind-driven circulation. The expressions thermohaline circulation (THC) and meridional overturning circulation (MOC) in the ocean are quite often used as synonyms although the latter includes all effects (wind, thermal, haline forcing) and describes the ocean transport in meridional direction. Another related expression is the ocean conveyor belt. This metaphor is motivated by the fact that the North Atlantic is the source of the deep limb of a global ocean circulation system [10]. If North Atlantic surface waters did not sink, the global ocean circulation would cease, currents would weaken or be redirected. The resulting reorganization would reconfigure climate patterns, especially in the Atlantic Ocean. One fundamental aspect of this circulation is the balance of two processes: cooling of the deep ocean at high latitudes, and heating of deeper levels from the surface through vertical mixing.

Definition of the Subject

The occurrence of abrupt change of climate at various time scales has attracted a great deal of interest for its theoretical and practical significance [2,3,9]. To some extent, a definition of what constitutes an abrupt climatic change depends on the sampling interval of the data being examined [28]. For the instrumental period covering approximately the last 100 years of annually or seasonally sampled data, an abrupt change in a particular climate variable will be taken to mean a statistically highly significant difference between adjacent 10-year sample means. In the paleoclimate context (i. e. on long time scales), an abrupt climate change can be in the order of decades to thousands of years. Since the climate dynamics can be often projected onto a limited number of modes or patterns of climate variability (e. g., [21,22]), the definition of abrupt climate change is also related to spatio-temporal patterns.

The concept of abrupt change of climate is therefore applied for different time scales. For example, changes in climatic regimes were described associated with surface temperature, precipitation, atmospheric circulation in North America during the 1920s and 1960s [19,75]. Sometimes, the term "climate jump" is used instead of "abrupt climate change", e. g. [92]. Flohn [25] expanded the concept of abrupt climate change to include both singular events and catastrophes such as the extreme El Niño of 1982/1983, as well as discontinuities in paleoclimate indices taken from ice cores and other proxy data. In the instrumental record covering the last 150 years, there is a well-documented abrupt shift of sea surface temperature and atmospheric circulation features in the Northern Hemisphere in the mid-1970s, e. g. [22,67,88]. Some of the best-known and best-studied widespread abrupt climate changes started and ended during the last deglaciation, most pronounced at high latitudes.

In his classic studies of chaotic systems, Lorenz has proposed a deterministic theory of climate change with his concept of the "almost-intransitivity" of the highly nonlinear climate systems. In this set of equations, there exists the possibility of multiple stable solutions to the governing equations, even in the absence of any variations in external forcing [51]. More complex models, e. g. [11,20] also demonstrated this possibility. On the other hand, variations in external forcing, such as the changes of incoming solar radiation, volcanic activity, deglacial meltwater, and increases of greenhouse gas concentration have also been proposed to account for abrupt changes in addition to climate intransitivity [9,25,38,41,49]. A particular climate change is linked to the widespread continental glaciation of Antarctica during the Cenozoic (65 Ma to present) at about 34 Ma, e. g. [93]. It should be noted that many facets of regional climate change are abrupt changes although the global means are rather smoothly changing.

Besides abrupt climate change as described in the time domain, we can find abrupt shifts in the frequency domain. A prominent example for an abrupt climate change in the frequency domain is the mid-Pleistocene transition or revolution (MPR), which is the last major "event" in a secular trend towards more intensive global glaciation that characterizes the last few tens of millions of years. The MPR is the term used to describe the transition between 41 ky (ky $= 10^3$ years) and 100 ky glacial-interglacial cycles which occurred about one million years ago (see a recent review in [61]). Evidence of this is provided by high-resolution oxygen isotope data from deep sea cores, e. g. [45,83].

Another example is the possibility of greenhouse gas-driven warming leading to a change in El Niño events. Modeling studies indicate that a strong enhancement of El Niño conditions in the future is not inconceivable [85]. Such a shift would have enormous consequences for both the biosphere and humans. The apparent phase shifts during the 1970s seems unique over this time period, and may thus represent a real climate shift although the available time series is probably too short to unequivocally prove that the shift is significant [90]. The inability to resolve questions of this kind from short instrumental time series provides one of the strongest arguments for extending the instrumental record of climate variability with well-dated, temporally finely resolved and rigorously calibrated proxy data.

Introduction

One view of climate change is that the Earth's climate system has changed gradually in response to both natural and human-induced processes. Researchers became intrigued by abrupt climate change when they discovered striking evidence of large, abrupt, and widespread changes preserved in paleoclimatic archives, the history of Earth's climate recorded in tree rings, ice cores, sediments, and other sources. For example, tree rings show the frequency of droughts, sediments reveal the number and type of organisms present, and gas bubbles trapped in ice cores indicate past atmospheric conditions.

The Earth's climate system is characterized by change on all time and space scales, and some of the changes are abrupt even relative to the short time scales of relevance to human societies. Paleoclimatic records show that abrupt climate changes have affected much or all of the Earth repeatedly over the last ice-age cycle as well as earlier – and these changes sometimes have occurred in periods as short as a few years, as documented in Greenland ice cores. Perturbations at northern high latitudes were spectacularly large: some had temperature increases of up to 10–20°C and a local doubling of precipitation within decades.

In the frequency domain, abrupt climate shifts are due to changes in the dominant oscillations (as in the case of the MPR), or due to a shift in the phase between different climate signals. As an example, the phase between the Indian Monsoon and ENSO exhibits significant shifts for the past 100 years [59].

The period of regular instrumental records of global climate is relatively short (100–200 years). Even so, this record shows many climatic fluctuations, some abrupt or sudden, as well as slow drifts in climate. Climatic changes become apparent on many temporal and spatial scales. Most abrupt climate changes are regional in their spatial extent. However, regional changes can have remote impacts due to atmospheric and oceanic teleconnections. Some of these shifts may be termed abrupt or sudden in that they represent relatively rapid changes in otherwise comparatively stable conditions, but they can also be found superimposed on other much slower climatic changes.

The definition of "abrupt" or "rapid" climate changes is therefore necessarily subjective, since it depends in large measure on the sample interval used in a particular study and on the pattern of longer-term variation within which the sudden shift is embedded. It is therefore useful to avoid a too general approach, but instead to focus on different types of rapid transitions as they are detected and modeled for different time periods. Although distinctions between types are somewhat arbitrary, together they cover a wide range of shifts in dominant climate mode on time scales ranging from the Cenozoic (the last 65 millions of years) to the recent and future climate.

A Mathematical Definition

Time Domain

Abrupt climate change is characterized by a transition of the climate system into a different state (of temperature, rainfall, and other variables) on a time scale that is faster than variations in the neighborhood (in time). Abrupt climate change could be related to a forcing or internally generated. Consider $x(t) \in \mathbb{R}^n$ as a multi-dimensional climate state variable (temperature, rainfall, and other variables). We define an abrupt climate shift of degree ϵ and amplitude B, if

$$\frac{\mathrm{d}}{\mathrm{d}t} x_i(t) \quad \text{can be approximated by a function} \quad \frac{B}{\pi} \frac{\epsilon}{x_i^2 + \epsilon^2} \tag{3}$$

for one $i \in \{1, \ldots, n\}$ in a time interval $[t_1, t_2]$. The case $\epsilon \to 0$ is called instantaneous climate shift, i. e. $x_i(t)$ can be approximated by the Heaviside step function. The degree of approximation can be specified by a proper norm.

An alternative way of defining an abrupt climate shift is through the identification of probable breaks in a time series (e. g., the surface temperature series). The formulation of a two-phase regression (TPR) test, e. g. [55,79], describing a series $x(t)$ is given by

$$x(t) = \mu_1 + \alpha_1 t + \epsilon(t) \quad \text{for } t \le c \tag{4}$$

$$x(t) = \mu_2 + \alpha_2 t + \epsilon(t) \quad \text{for } t > c\,. \tag{5}$$

Under the null hypothesis of no changepoint, the two phases of the regression should be statistically equivalent and both the difference in means $\mu_{1,2}$, and the difference in slopes, $\alpha_{1,2}$, should be close to zero for each possible changepoint c.

In a stochastic framework one may use an appropriate stochastic differential equation (Langevin equation)

$$\frac{\mathrm{d}}{\mathrm{d}t} x(t) = f(x) + g(x)\xi\,, \tag{6}$$

where ξ is a stationary stochastic process and the functions $f, g\colon \mathbb{R}^n \to \mathbb{R}^n$ describe the climate dynamics. Abrupt climate change can be defined as a transition of a short period of time $[t_1, t_2]$, where the probability of an event is larger than a threshold. The properties of the random force are described through its distribution and its correlation properties at different times. In the Ornstein–Uhlenbeck process ξ is assumed to have a Gaussian distribution of zero average,

$$\langle \xi(t) \rangle = 0 \tag{7}$$

and to be δ-correlated in time,

$$\langle \xi(t)\xi(t+\tau) \rangle = \delta(\tau)\,. \tag{8}$$

The brackets indicate an average over realizations of the random force. For a Gaussian process only the average and second moment need to be specified since all higher moments can be expressed in terms of the first two. Note that the dependence of the correlation function on the time difference τ assumes that ξ is a stationary process.

The probability density $p(x, t)$ for the variable $x(t)$ in (6) obeys the Fokker–Planck equation

$$\partial_t p = -\frac{\partial}{\partial x}\left[f(x)p\right] + \frac{\partial}{\partial x}\left[g(x)\frac{\partial}{\partial x}\left\{g(x)p\right\}\right]. \tag{9}$$

Its stationary probability density of (6) is given by

$$p_{st}(x) = \aleph \exp\left(-2\int_{x_0}^{x} \frac{f(y) - g(y)g'(y)}{g(y)^2}\,\mathrm{d}y\right), \tag{10}$$

where $\aleph$ is a normalization constant. $g'(y)$ stands for the derivative of g with respect to its argument. The extrema x_m of the steady state density obey the equation

$$f(x_m) - g(x_m)g'(x_m) = 0 \qquad (11)$$

for $g(x_m) \neq 0$. Here is the crux of the noise-induced transition phenomenon: one notes that this equation is not the same as the equation $f(x_m) = 0$ that determines the steady states of the system in the absence of multiplicative noise. As a result, the most probable states of the noisy system need not coincide with the deterministic stationary states. More importantly, new solutions may appear or existing solutions may be destabilized by the noise. These are the changes in the asymptotic behavior of the system caused by the presence of the noise, e. g. [84].

Climate Variability and Climate Change

The temporal evolution of climate can be expressed in terms of two basic modes: the forced variations which are the response of the climate system to changes in the external forcing $F(x,t)$ (mostly called climate change), and the free variations owing to internal instabilities and feedbacks leading to non-linear interactions among the various components of the climate system [68] (mostly called climate variability). The external causes $F(x,t)$, operate mostly by causing variations in the amount of solar radiation received by or absorbed by the Earth, and comprise variations in both astronomical (e. g. orbital parameters) and terrestrial forcings (e. g. atmospheric composition, aerosol loading). For example, the diurnal and seasonal variations in climate are related to external astronomical forcings operating via solar radiation, while ice ages are related to changes in Earth orbital parameters. Volcanic eruptions are one example of a terrestrial forcing which may introduce abrupt modifications over a period of 2 or 3 years. The more rapid the forcing, the more likely it is that it will cause an abrupt change. The resulting evolution may be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = f(x) + g(x)\xi + F(x,t)\,. \qquad (12)$$

$F(x,t)$ is independent of x if the forcing does not depend on climate (external forcing).

The internal free variations within the climate system are associated with both positive and negative feedback interactions between the atmosphere, oceans, cryosphere and biosphere. These feedbacks lead to instabilities or oscillations of the system on all time scales, and can either operate independently or reinforce external forcings. Investigations of the properties of systems which are far from equilibrium show that they have a number of unusual properties. In particular, as the distance from equilibrium increases, they can develop complex oscillations with both chaotic and periodic characteristics. They also may show bifurcation points where the system may switch between various regimes. Under non-equilibrium conditions, local events have repercussions throughout the whole system. These long-range correlations are at first small, but increase with distance from equilibrium, and may become essential at bifurcation points.

When applying (12), different concepts of climate change are in the literature. Quite often, the dynamics is governed by the following stochastic differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{\mathrm{d}}{\mathrm{d}x}U(x) + \sigma\xi + F(t) \qquad (13)$$

with potential

$$U(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x\,. \qquad (14)$$

If the potential is quadratic and $F(t) = 0$, the Orstein–Uhlenbeck process is retained. In contrast, a bistable non-linear system with two minima in $U(x)$ has been assumed in which shifts between the two distinctly different states are triggered randomly by stochastic forcing, e. g. [7]. In such a system, climate variability and change in the potential can interact due to stochastic resonance [1,7]. Stochastic resonance occurs when the signal-to-noise ratio of a non-linear device is maximized for a moderate value of noise intensity σ. It often occurs in bistable and excitable systems with sub-threshold inputs. For lower noise intensities, the signal does not cause the device to cross threshold, so little signal is passed through it. For large noise intensities, the output is dominated by the noise, also leading to a low signal-to-noise ratio. For moderate intensities, the noise allows the signal to reach threshold, but the noise intensity is not so large as to swamp it.

Strictly speaking, stochastic resonance occurs in bistable systems, when a small periodic force $F(t)$ (which is external) is applied together with a large wide-band stochastic force $\sigma\xi$ (which is internal). The system response is driven by the combination of the two forces that compete/cooperate to make the system switch between the two stable states. The degree of order is related to the amount of periodic function that it shows in the system response. When the periodic force is chosen small enough in order to not make the system response switch, the presence of a non-negligible noise is required for it to happen. When the noise is small very few switches occur, mainly at random with no significant periodicity in the system response. When the noise is very strong a large number of switches occur for each period of the periodic force and the system response does not show remarkable periodicity. Quite surprisingly, between these two conditions, there exists an optimal value of the noise that cooperatively concurs with the periodic forcing in order to make almost exactly one switch per period (a maximum in the signal-to-noise ratio).

Furthermore, non-linear oscillators have been proposed where the timing of the deterministic external forcing is crucial for generating oscillations [51,77,78]. Some aspects of non-equilibrium systems can be found in the climatic system. On the climatological scale, it exhibits abrupt jumps in the long-term rate of temperature change, which are often associated with changes in circulation patterns.

Frequency Domain

In the frequency domain, there are different ways to describe abrupt climate change. A stationary process exhibits an autocovariance function of the form

$$\mathrm{Cov}(\tau) = \langle (x(t+\tau) - \langle x \rangle)(x(t) - \langle x \rangle) \rangle \tag{15}$$

where $\langle\rangle$ denotes the expectation or the statistical mean. Normalized to the variance (i. e. the autocovariance function at $\tau = 0$) one gets the autocorrelation function $C(\tau)$:

$$C(\tau) = \mathrm{Cov}(\tau) / \mathrm{Cov}(0) \,. \tag{16}$$

Many stochastic processes in nature exhibit short-range correlations, which decay exponentially:

$$C(\tau) \sim \exp(-\tau/\tau_0) \,, \quad \text{for } \tau \to \infty \,. \tag{17}$$

These processes exhibit a typical time scale τ_0.

As the frequency domain counterpart of the autocovariance function of a stationary process, one can define the spectrum as

$$S(\omega) = \widehat{\mathrm{Cov}(\tau)} \,, \tag{18}$$

where the hat denotes the Fourier transformation. However, geophysical processes are furthermore often non-stationary. In this regard, the optimal method is continuous wavelet analysis as it intrinsically adjusts the time resolution to the analyzed scale, e. g. [16,59].

Wavelet Spectra A major question concerns the significance testing of wavelet spectra. Torrence and Compo [86] formulated pointwise significance tests against reasonable background spectra. However, Maraun and Kurths [58] pointed out a serious deficiency of pointwise significance testing: Given a realization of white noise, large patches of spurious significance are detected, making it – without further insight – impossible to judge which features of an estimated wavelet spectrum differ from background noise and which are just artifacts of multiple testing. Under these conditions, a reliable corroboration of a given hypothesis is impossible. This demonstrates the necessity to study the significance testing of continuous wavelet spectra in terms of sensitivity and specificity. Given the set of all patches with pointwise significant values, areawise significant patches are defined as the subset of additionally areawise significant wavelet spectral coefficients given as the union of all critical areas that completely lie inside the patches of pointwise significant values. Whereas the specificity of the areawise test appears to be – almost independently of the signal-to-noise ratio – close to one, that of the pointwise test decreases for high background noise, as more and more spurious patches appear [58].

Eigenvalues and Pseudospectrum Another spectral method characterizing the abruptness of climate change is related to the resonance of the linear system (1). As we will see later in the context of atmosphere and ocean instabilities, an eigenvalue analysis is inappropriate in describing the dynamics of the system (12). Inspection of many geophysical systems shows that most of the systems fail the normality condition

$$A\,A^{\dagger} = A^{\dagger}\,A \,, \tag{19}$$

where † denotes the adjoint-complex operator. If a matrix is far from normal, its eigenvalues (the spectrum) have little to do with its temporal evolution [71,87]. More about the dynamics can be learned by examining the pseudospectrum of A in the complex plane. The ϵ-pseudospectrum of operator A is defined by two equivalent formulations:

$$\begin{aligned} \Lambda_\epsilon(A) &= \{ z \in \mathbb{C} : \|(zI - A)^{-1}\| \geq \epsilon^{-1} \} \\ &= \{ z \in \mathbb{C} : [\text{ smallest singular value of} \\ &\qquad\qquad (zI - A)] \leq \epsilon \} \,. \end{aligned} \tag{20}$$

This set of values z in the complex plane are defined by contour lines of the resolvent $(zI - A)^{-1}$. The resolvent determines the system's response to a forcing as supplied by external forcing $F(x, t)$, stochastic forcing $g(x)\xi$, or initial/boundary conditions. The pseudospectrum reflects the robustness of the spectrum and provides information about instability and resonance. One theorem is derived from Laplace transformation stating that transient growth is related to how far the ϵ-pseudospectrum extends into the right half plane:

$$||\exp(A\,t)|| \geq \frac{1}{\epsilon} \sup_{z \in \Lambda_\epsilon(A)} \text{Real}(z)\,. \tag{21}$$

In terms of climate theory, the pseudospectrum indicates resonant amplification. Maximal amplification is at the poles of $(zI - A)^{-1}$, characterized by the eigenfrequencies. In a system satisfying (19), the system's response is characterized solely by the proximity to the eigenfrequencies. In the non-normal case, the pseudospectrum shows large resonant amplification for frequencies which are not eigenfrequencies. This transient growth mechanism is important for both initial value and forced problems.

Earth System Modeling and Analysis

Hierarchy of Models

Modeling is necessary to produce a useful understanding of abrupt climate processes. Model analyses help to focus research on possible causes of abrupt climate change, such as human activities; on key areas where climatic thresholds might be crossed; and on fundamental uncertainties in climate-system dynamics. Improved understanding of abrupt climatic changes that occurred in the past and that are possible in the future can be gained through climate models. A comprehensive modeling strategy designed to address abrupt climate change includes vigorous use of a hierarchy of models, from theory and conceptual models through models of intermediate complexity, to high-resolution models of components of the climate system, to fully coupled earth-system models. The simpler models are well-suited for use in developing new hypotheses for abrupt climate change. Model-data comparisons are needed to assess the quality of model predictions. It is important to note that the multiple long integrations of enhanced, fully coupled Earth system models required for this research are not possible with the computer resources available today, and thus, these resources are currently being enhanced.

Feedback

One particularly convincing example showing that the feedbacks in the climate system are important is the drying of the Sahara about 5000 years before present which is triggered by variations in the Earth's orbit around the sun. Numerous modeling studies, e. g. [31], suggest that the abruptness of the onset and termination of the early to mid-Holocene humid period across much of Africa north of the equator depends on the presence of non-linear feedbacks associated with both ocean circulation and changes in surface hydrology and vegetation, e. g. [18]. Without including these feedbacks alongside gradual insolation forcing, it is impossible for existing models to come even close to simulating the rapidity or the magnitude of climatic change associated with the extension of wetlands and plant cover in the Sahara/Sahel region prior to the onset of desiccation around 5000 years before present.

Climate Archives and Modeling

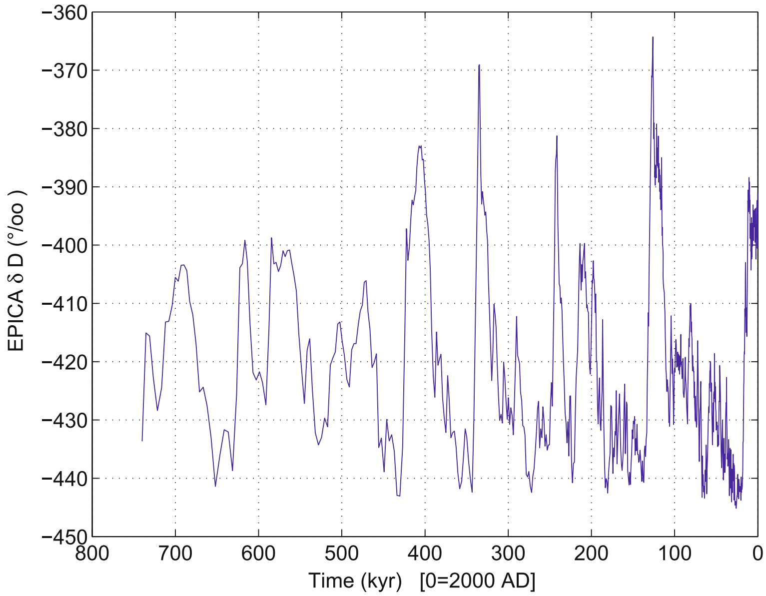
Systematic measurements of climate using modern instruments have produced records covering the last 150 years. In order to reconstruct past variations in the climate system further back in time, scientists use natural archives of climatic and environmental changes, such as ice cores, tree rings, ocean and lake sediments, corals, and historical evidence. Scientists call these records proxies because, although they are not usually direct measures of temperature or other climatic variables, they are affected by temperature, and using modern calibrations, the changes in the proxy preserved in the fossil record can be interpreted in terms of past climate.

Ice core data, coral data, ring width of a tree, or information from marine sediments are examples of a proxy for temperature, or in some cases rainfall, because the thickness of the ring can be statistically related to temperature and/or rainfall in the past. The most valuable proxies are those that can be scaled to climate variables, and those where the uncertainty in the proxy can be measured. Proxies that cannot be quantified in terms of climate or environment are less useful in studying abrupt climate change because the magnitude of change cannot be determined. Quite often, the interpretation of proxy data is already a model of climate change since it involves constraints (dating, representativeness etc.). Uncertainties in the proxies, and uncertainties in the dating, are the main reasons that abrupt climate change is one of the more difficult topics in the field of paleoclimatology.

Example: Glacial-Interglacial Transitions

Astronomical Theory of Ice Ages

Over the past half million years, marine, polar ice core and terrestrial records all highlight the sudden and dramatic nature of glacial terminations, the shifts in global climate that occurred as the world passed from dominantly glacial to interglacial conditions, e. g. [23,69]. These climate transitions, although probably of relatively minor relevance to the prediction of potential future rapid climate change, do provide the most compelling evidence available in the historical record for the role of greenhouse gas, oceanic



Abrupt Climate Change Modeling, Figure 1
Oxygen isotope record from a southern hemisphere ice core [23] showing the glacial-interglacial changes. Note the asymmetry: the state is longer in the cold (glacials) phases than in the warm phases (interglacials)

and biospheric feedbacks as non-linear amplifiers in the climate system. It is such evidence of the dramatic effect of non-linear feedbacks that shows relatively minor changes in climatic forcing may lead to abrupt climate response.

A salient feature of glacial-interglacial climate change is furthermore its asymmetry (Fig. 1). Warmings are rapid, usually followed by slower descent into colder climate. Given the symmetry of orbital forcings $F(t)$, the cause of rapid warming at glacial "terminations" must lie in a climate feedback [37,65]. Clearly, the asymmetric feedback is due to the albedo (reflectance) of ice and snow changing from high values under glacial climates to low values under warm climates. The albedo feedback helps explain the rapidity of deglaciations and their beginnings in spring and summer. Increased absorption of sunlight caused by lower albedo provides the energy for rapid ice melt. The build-up of snow and ice takes much longer than melting.

Many simplified climate models consist of only a few coupled ordinary differential equations controlled by carefully selected parameters. It is generally acknowledged that the "best" models will be those that contain a minimum of adjustable parameters [77] and are robust with respect to changes in those parameters. Rial [72] formulated a logistic-delayed and energy balance model to understand the saw-tooth shape in the paleoclimate record: A fast warming-slow cooling is described by

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = R\left(1 - \frac{x(t-\tau)}{K(t)}\right)x(t-\tau) \qquad (22)$$

$$C\frac{\mathrm{d}}{\mathrm{d}t}T(t) = Q\,(1-\alpha(x)) - (A + BT) \qquad (23)$$

with $x(t)$ for the normalized ice extent, τ time delay, $K(t) = 1 + e(t)T(t)$ carrying capacity, $1/R$ response time of the ice sheet, $T(t)$ global mean temperature, $\alpha(x)$ planetary albedo, external parameter $e(t)$, and $R\tau$ bifurcation parameter. A, B, C, Q are constants for the energy balance of the climate. The equation is calibrated so that for $x(t) = 1$ the albedo $\alpha(x) = 0.3$ and $T(t) = 15$°C. With (23), saw-toothed waveforms and frequency modulation can be understood [72]. The delayed equation yields damped oscillations of $x(t)$ about the carrying capacity for small τ. If τ becomes long compared to the natural response time of the system, the oscillations will become strong, and will grow in amplitude, period and duration. As in the logistic equation for growth, here the product $R\tau$ is a bifurcation parameter, which when crossing the threshold value $\pi/2$ makes the solutions undergo a Hopf bifurcation and settle to a stable limit cycle with fundamental period $\sim 4\tau$ [73].

The astronomical theory of ice ages – also called Milankovitch theory [62] – gained the status of a paradigm for explaining the multi-millennial variability. A key element of this theory is that summer insolation at high latitudes of the northern hemisphere determines glacial-interglacial transitions connected with the waxing and waning of large continental ice sheets, e. g. [33,37], the dominant signal in the climate record for the last million years. Climate conditions of glacials and interglacials are very different. During the Last Glacial Maximum, about 20,000 years before present, surface temperature in the north Atlantic realm was 10–20°C lower than today [13]. A recent study of Huybers and Wunsch [36] has shown that the most simple system for the phase of ice volume $x(t)$ is given by

$$x(t+1) = x(t) + \sigma\xi \qquad (24)$$

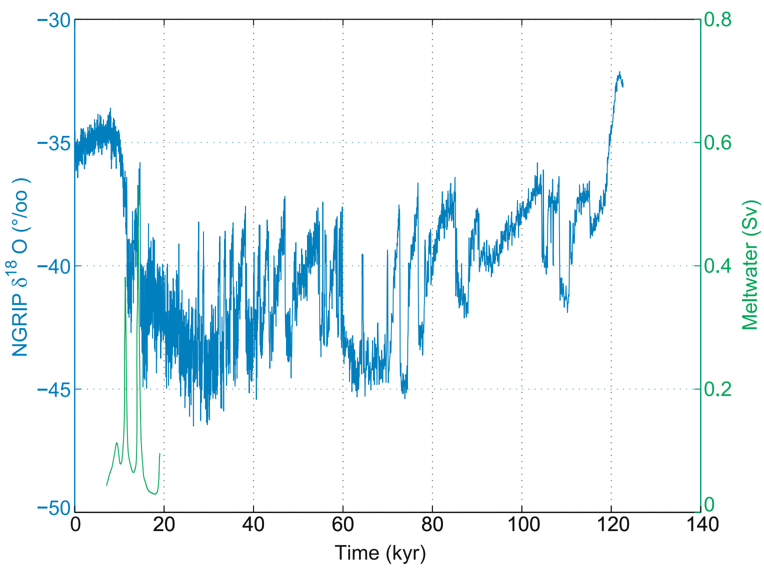
with ξ a Gaussian white noise process, but with mean $\mu = 1$, and $\sigma = 2$. ξ represents the unpredictable background weather and climate variability spanning all time scales out to the glacial/interglacial. This highly simplified model posits 1-ky steps in ice volume $x(t)$. The non-zero mean biases the Earth toward glaciation. Once $x(t)$ reaches a threshold, a termination is triggered, and ice-volume is linearly reset to zero over 10 ky. The following threshold condition for a termination makes it more likely for a termination of ice volume to occur when obliquity $\Theta(t)$ is large:

$$x(t) \geq T_0 - a\Theta(t)\,. \qquad (25)$$

$\Theta(t)$ has a frequency of about 41 ky, and is furthermore normalized to zero mean with unit variance. The other parameters are: amplitude $a = 15$, $T_0 = 105$. Furthermore, the initial ice volume at 700 ky before present is set to $x(t = -700) = 30$. Equation (24) resembles an order-one autoregressive process, similar to (2), plus the threshold condition (25). Models like (24), (25) are not theories of climate change, but rather attempts at efficient kinematic descriptions of the data, and different mechanisms can be consistent with the limited observational records. In the next section, the process of deglaciation is modeled in a three-dimensional model including the spatial dimension.

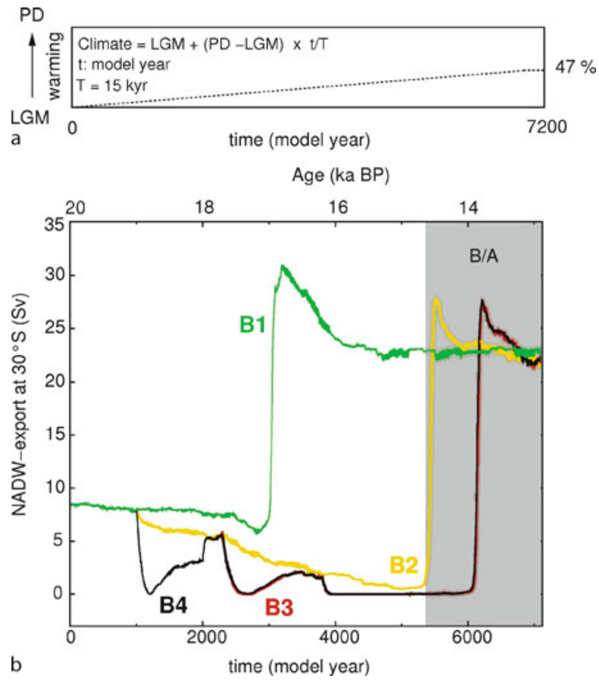
Deglaciation

The question is what causes the abrupt warming at the onset of the Boelling as seen in the Greenland ice cores (Fig. 2). There is a clear antiphasing seen in the deglaciation interval between 20 and 10 ky ago: During the first half of this period, Antarctica steadily warmed, but little change occurred in Greenland. Then, at the time when Greenland's climate underwent an abrupt warming, the warming in Antarctica stopped. Knorr and Lohmann [42], also summarizing numerous modeling studies for deglaciation, describe how global warming (which may be induced by greenhouse gases and feedbacks) can induce a rapid intensification of the ocean cir-



Abrupt Climate Change Modeling, Figure 2
Oxygen isotope record from a Greenland ice core record [64] using an updated time scale for this record [23]. *Green*: Sea-level derived rate of deglacial meltwater discharge [24] which is strong after deglacial warming

culation (Fig. 3). During the Boelling/Alleroed, a sudden increase of the northward heat transport draws more heat from the south, and leads to a strong warming in the north. This "heat piracy" from the South Atlantic has been formulated by Crowley [15]. A logical consequence of this heat piracy is the Antarctic Cold Reversal (ACR) during the Northern Hemisphere warm Boelling/Alleroed. This particular example shows that an abrupt climate change of the ocean circulation (with large climate impacts in the North Atlantic) is related to a smooth global warming. To understand the dynamical behavior of the system, the concept of hysteresis is applied, using the global warming after the last ice ages as the control parameter [42]. The system exhibits multiple steady states (Fig. 4): a weak glacial ocean circulation and a stronger circulation (which is comparable in strength to the modern mode of operation). Deglacial warming induces a transition from a weak glacial THC state to a stronger THC state, characterizing the abrupt warming during the deglaciation.



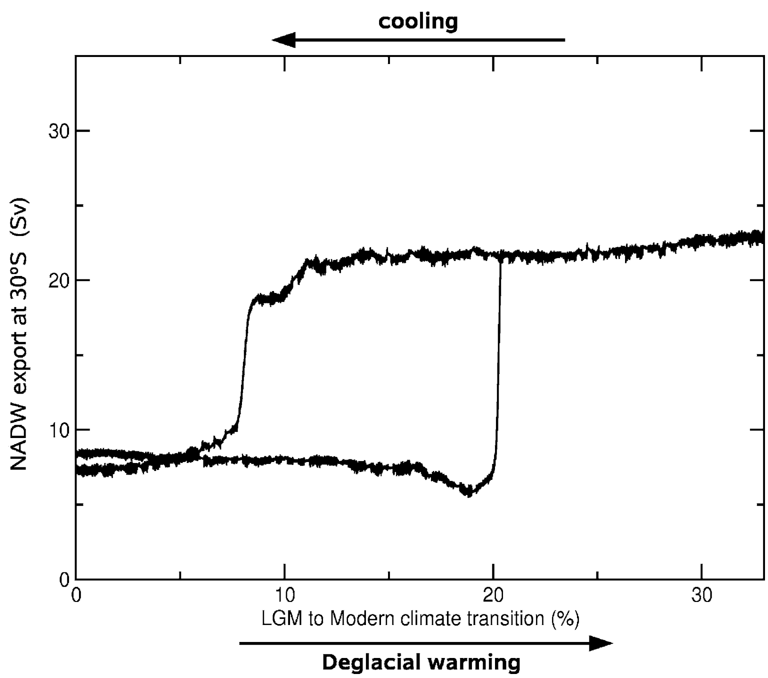
Abrupt Climate Change Modeling, Figure 3
Forcing and model response of the ocean overturning rate. a The background climate conditions are linearly interpolated between glacial (LGM), and modern (PD), conditions. Gradual warming is stopped after 7000 model years, which is related to ∼ 47% of the total warming. b Circulation strength (export at 30°S) versus time. The *green curve* B1 represents the experiment without any deglacial freshwater release to the North Atlantic. Experiments B2 (*yellow curve*), B3 (*red curve*), and B4 (*black curve*), exhibit different successions of deglacial meltwater pulse scenarios to the North Atlantic [42]

Millennial Climate Variability

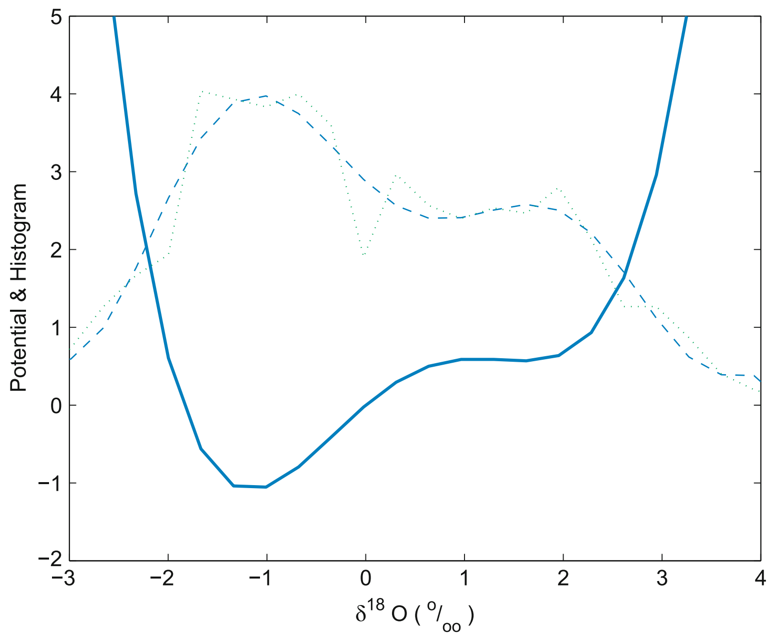
Within glacial periods, and especially well documented during the last one, spanning from around 110 to 11.6 ky ago, there are dramatic climate oscillations, including high-latitude temperature changes approaching the same magnitude as the glacial cycle itself, recorded in archives from the polar ice caps, high to middle latitude marine sediments, lake sediments and continental loess sections. These oscillations are usually referred to as the Dansgaard–Oeschger Cycle and occur mostly on 1 to 2 ky time scales, e. g. [6], although regional records of these transitions can show much more rapid change. The termination of the Younger Dryas cold event, for example, is manifested in ice core records from central Greenland as a near doubling of snow accumulation rate and a temperature shift of around 10°C occurring within a decade with world-wide teleconnections. One hypothesis for explaining these climatic transitions is that the ocean thermohaline circulation flips between different modes, with warm intervals reflecting periods of strong deep water formation in the northern North Atlantic and vice versa [29]. As an alternative approach, one can estimate the underlying dynamics (13), (14) directly from data [43]. The method is based on the unscented Kalman filter, a non-linear extension of the conventional Kalman filter. This technique allows one to consistently estimate parameters in deterministic and stochastic non-linear models. The optimization yields for the coefficients $a_4 = 0.13 \pm 0.01$, $a_3 = -0.27 \pm 0.02$, $a_2 = -0.36 \pm 0.08$, and $a_1 = 1.09 \pm 0.23$. The dynamical noise level of the system σ is estimated to be 2.4. The potential is highly asymmetric and degenerate (that is, close to a bifurcation): there is one stable cold stadial state and one indifferently stable warm interstadial state (Fig. 5). This seems to be related to the fact that the warm intervals are relatively short-lasting.

Coming back to the ice cores and a potential linkage of the hemispheres, Stocker and Johnson [81] proposed a conceptual model linking the isotopic records from Antarctica and Greenland. The basis is an energy balance with temperatures in the North and South Atlantic Ocean, as well as a "southern heat reservoir". It is assumed that the change in heat storage of a "southern heat reservoir" T_S is given by the temperature difference between the reservoir T_S and the Southern Ocean temperature T, with a characteristic time scale τ:

$$\frac{d}{dt} T_S(t) = \frac{1}{\tau} \left[T - T_S \right] \; . \tag{26}$$



Abrupt Climate Change Modeling, Figure 4
Hysteresis loop of the ocean overturning strength (*black curve*) with respect to slowly varying climate background conditions. The transition values are given in % of a full glacial-interglacial transition [42]



Abrupt Climate Change Modeling, Figure 5
Potential derived from the data (*solid*) together with probability densities of the model (*dashed*) and the data (*dotted*)

T_N denotes the time-dependent temperature anomaly of the North Atlantic. The Southern Ocean temperature T is assumed to be $-T_N$ according to the bipolar seesaw (North Atlantic cold $\leftrightarrow$ South Atlantic warm). Using Laplace transform, one can solve for T_S

$$T_S = -\frac{1}{\tau}\int_0^t T_N(t-t')\exp(-t'/\tau)\mathrm{d}t' + T_S(0)\exp(-t/\tau)\,. \tag{27}$$

The reservoir temperature is therefore a convolution of the northern temperature using the time scale τ ranging from 100 to 4000 years. Equation (27) demonstrates that T_S and T_N will have entirely different time characteristics. Abrupt changes in the north appear damped and integrated in time in the southern reservoir. A sudden reduction in the thermohaline circulation causes a cooling in the North Atlantic and a warming in the South, a situation similar to the Younger Dryas period [80], see also Fig. 2.

Example: Cenozoic Climate Cooling

Antarctic Glaciation

During the Cenozoic (65 million years ago (Ma) to present), there was the widespread glaciation of the Antarctic continent at about 34 Ma, e. g. [93]. Antarctic glaciation is the first part of a climate change from relatively warm and certainly ice-free conditions to massive ice sheets in both, the southern and northern hemispheres [44]. Opening of circum-Antarctic seaways is one of the factors that have been ascribed as a cause for Antarctic climate change so far [40,93]. Besides gateway openings, the atmospheric carbon dioxide concentration is another important factor affecting the evolution of the Cenozoic climate [17,93]. As a third component in the long-term evolution of Antarctic glaciation, land topography is able to insert certain thresholds for abrupt ice sheet build-up. Whereas tectonics, land topography, and long-term Cenozoic CO_2-decrease act as preconditioning for Antarctic land ice formation, the cyclicities of the Earth's orbital configuration are superimposed on shorter time scales and may have served as the ultimate trigger and pacemaker for ice-sheet growth at the Eocene-Oligocene boundary around 34 Ma [14].

DeConto and Pollard [17] varied Southern Ocean heat transport to mimic gateway opening instead of an explicit simulation of ocean dynamics. They found a predominating role of pCO_2 in the onset of glaciation instead of a dominating tectonic role for "thermal isolation".

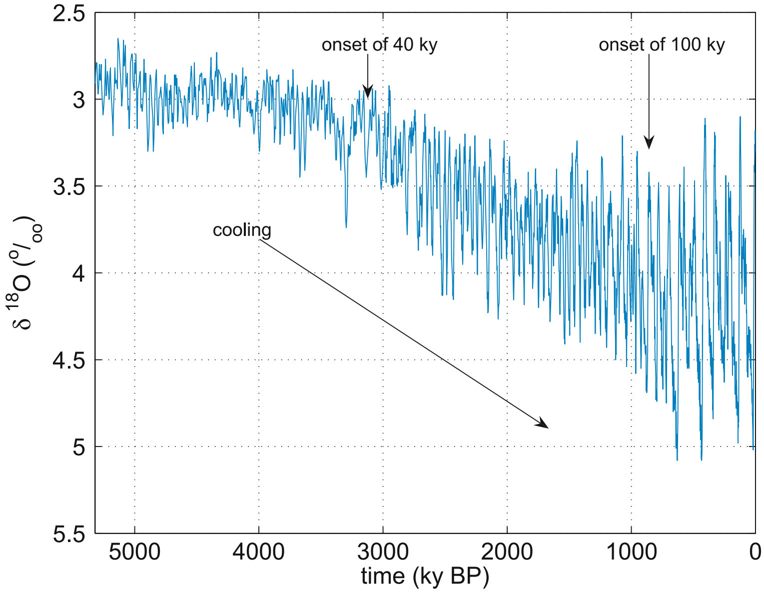
Mid-Pleistocene Revolution

Glaciation in the Northern Hemisphere lagged behind, with the earliest recorded glaciation anywhere in the Northern Hemisphere occurring between 10 and 6 Ma and continuing through to the major increases in global ice volume around 2–3 Ma [60]. A recent compilation of 57 globally distributed records [45] is shown in Fig. 6. Let us focus now on the mid-Pleistocene transition or revolution (MPR), describing the transition from 41 ky to 100 ky glacial-interglacial cycles.

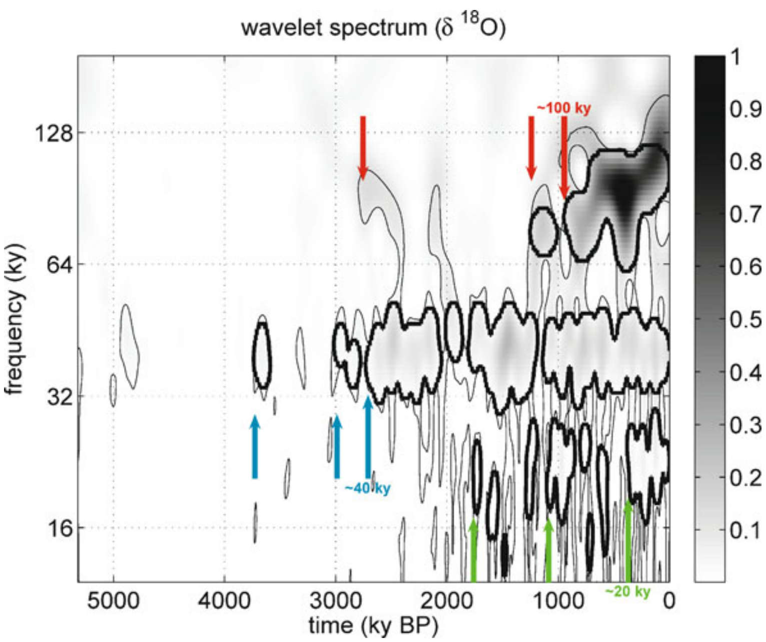
Milankovitch [62] initially suggested that the critical factor was total summer insolation at about 65°N, because for an ice sheet to grow some additional ice must survive each successive summer. In contrast, the Southern Hemisphere is limited in its response because the expansion of ice sheets is curtailed by the Southern Ocean around Antarctica. The conventional view of glaciation is thus that low summer insolation in the temperate North Hemisphere allows ice to survive summer and thus start to build up on the northern continents. If so, how then do we account for the MPR? Despite the pronounced change in Earth system response evidenced in paleoclimatic records, the frequency and amplitude characteristics of the orbital parameters which force long-term global climate change, e. g., eccentricity ($\sim$ 100 ky), obliquity ($\sim$ 41 ky) and precession ($\sim$ 21 and $\sim$ 19 ky), do not vary during the MPR [8]. This suggests that the cause of change in response at the MPR is internal rather than external to the global climate system.

The result of a wavelet spectral analysis (Fig. 7) suggests several abrupt climate changes in the frequency domain (shown as schematic arrows in the figure). These abrupt climate shifts represent major reorganizations in the climate system. Some of them are possibly linked to the development of Northern Hemisphere ice volume. The MPR marked a prolongation to and intensification of the $\sim$ 100 ky glacial-interglacial climate. Not only does the periodicity of glacial-interglacial cycles increase going through the MPR, but there is also an increase in the amplitude of global ice volume variations.

It is likely that the MPR is a transition to a more intense and prolonged glacial state, and associated subsequent rapid deglaciation becomes possible. The first occurrence of continental-scale ice sheets, especially on Greenland, is recorded as ice-rafted detritus released from drifting icebergs into sediments of the mid- and high-latitude ocean. After a transient precursor event at 3.2 Ma, signals of large-scale glaciations suddenly started in the subpolar North Atlantic in two steps, at 2.9 and 2.7 Ma, e. g. [5].



Abrupt Climate Change Modeling, Figure 6
A compilation of 57 globally distributed records by Lisiecki and Raymo [45]: The $\delta^{18}O$ record reflects mainly the climate variables temperature and ice volume



Abrupt Climate Change Modeling, Figure 7
Lisiecki and Raymo [45]: The corresponding wavelet sample spectrum calculated using Morlet wavelet with $\omega_0 = 6$. *Thin* and *thick lines* surround pointwise and areawise significant patches, respectively

The ice volume increase may in part be attributed to the prolonging of glacial periods and thus of ice accumulation. The amplitude of ice volume variation is also accentuated by the extreme warmth of many interglacial periods. Thus, a colder climate with larger ice sheets should have the possibility of a greater sudden warming [45]. The MPR therefore marks a dramatic sharpening of the contrast between warm and cold periods. Note however, that the amount of energy at the 40 ka period is hardly changed in the time after 1 Ma, and notably, one sees the addition of energy at longer periods, without any significant reduction in obliquity-band energy. After about 1 Ma, large glacial-interglacial changes begin to occur on an approximately 100 ka time scale (but not periodically) superimposed upon the variability which continues largely unchanged [91]. Why did 100 ka glacial-interglacials also become possible in addition to the ice volume variability? Lowering of global CO_2 below some critical threshold, or changes in continental configuration, or atmospheric circulation patterns, or all together, are among the conceivable possibilities, e. g. [70].

Examples: Transient Growth

The former examples show the power of the combination of models, data analysis, and interpretation for abrupt climate change. In the next two examples, it is shown how important the transient growth mechanism is for abrupt climate change.

Conceptual Model of the Ocean Circulation

In this section, a category of the non-linear models following the simple thermohaline model of Stommel [82] is analyzed. The common assumption of these box models is that the oceanic overturning rate Φ can be expressed by the meridional density difference:

$$\Phi = -c(\alpha \Delta T - \beta \Delta S) \,, \tag{28}$$

where α and β are the thermal and haline expansion coefficients, c is a tunable parameter, and Δ denotes the meridional difference operator applied to the variables temperature T and salinity S, respectively. Stommel [82] considered a two-box ocean model where the boxes are connected by an overflow at the top and a capillary tube at the bottom, such that the capillary flow is directed from the high-density vessel to the low-density vessel following (28).

The equations for temperature T and salinity S are the heat and salt budgets using an upstream scheme for the advective transport and fluxes with the atmosphere:

$$\frac{\mathrm{d}}{\mathrm{d}t} T = -\frac{\Phi}{V} \Delta T - \frac{F_{\mathrm{oa}}}{\rho_0 c_p h} \tag{29}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} S = -\frac{\Phi}{V} \Delta S - \frac{S_0}{h}(P - E) \,, \tag{30}$$

where V is the volume of the box with depth h, and $(P-E)$ denotes the freshwater flux (precipitation minus evaporation plus runoff). F_{oa} is the heat flux at the ocean-atmosphere interface, S_0 is a reference salinity, and $\rho_0 c_p$ denotes the heat capacity of the ocean.

Denoting furthermore $x \in \mathbb{R}^2$ for the anomalies of $(\Delta T, \Delta S)$, Lohmann and Schneider [48] have shown the evolution equation is of the following structure:

$$\frac{\mathrm{d}}{\mathrm{d}t} x = Ax + \langle b | x \rangle \, x \,. \tag{31}$$

The brackets $\langle | \rangle$ denote the Euclidean scalar product. This evolution Equation (31) can be transferred to a

$$x(t) = \frac{1}{\gamma(t)} \exp(At) x_0 \,, \tag{32}$$

with a scaling function $\gamma(t, x_0)$. The models of Stommel [82], and many others are of this type, and their dynamics are therefore exactly known.

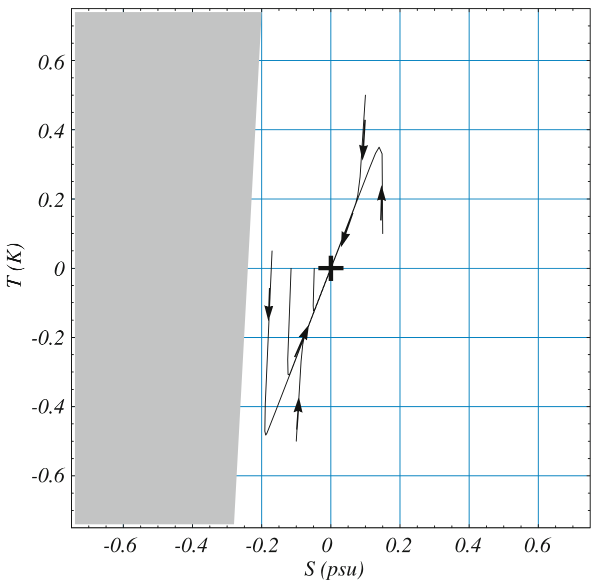
It is worth knowing that (29), (30) is equivalent to the multi-dimensional Malthus–Verhulst model (also known as the logistic equation), which was originally proposed to describe the evolution of a biological population. Let x denote the number (or density) of individuals of a certain population. This number will change due to growth, death, and competition. In the simplest version, birth and death rates are assumed proportional to n, but accounting for limited resources and competition it is modified by $(1 - x)$:

$$\frac{\mathrm{d}}{\mathrm{d}t} x(t) = a(1 - x)x \,. \tag{33}$$

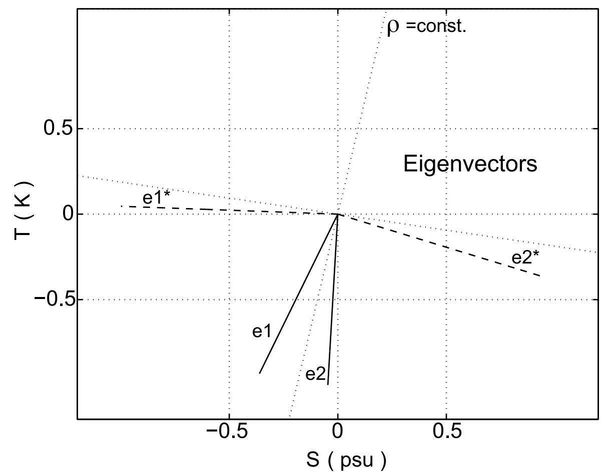
In climate, the logistic equation is important for Lorenz's [52] error growth model: where $x(t)$ is the algebraic forecast error at time t and a is the linear growth rate.

It is useful to analyze the dynamics in the phase space spanned by temperature and salinity anomalies and investigate the model sensitivity under anomalous high latitude forcing, induced as an initial perturbation. The lines in Fig. 8 are phase space trajectories after perturbations of different magnitude have been injected into the North Atlantic. We notice that for most trajectories, the distances from zero $(0, 0)$ increase temporarily, where the maximal distance from zero is after a decade. After about 10 years the trajectories in Fig. 8 point into a "mixed temperature/salinity direction", denoted further as e_1.

Figure 8 implies that the adjustment of the THC involves two phases: A fast thermal response and a slower response on the e_1-direction. The vector e_1 is identical with the most unstable mode in the system. Because the scaling function $\gamma(t)$ acts upon both temperature and salinity (32), the evolution of the non-linear model can be well characterized by the eigenvectors of the matrix A, which is discussed in the following.



Abrupt Climate Change Modeling, Figure 8
The basin of attraction (*white area*) and the dynamics in the thermohaline phase space. With initial conditions outside the gray area, the trajectories converge asymptotically to the origin corresponding to the thermally driven solution of the THC. Due to the fast thermal response during the first decade of relaxation, the distance of the trajectories from zero can increase temporarily



Abrupt Climate Change Modeling, Figure 9
Eigenvectors e_1, e_2, and adjoint eigenvectors e_1^*, e_2^* of the tangent linear operator $A^\dagger$. The *dotted lines* show the line of constant density and the perpendicular

In our system, the operator A of the box model is found to be non-normal, and the eigenvectors are not orthogonal. One eigenvalue (e_2) is closely related to temperature anomalies, whereas the other (e_1) is a "mixed temperature/salinity eigenvector" (Fig. 9). The eigenvectors of the adjoint matrix $A^\dagger$ are denoted by e_1^* and e_2^*, respectively. For the non-normal matrix A, the eigenvectors of A and $A^\dagger$ do not coincide, but fulfill the "biorthogonality condition":

$$e_1^* \perp e_2 \quad \text{and} \quad e_2^* \perp e_1 \,. \tag{34}$$

Both eigenvalues $\lambda_{1,2}$ are real and negative. Because of $\lambda_2 < \lambda_1$, the first term dominates for long time scales and the second for short time scales. Using the biorthogonality condition, we get furthermore the coefficients

$$c_i = \frac{\langle e_i^* | x_0 \rangle}{\langle e_i^* | e_i \rangle} \qquad \text{for } i = 1, 2 \,. \tag{35}$$

A perturbation is called "optimal", if the initial error vector has minimal projection onto the subspace with fastest decaying perturbations, or equivalently if the coefficient c_1 is maximal. This is according to (35) equivalent to x_0 pointing into the direction of e_1^*. This unit vector e_1^* is called the "biorthogonal" [66] to the most unstable eigenvector e_1 which we want to excite. In order to make a geometrical picture for the mathematical considerations, assume that the tail of the vector x_0 is placed on the e_1-line and its tip on the e_2-line. This vector is stretched maximally because the tail decays to zero quickly, whereas the tip is hardly unchanged due to the larger eigenvalue λ_1. The most unstable mode e_1 and its biorthogonal e_1^* differ greatly from each other, and the perturbation that optimally excites the mode bears little resemblance to the mode itself.

It is remarkable that the optimal initial perturbation vector e_1^* does not coincide with a perturbation in sea surface density at high latitudes, which would reside on the dotted line perpendicular to $\rho = \text{const}$ in Fig. 9. Even when using a space spanned by $(\alpha T, \beta S)$ instead of (T, S), to take into account the different values for the thermal and haline expansion coefficients, vector e_1^* is much more dominated by the scaled salinity anomalies than the temperature anomalies of the high latitudinal box.

Numerical simulations by Manabe and Stouffer [57] showed, for the North Atlantic, that between two and four times the preindustrial CO_2 concentration, a threshold value is passed and the thermohaline circulation ceases completely. One other example of early Holocene rapid climate change is the "8200-yr BP" cooling event recorded in the North Atlantic region possibly induced by freshwater. One possible explanation for this dramatic regional cooling is a shutdown in the formation of deep water in the northern North Atlantic due to freshwater input caused by catastrophic drainage of Laurentide lakes [4,46]. The theoretic considerations and these numerical experiments suggest that formation of deep water in the North Atlantic is highly sensitive to the freshwater forcing.

Resonance in an Atmospheric Circulation Model

An atmospheric general circulation model PUMA [26] is applied to the problem. The model is based on the multi-level spectral model described by Hoskins and Simmons [35]. For our experiments we chose five vertical levels and a T21 horizontal resolution. PUMA belongs to the class of models of intermediate complexity [12]; it has been used to understand principle feedbacks [56], and dynamics on long time scales [76]. For simplicity, the equations are scaled here such that they are dimensionless. The model is linearized about a zonally symmetric mean state providing for a realistic storm track at mid-latitudes [27]. In a simplified version of the model and calculating the linear model A with $n = 214$, one can derive the pseudospectrum. Figure 10 indicates resonances besides the poles (the eigenvalues) indicated by crosses. The Im(z)-axis shows the frequencies, the Re(z)-axis the damping/amplification of the modes. Important modes for the climate system are those with $-0.5 < \text{Im}(z) < 0.5$ representing planetary Rossby waves. The basic feature is that transient growth of initially small perturbations can occur even if all the eigenmodes decay exponentially. Mathematically, an arbitrary matrix A can be decomposed as a sum

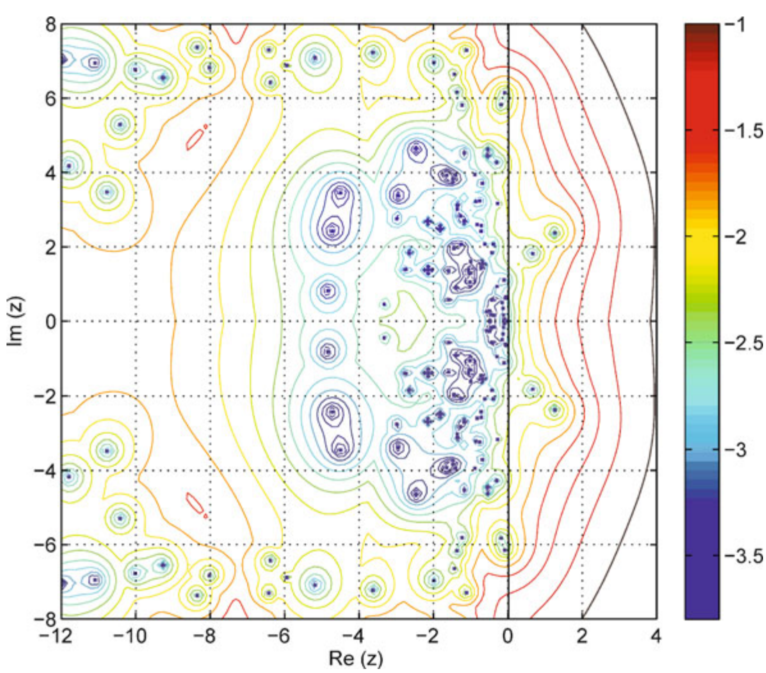
$$A = D + N \tag{36}$$

where A is diagonalizable, and N is nilpotent (there exists an integer $q \in \mathbb{N}$ with $N^q = 0$), and D commutes with N (i. e. $DN = NA$). This fact follows from the Jordan–Chevalley decomposition theorem. This means we can compute the exponential of (A t) by reducing to the cases:

$$\exp(At) = \exp((D + N)t) = \exp(Dt)\exp(Nt) \tag{37}$$

where the exponential of Nt can be computed directly from the series expansion, as the series terminates after a finite number of terms. Basically, the number $q \in \mathbb{N}$ is related to the transient growth of the system ($q = 1$ means no transient growth).

The resonant structures are due to the mode interaction: It is not possible to change one variable without the others, because they are not orthogonal. Interestingly, one can also compute the $A^\dagger$ model, showing the optimal



Abrupt Climate Change Modeling, Figure 10
Contours of $\log_{10}(1/\epsilon)$. The figure displays resonant structures of the linearized atmospheric circulation model. The modes extend to the right half plane and are connected through resonant structures, indicating the transient growth mechanism inherent in atmospheric dynamics

perturbation of a mode e_i through its biorthogonal vector (35).

The analysis indicates that non-normality of the system is a fundamental feature of the atmospheric dynamics. This has consequences for the error growth dynamics, and instability of the system, e. g. [48,66]. Similar features are obtained in shear flow systems [71,87] and other hydrodynamic applications. This transient growth mechanism is important for both initial value and forced problems of the climate system.

Future Directions

Until now, details of abrupt climate change are not well known to accurately predict it. With better information, the society could take more confident action to reduce the potential impact of abrupt changes on agriculture, water resources, and the built environment, among other impacts. A better understanding of sea-ice and glacier stability, land-surface processes, and atmospheric and oceanic circulation patterns is needed. Moreover, to effectively use any additional knowledge of these and other physical processes behind abrupt climate change, more sophisticated ways of assessing their interactions must be developed, including:

Better models. At present, the models used to assess climate and its impacts cannot simulate the size, speed, and extent of past abrupt changes, let alone predict future abrupt changes. Efforts are needed to improve how the mechanisms driving abrupt climate change are represented in these models and to more rigorously test models against the climate record.

More theory. There are concepts to find the underlying dynamical system, to derive a theory from a high-order to low-order description similar to what is done in statistical physics (Mori–Zwanzig approach [63,94], Master equation), or in stochastic differential equations. A systematic reduction of the complex system into fewer degrees of freedom shall bring a deeper level of understanding about the underlying physics. A systematic approach was suggested by Saltzman [77]. Spectral and pseudo-spectral concepts have not been used too much in climate theory. There is a variety of phenomenological stochastic models in which non-linearity and fluctuations coexist, and in which this coexistence leads to interesting phenomena that would not arise without the complex interplay.

Paleoclimatic data. More climate information from the distant past would go a long way toward strengthening our understanding of abrupt climate changes and models of past climate. In particular, an enhanced effort is needed to expand the geographic coverage, temporal resolution, and variety of paleoclimatic data. Although the present climate has no direct analogon to the past [54], the dynamical interpretation of data will improve the understanding of thresholds and non-linearities in the Earth system.

Appropriate statistical tools. Because most statistical calculations at present are based on the assumption that climate is not changing but is stationary, they have limited value for non-stationary (changing) climates and for climate-related variables that are often highly skewed by rapid changes over time such as for abrupt-change regimes. Available statistical tools themselves need to be adapted or replaced with new approaches altogether to better reflect the properties of abrupt climate change.

Synthesis. Physical, ecological, and human systems are complex, non-linear, dynamic and imperfectly understood. Present climate change is producing conditions outside the range of recent historical experience and observation, and it is unclear how the systems will interact with and react to further climate changes. Hence, it is crucial to be able to better understand and recognize abrupt climate changes quickly. This capability will involve improved monitoring of parameters that describe climatic, ecological, and economic systems. Some of the desired data are not uniquely associated with abrupt climate change and, indeed, have broad applications. Other data take on particular importance because they concern properties or regions implicated in postulated mechanisms of abrupt climate change. Research to increase our understanding of abrupt climate change should be designed specifically within the context of the various mechanisms thought to be involved. Focus is required to provide data for process studies from key regions where triggers of abrupt climate change are likely to occur, and to obtain reliable time series of climate indicators that play crucial roles in the postulated mechanisms. Observations could enable early warning of the onset of abrupt climate change. New observational techniques and data-model comparisons will also be required.

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