Climate System II

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2020/2021

Lecture 9: Climate diagnostics, EBM
With help of Christian Stepanek

Oral exam (ca. 25 min)

Stimmabgaben zur Umfrage

	Februar 2021			März 2021		
	Di 23	Mi 24	Do 25	Mo 1	Di 2	Mo 8
Nikita kasrekar					✓	
JAMS NOUFAL C A				(✓)	(✓)	✓
Benjamin Owusu	✓	✓	✓	✓	✓	✓
Costanza Civale				(✓)	✓	✓
Richard Alawode	✓	✓	✓			
Arunabh						✓
Nandu					(✓)	✓
Stanley Agu					✓	✓
Ihr Name	✓ (✓) ⊘	✓ (✓) ⊘	✓ (✓) ⊘	✓ (✓) ⊘	✓ (✓) ⊘	✓ (✓) ⊘
Insgesamt 8 Stimmen	2	2	2	1 (+2)	4 (+2)	★6

Speichern

https://terminplaner.dfn.de/ojwbkEYs4pWOsBrG

Repeat

Configuration of the Earth‘s orbit 9000 years ago

Today:
Perihelion in January

Tilt of the earth‘s axis:
23.5°

9000 years ago::
Perihelion in July

Tilt of the earth‘s axis:
24.0°

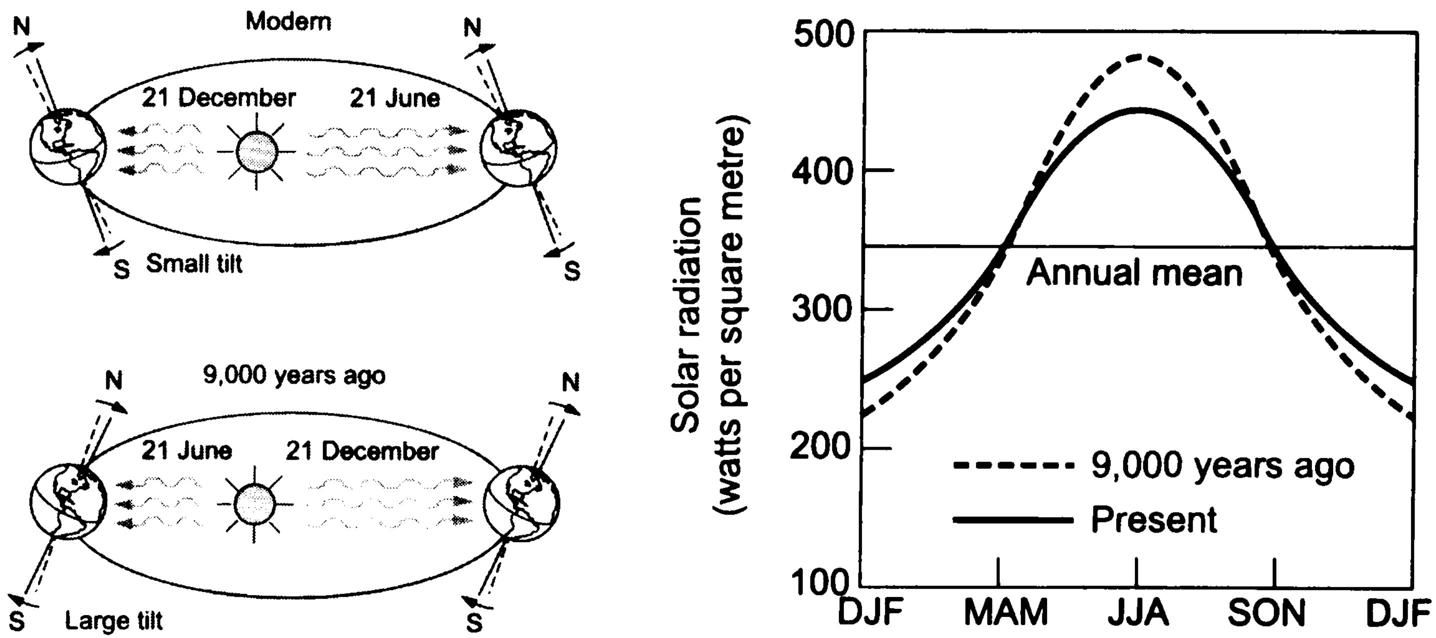
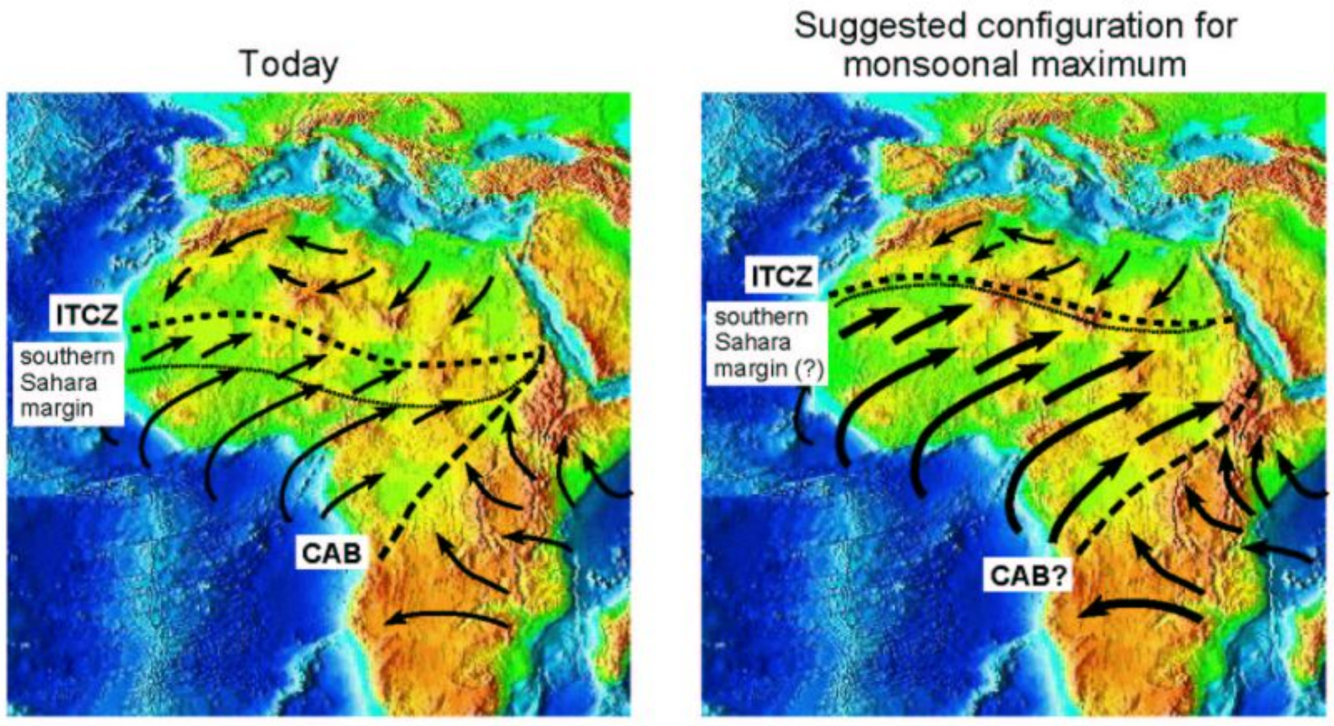


Fig. 5.19: Changes in the **Earth's elliptical orbit** from the present configuration to **9,000 years ago**.(left)

Changes in the average **solar radiation during the year** over the **northern hemisphere** (right).
The incoming solar energy averaged over the northern hemisphere was ca. 7 % greater in July
and correspondingly less in January.

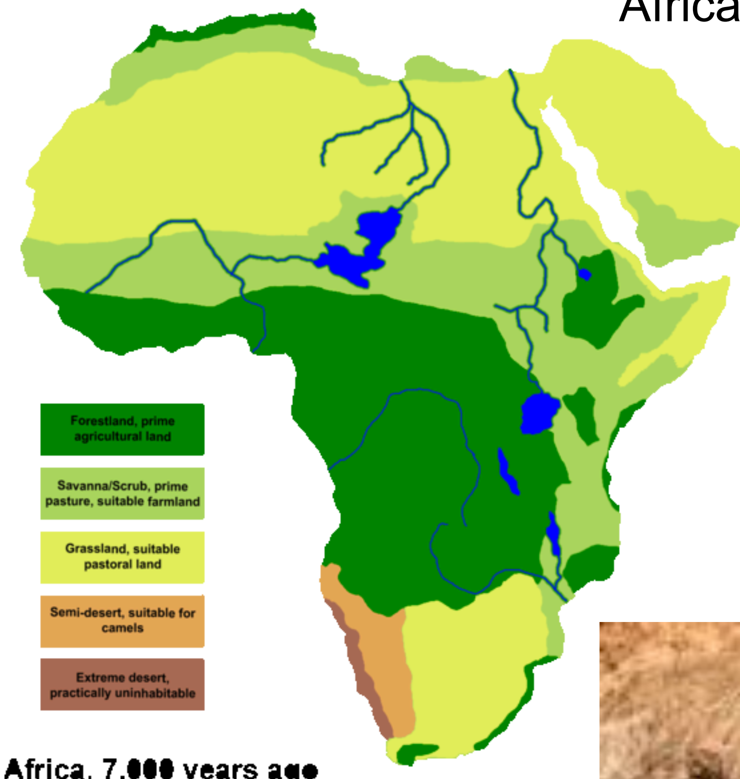
Urquelle:J.E. Kutzbach in „Climate System Modelling“ (1992). Quelle:/ Houghton, J.: „ Global Warming“ (1997), fig 5.19; p.82/

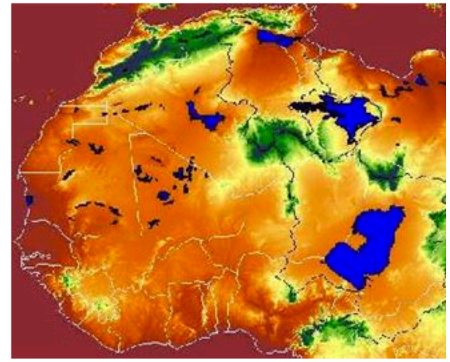
Precession: Effect on climate

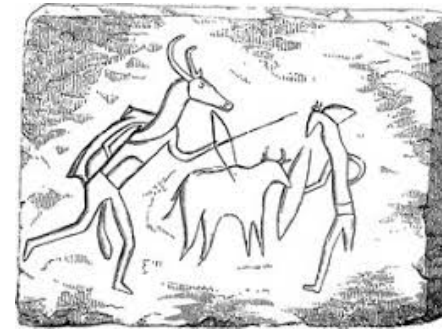


Rough locations of the Intertropical Convergence Zone (ITCZ), the Congo Air Boundary (CAB), and the southen margin of the Sahara Desert for the present-day, and for the monsoonal maximum.

African Humid Period (AHP)

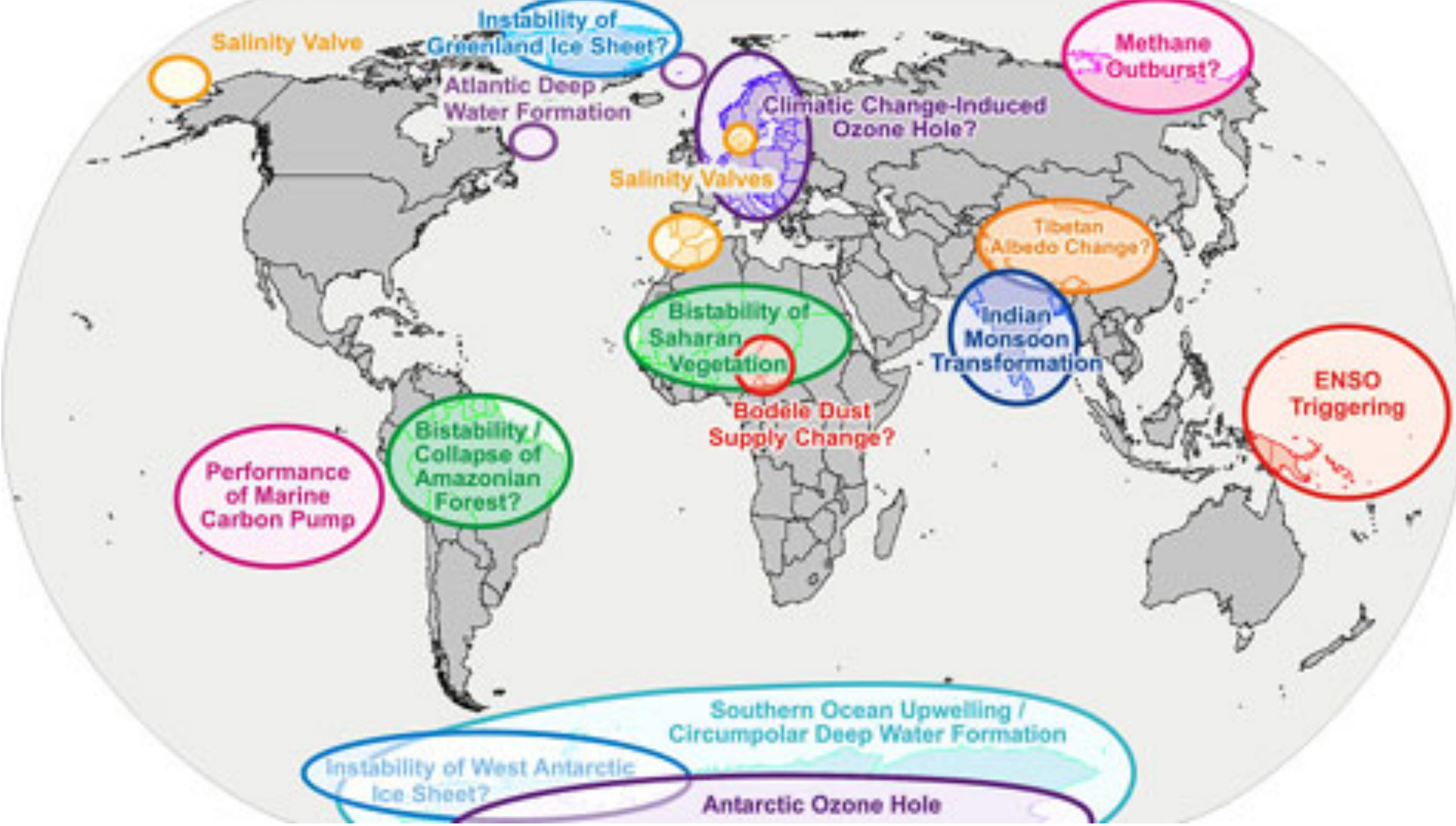






Exercise 1

The obliquity is the angle between an object's rotational axis and its orbital axis. Earth's obliquity oscillates between 22.1 and 24.5 degrees on a 41,000-year cycle; the earth's mean obliquity is currently 23.4 degrees and decreasing. The Earth radius is 6,371 km. How many meters per year is the movement of the Tropic of Cancer due to obliquity changes?

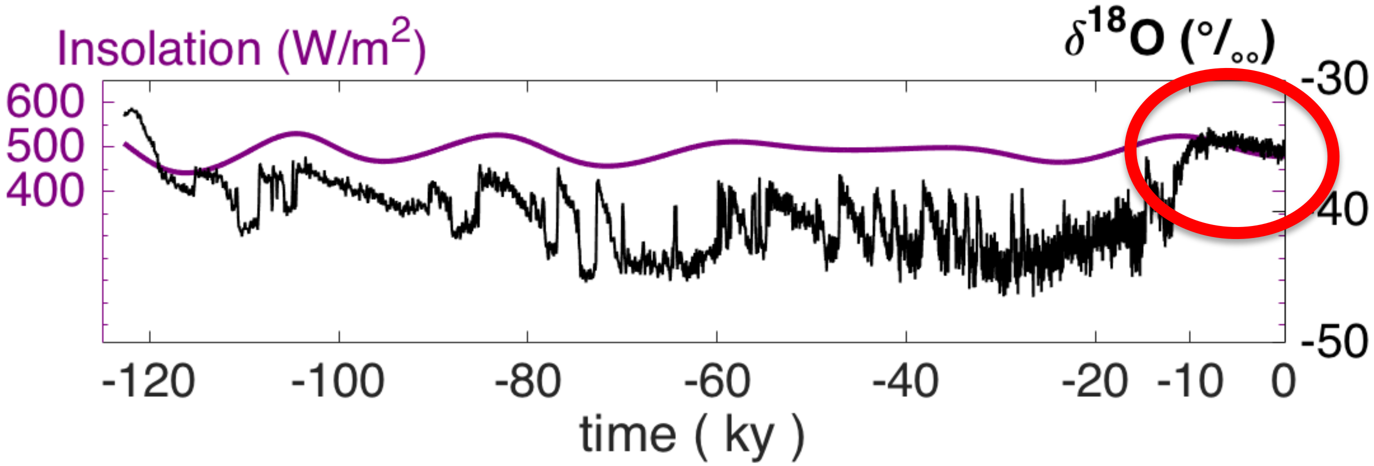


Map of potential policy-relevant tipping elements in the climate system, and overlain on global population density. Subsystems indicated could exhibit threshold-type behavior in response to anthropogenic climate forcing, where a small perturbation at a critical point qualitatively alters the future fate of the system. We exclude from the map systems in which any threshold appears inaccessible this century (e.g., East Antarctic Ice Sheet) or the qualitative change would appear beyond this millennium (e.g., marine methane hydrates). Question marks indicate systems whose status as tipping elements is particularly uncertain.

Abrupt Changes: Millennial variability

Local insolation
60° N June

Greenland ice core
“temperature“



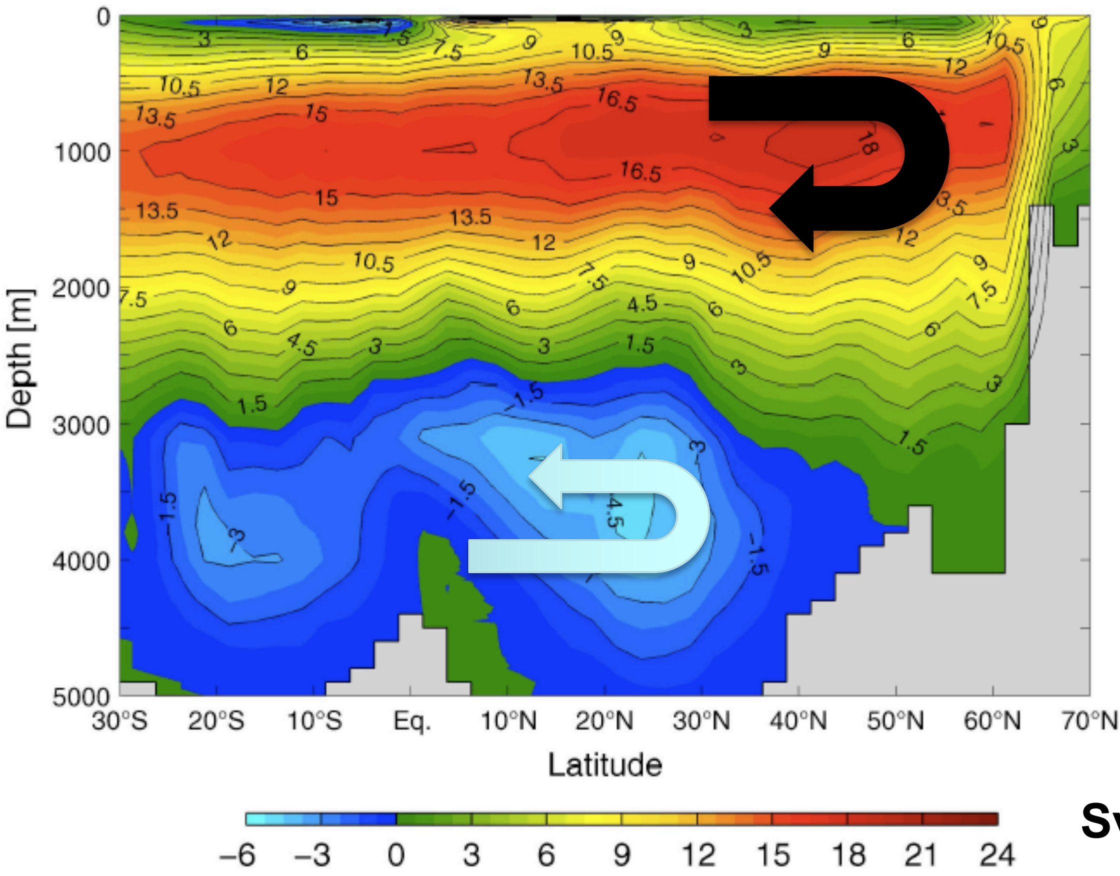
Holocene

Difference between stable Holocene and unstable glacial can not be explained by insolation

NGRIP members, 2004

Meridional overturning circulation

Atlantic Ocean deep sea circulation



NADW: 18 Sv

AABW: 4 Sv

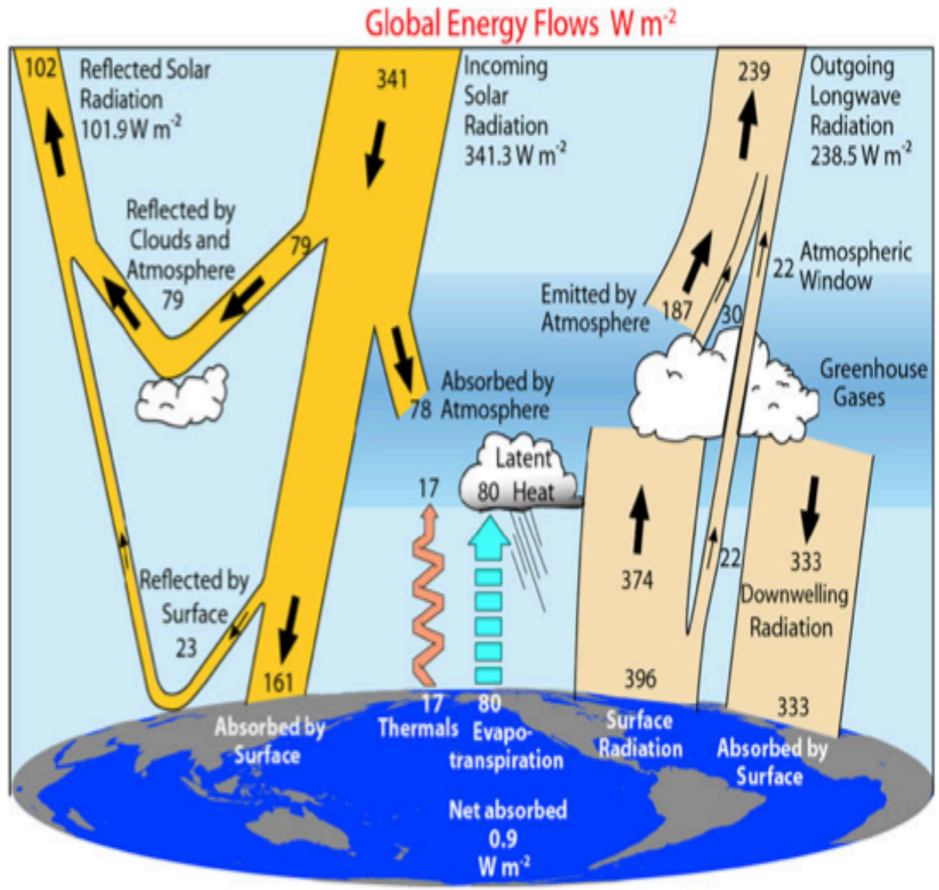
Sv=10^6 m^3/s

boring for the hundredth time

Energy Budget

CHANGE IN STORAGE = IN – OUT

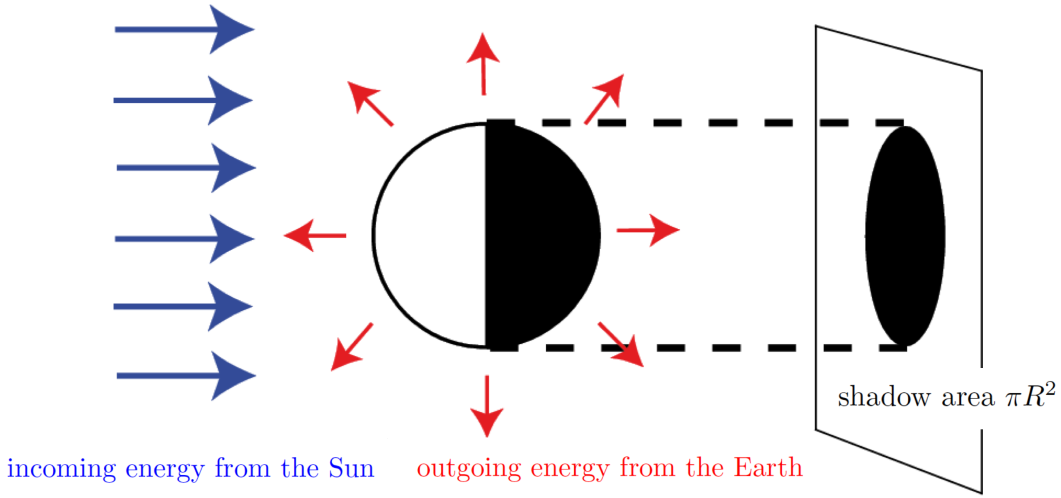
- many papers discuss an imbalance in this equation, which results in missing energy



(Trenberth & Fasullo, 2012)

Energy balance model

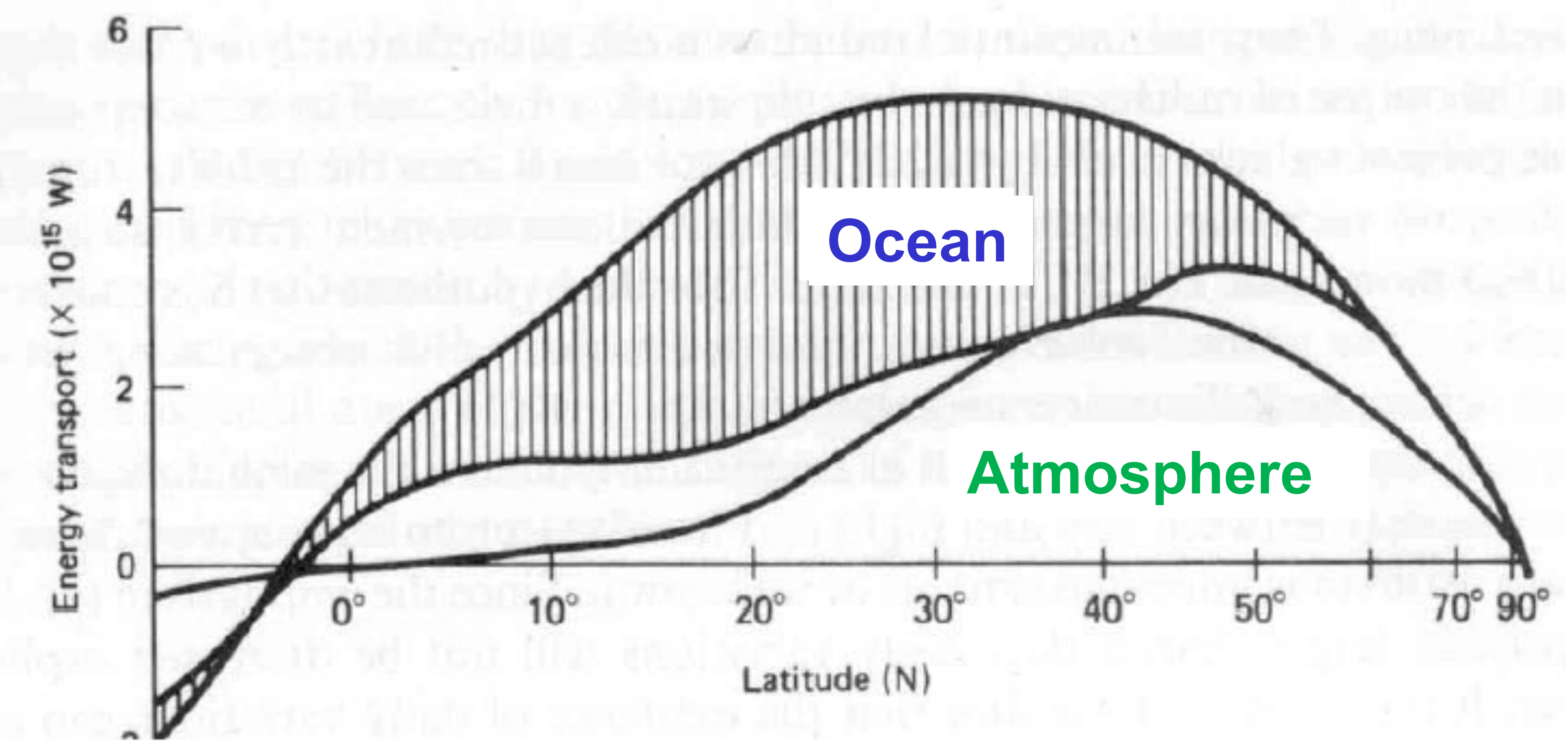
$$(1-\alpha)S\pi R^2 = 4\pi R^2 \epsilon\sigma T^4$$



$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

boring for the hundredth time, but …

Northward Heat Transport



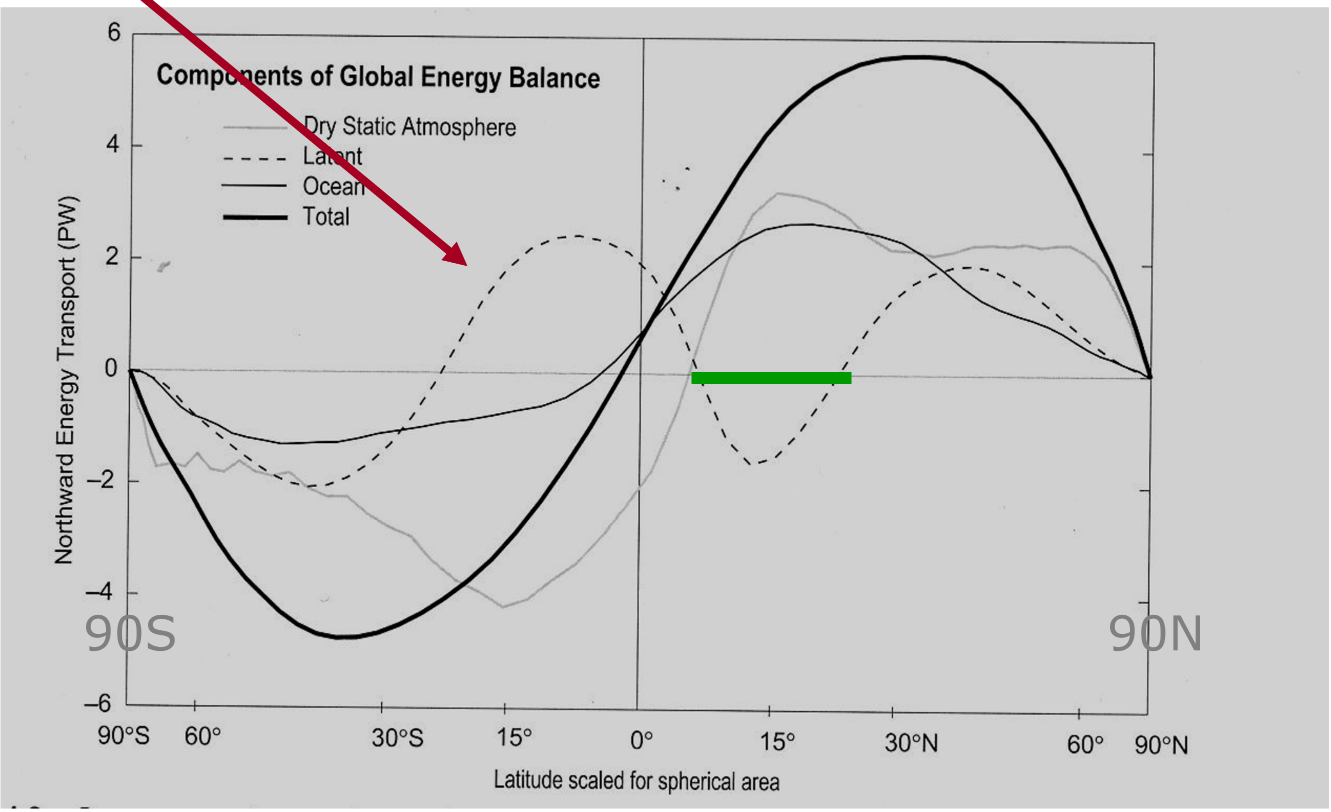
nach Von der Haar & Ort; Quelle: Gill

Global meridional heat transport divides roughly equally into 3 modes:

1. atmosphere (dry static energy)
2. ocean (sensible heat)
3. water vapor/latent heat transport

The three modes of poleward transport are comparable in amplitude, and distinct in character (sensible heat flux divergence focused in tropics, latent heat flux divergence focus in the subtropics)

(residual method, TOA radiation 1985-89 and ECMWF/NMC atmos obs)



$$C_p \partial_t T = \nabla \cdot HT + (1-\alpha) S(\varphi, t) - \epsilon \sigma T^4$$

$$HT = -k \nabla T$$

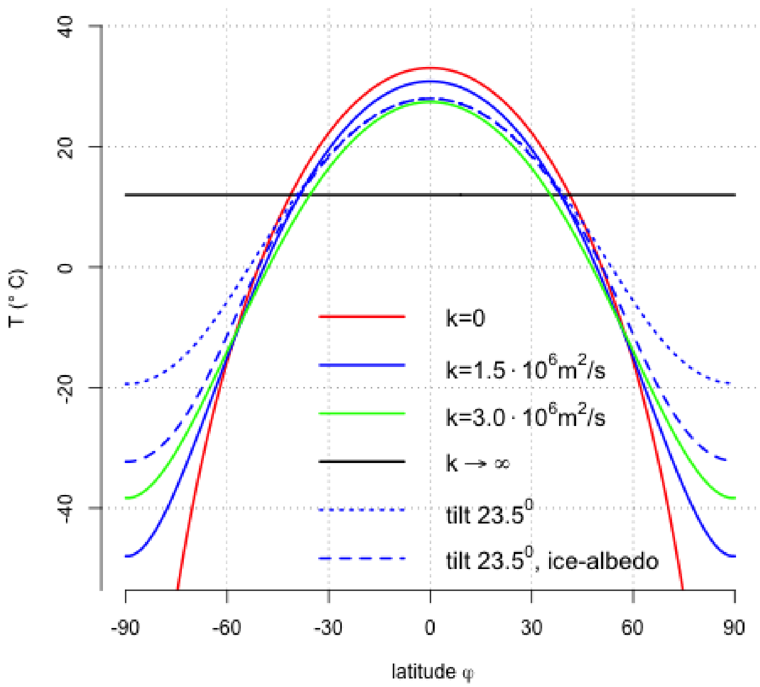


Figure 4. Equilibrium temperature of (15) using different diffusion coefficients. The blue lines use $1.5 \cdot 10^6 m^2/s$ with no tilt (solid line), a tilt of 23.5° (dotted line), and as the dashed line a tilt of 23.5° and ice-albedo feedback using the respresentation of Sellers (1969). Except for the dashed line, the global mean values are identical to the value calculated in (12). Units are °C.

In the exercise, long-wave radiation as A + BT

practical

Exercise 1

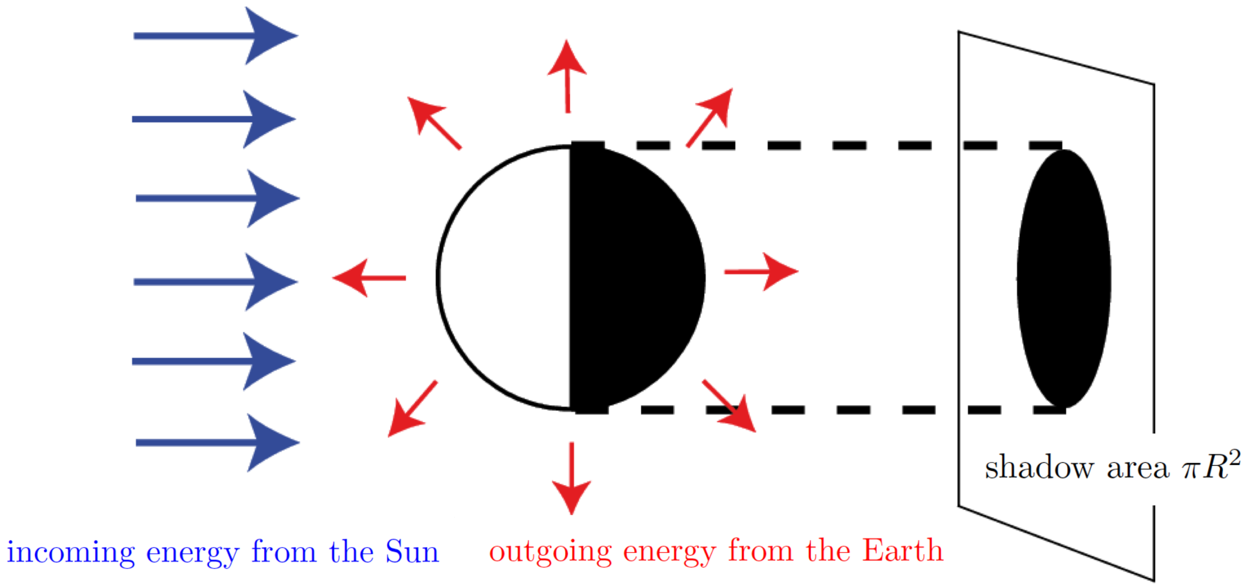
EBM analysis

- https://1drv.ms/u/s!AnZSDMNwdkDMgbx6sr3gVubSIqlYVw?e=MacPeK

Energy balance model

$$(1-\alpha)S\pi R^2 = 4\pi R^2 \epsilon\sigma T^4$$

$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$



Simple models are helpful for understanding of

- Critical parameters
- Concepts of climate

Incoming radiation

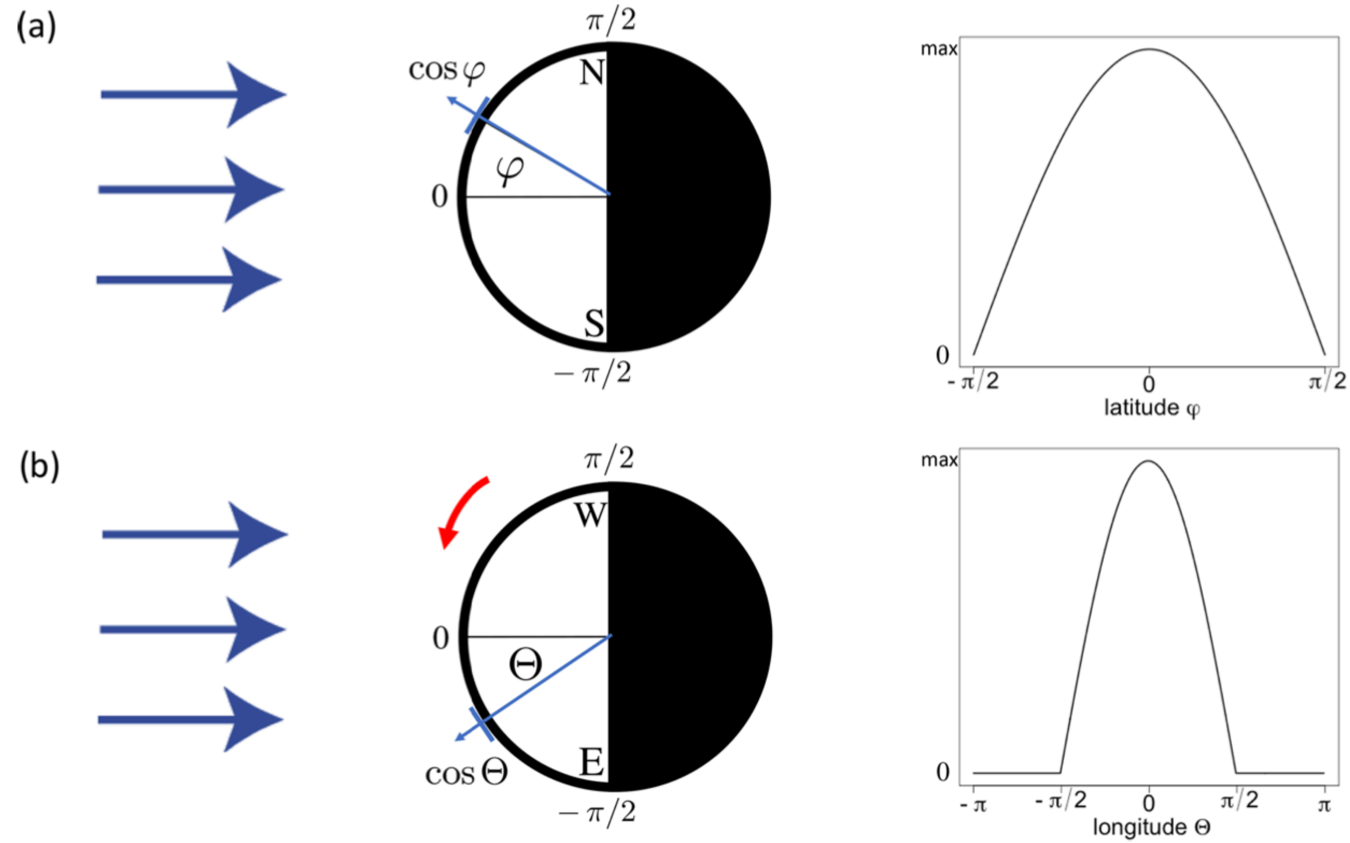


Figure 2. Latitudinal **(a)** and longitudinal **(b)** dependence of the incoming shortwave radiation. On the right-hand side, the insolation as a function of latitude φ and longitude Θ with maximum insolation $(1-\alpha)S$ is shown. See the text for the details.

What we really want is the mean of the temperature $\overline{T}$.
fourth root of (4):

$$T = \sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}} \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$$

$$T(\varphi) = \frac{\sqrt{2}}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\Theta)^{1/4} d\Theta}_{\sqrt{\pi}\Gamma(5/8)/\Gamma(9/8)} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} (\cos\varphi)^{1/4}$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}} \frac{\Gamma(5/8)}{\Gamma(9/8)}}_{\approx 0.608} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} (\cos\varphi)^{1/4}$$

When we integrate this over the latitudes, we obtain

$$\overline{T} = \frac{1}{2}\int_{-\pi/2}^{\pi/2} T(\varphi)\cos\varphi\, d\varphi$$

$$= \frac{1}{2}\frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}$$

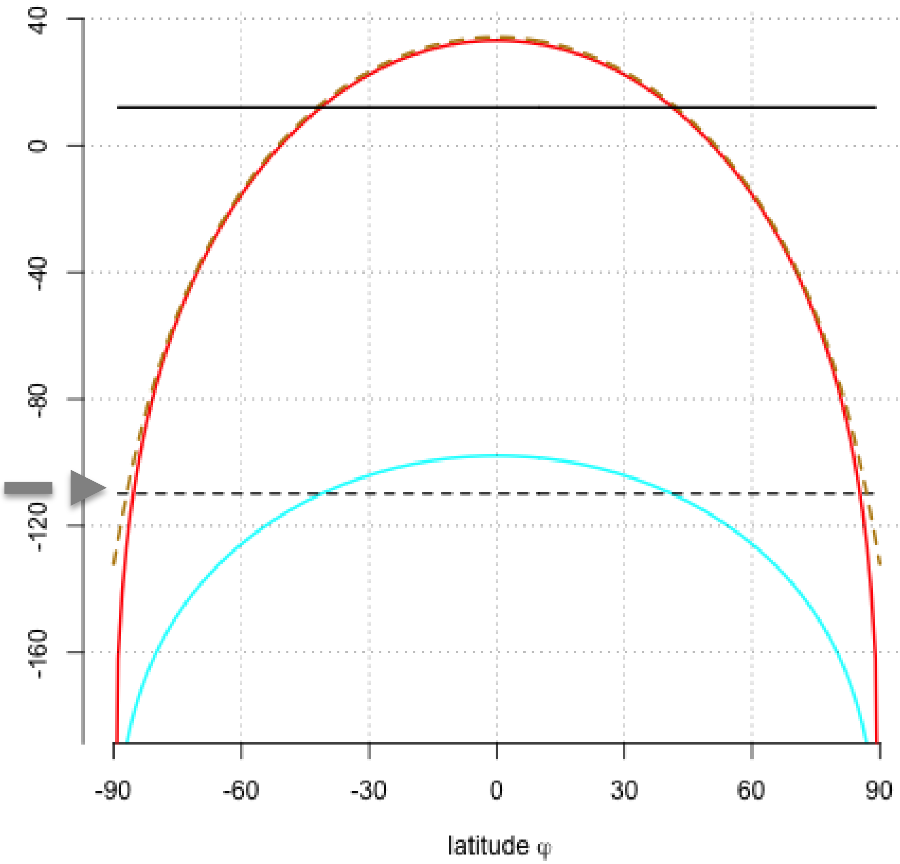
$$= \underbrace{\frac{1}{2}\frac{1}{\sqrt{2}}\frac{\Gamma(5/8)}{\Gamma(13/8)}}_{\frac{\sqrt{2}}{4}\,\frac{8}{5}=0.4\sqrt{2}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

When we integrate this over the latitudes, we obtain

$$\overline{T} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} T(\varphi) \cos\varphi \, d\varphi$$

$$= \frac{1}{2} \frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}$$

$$= \underbrace{\frac{1}{2} \frac{1}{\sqrt{2}} \frac{\Gamma(5/8)}{\Gamma(13/8)}}_{\frac{\sqrt{2}}{4} \frac{8}{5} = 0.4\sqrt{2}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad . \quad (8)$$



https://esd.copernicus.org/articles/11/1195/2020/esd-11-1195-2020.pdf

Heat capacity term

The energy balance shall take the heat capacity into account:

$$C_p\,\partial_t T \;=\; (1-\alpha) S \cos\varphi \cos\Theta \quad \times\, 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \;-\; \epsilon\sigma T^4 \tag{9}$$

The energy balance (9) is integrated over the longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int_0^{2\pi} T(t)\,d\Theta \qquad \text{with} \qquad \tilde{T}^4 \approx \frac{1}{2\pi}\int_0^{2\pi} T^4\,d\Theta$$

and therefore

$$\begin{aligned} C_p\,\partial_{\tilde{t}}\tilde{T} \;&=\; (1-\alpha) S \cos\varphi \cdot \frac{1}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} \cos\Theta\,d\Theta}_{2} - \epsilon\sigma\tilde{T}^4 \\ &=\; (1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \end{aligned} \tag{10}$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \tag{11}$$

shown in Fig. 2 as the read line

The new solution

The energy balance shall take the heat capacit

$$C_p\,\partial_t T \;=\; (1-\alpha) S \cos\varphi \cos\Theta$$

The ene

longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int\limits_0^{2\pi} T(t)\, d\Theta \qquad \text{with}$$

and therefore

$$C_p\,\partial_{\tilde{t}}\tilde{T} \;=\; (1-\alpha) S \cos\varphi \cdot \frac{1}{2\pi}\int\limits_{-\pi/2}^{\pi/2} \mathrm{c}$$

$$= \; (1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \qquad (10)$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \qquad (11)$$

shown in Fig. 2 as the read line

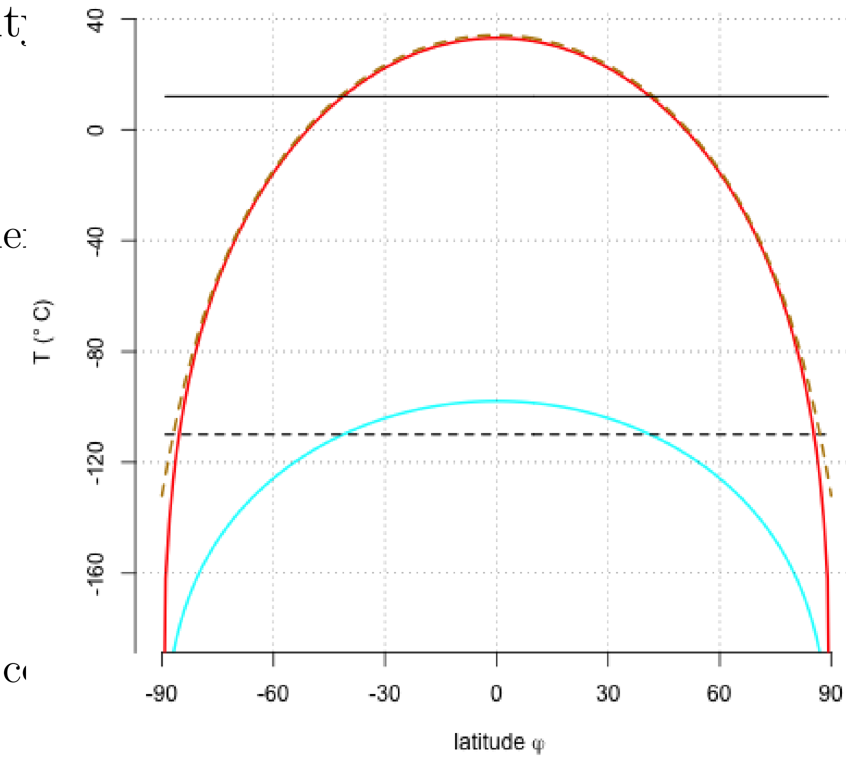


Figure 2. Latitudinal temperatures of the EBM with zero heat capacity (7) in cyan (its mean as a dashed line), the global approach (3) as solid black line, and the zonal and time averaging (11) in red. The dashed brownish curve shows the numerical solution by taking the zonal mean of (9).

$$\overline{\tilde{T}} = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad \underbrace{\frac{1}{2}\int\limits_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{1.862}$$

$$= \sqrt[4]{\frac{4}{\pi}} \frac{1.862}{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.989 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \qquad (12)$$

Therefore, $\overline{\tilde{T}} = 285 \approx 288$ K, very similar as in (1). A numerical solution of (9) is shown as the brownish dashed line in Fig. 2 where the diurnal cycle has been taken into account and $C_p = C_p^a$ has been chosen as the atmospheric heat capacity

$$C_p^a = c_p p_s / g = 1004\, JK^{-1}kg^{-1} \cdot 10^5 Pa/(9.81 ms^{-2}) = 1.02 \cdot 10^7 JK^{-1}m^{-2}$$

which is the specific heat at constant pressure c_p times the total mass p_s/g. p_s is the surface pressure and g the gravity. The temperature $\overline{T}$ is 286 K, again close to 288 K.

Heat capacity C_p/C_p^a

The effect of heat capacity is systematically analyzed in Fig. 3. The temperatures are relative insensitive for a wide range of C_p. We find a severe drop in temperatures for heat capacities below 0.01 of the atmospheric heat capacity C_p^a.

2

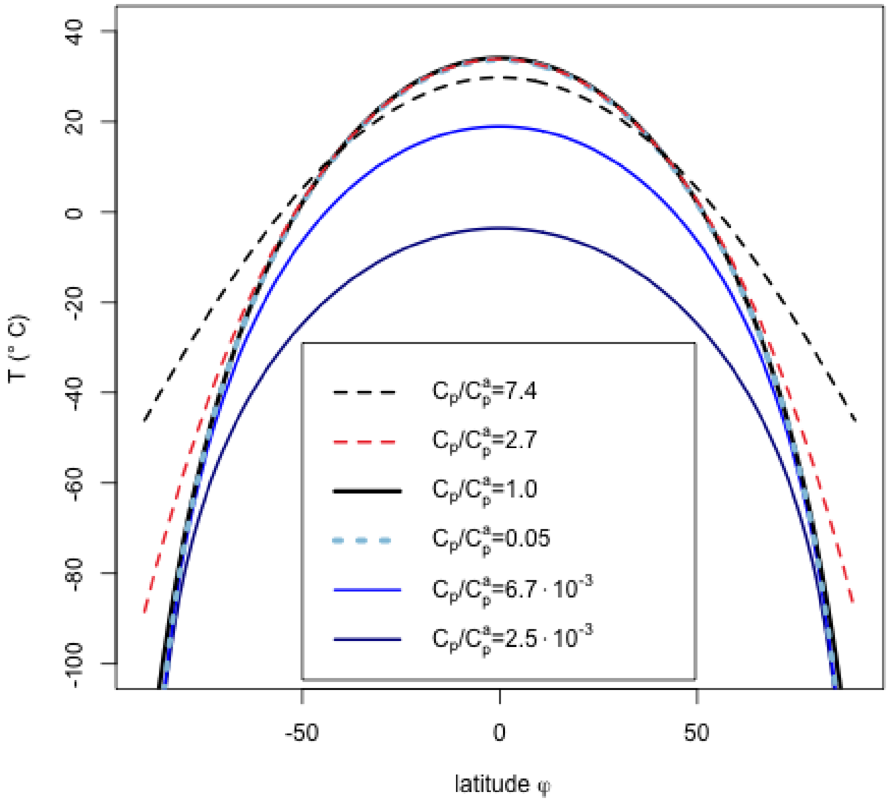


Figure 3. Temperature depending on C_p when solving (9) numerically. The reference heat capacity is the atmospheric heat capacity $C_p^a = 1.02 \cdot 10^7 JK^{-1}m^{-2}$. The climate is insensitive to changes in heat capacity $C_p \in [0.05 \cdot C_p^a, 2.0 \cdot C_p^a]$.