9. Climate variability and analysis

Climate System II

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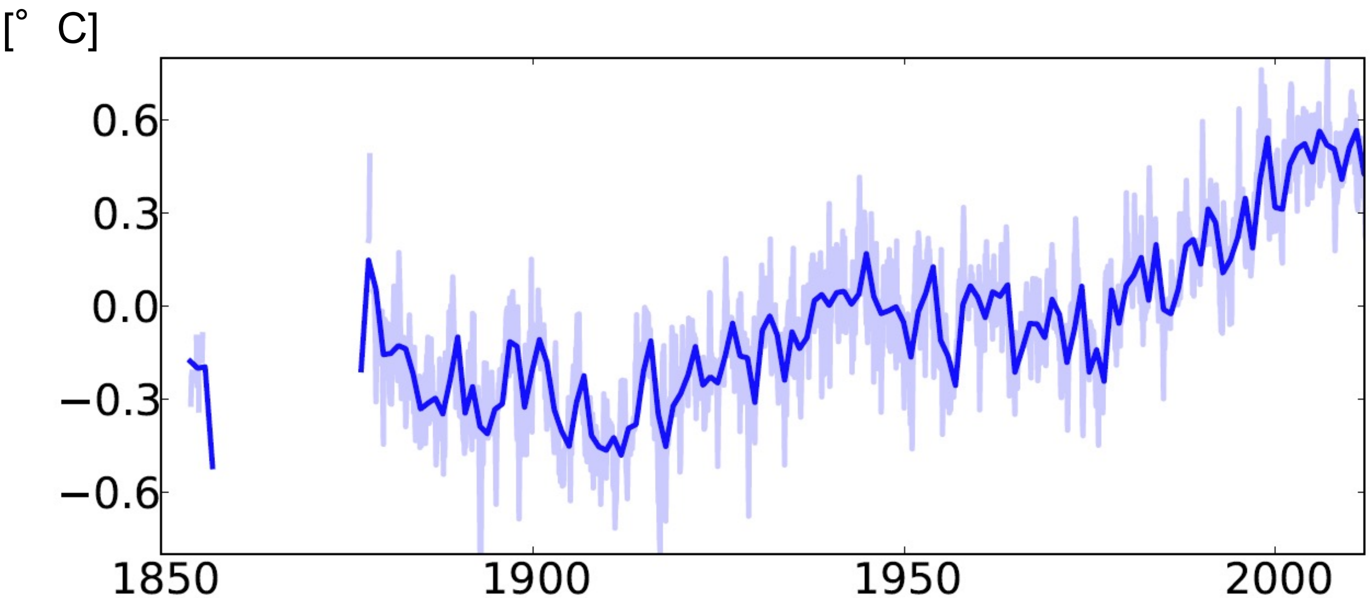
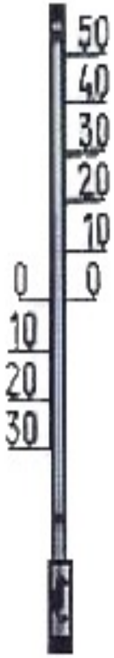
Dec 21, 2021

Climate Trends at different Timescales

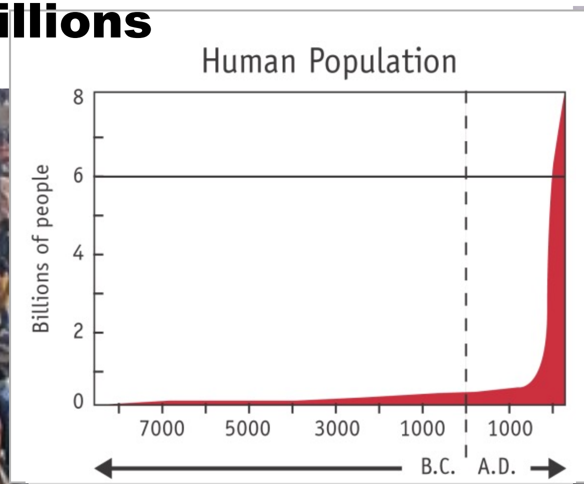
Temperature of the last **150 years** (instrumental data)

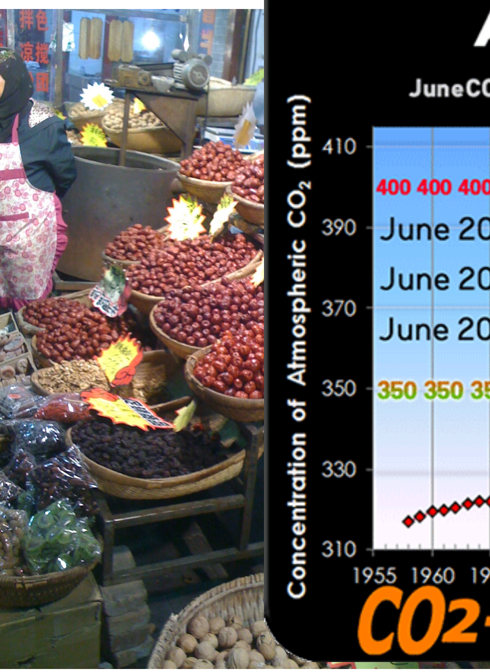
Northern Hemisphere Temp. anomaly
HadCRU

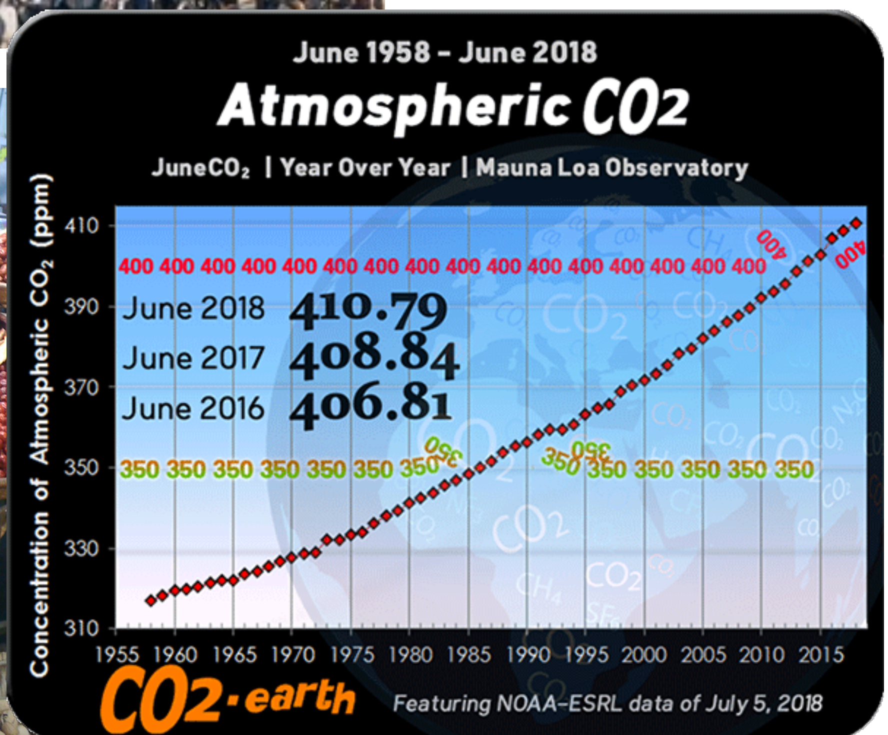




Human Population: 7 billions







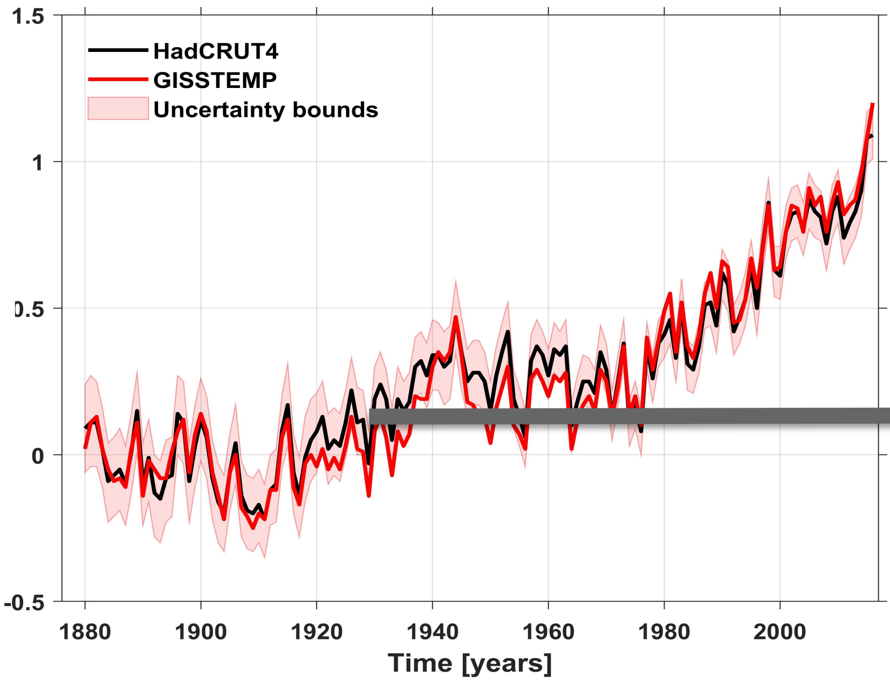
CO_2 Increase:

Land cover: 22%

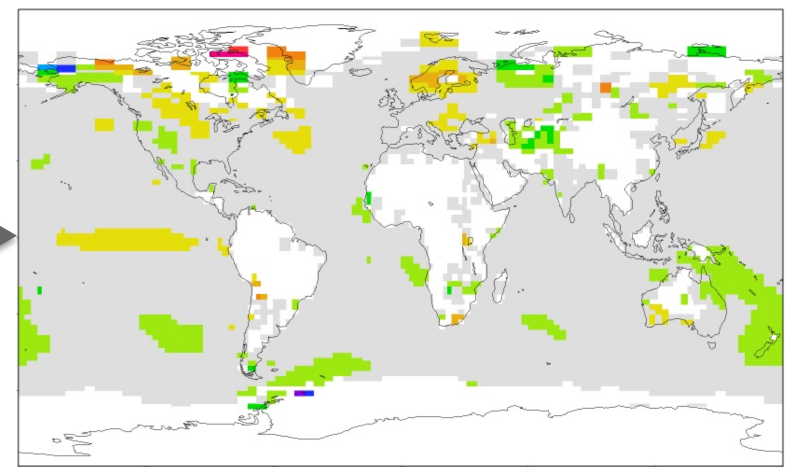
CO_2-Emissions: 78%

Motivation: Observational Record

Temperature anomaly [° C]



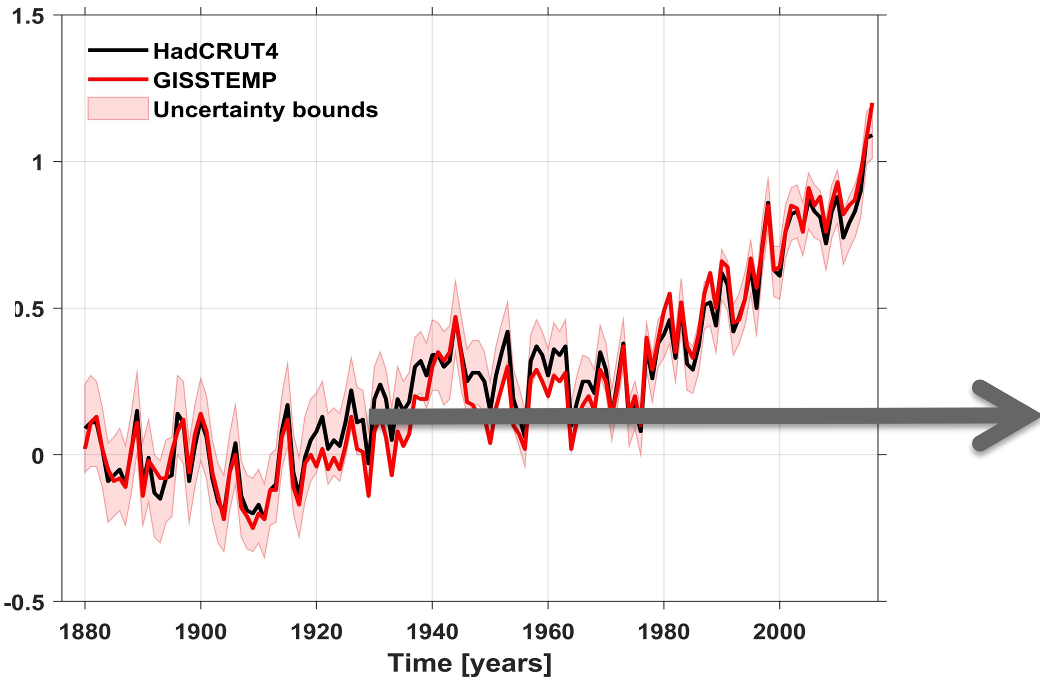
Uncertainty largely due to missing information at high latitudes



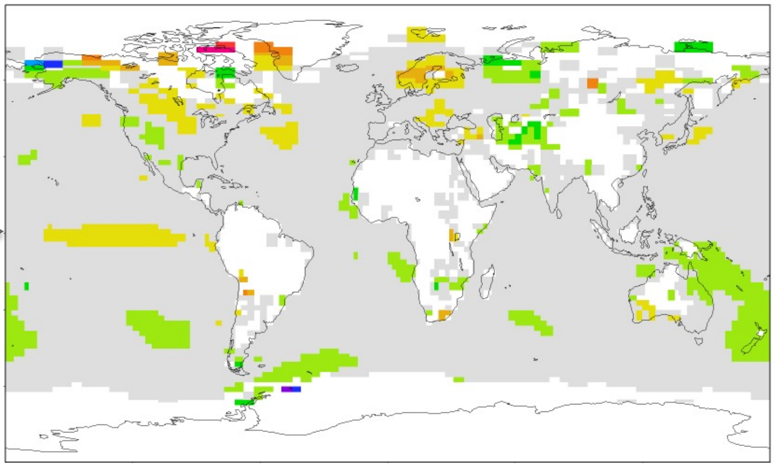
Temperature Anomaly 1930
White areas: not enough data

Motivation: Observational Record

Temperature anomaly [° C]



Uncertainty largely due to missing information at high latitudes

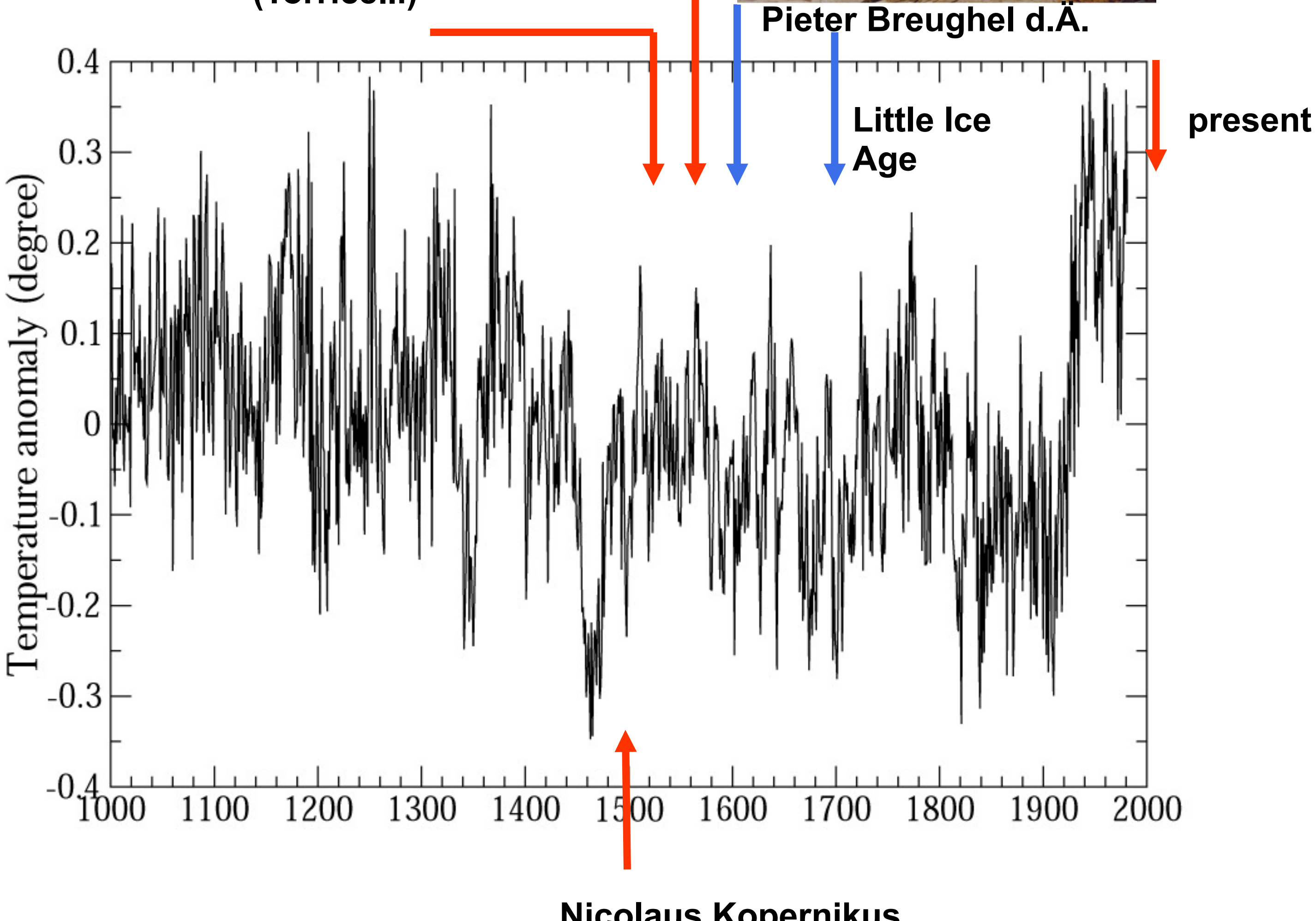


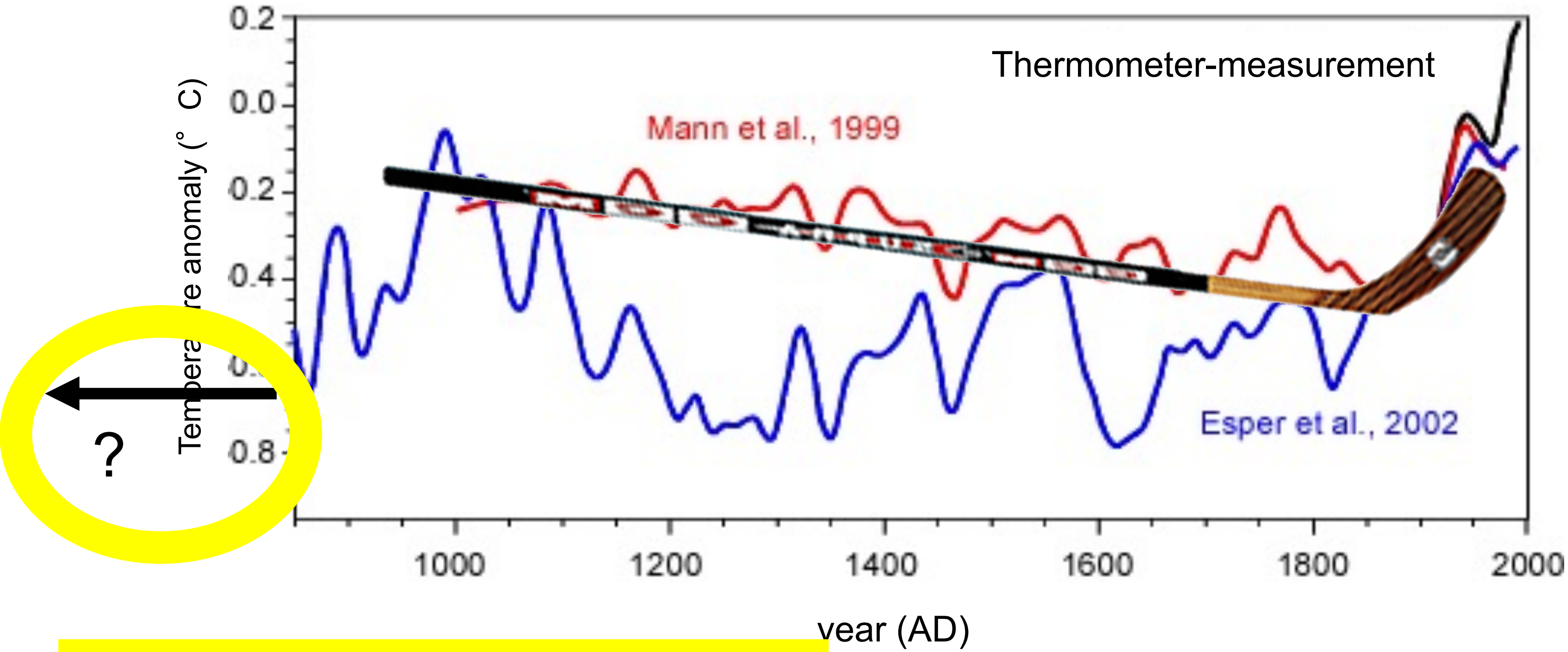
Temperature Anomaly 1930
White areas: not enough data

Climate variability beyond the instrumental record: Decadal, centennial, millennial

History

last **1000 Years**

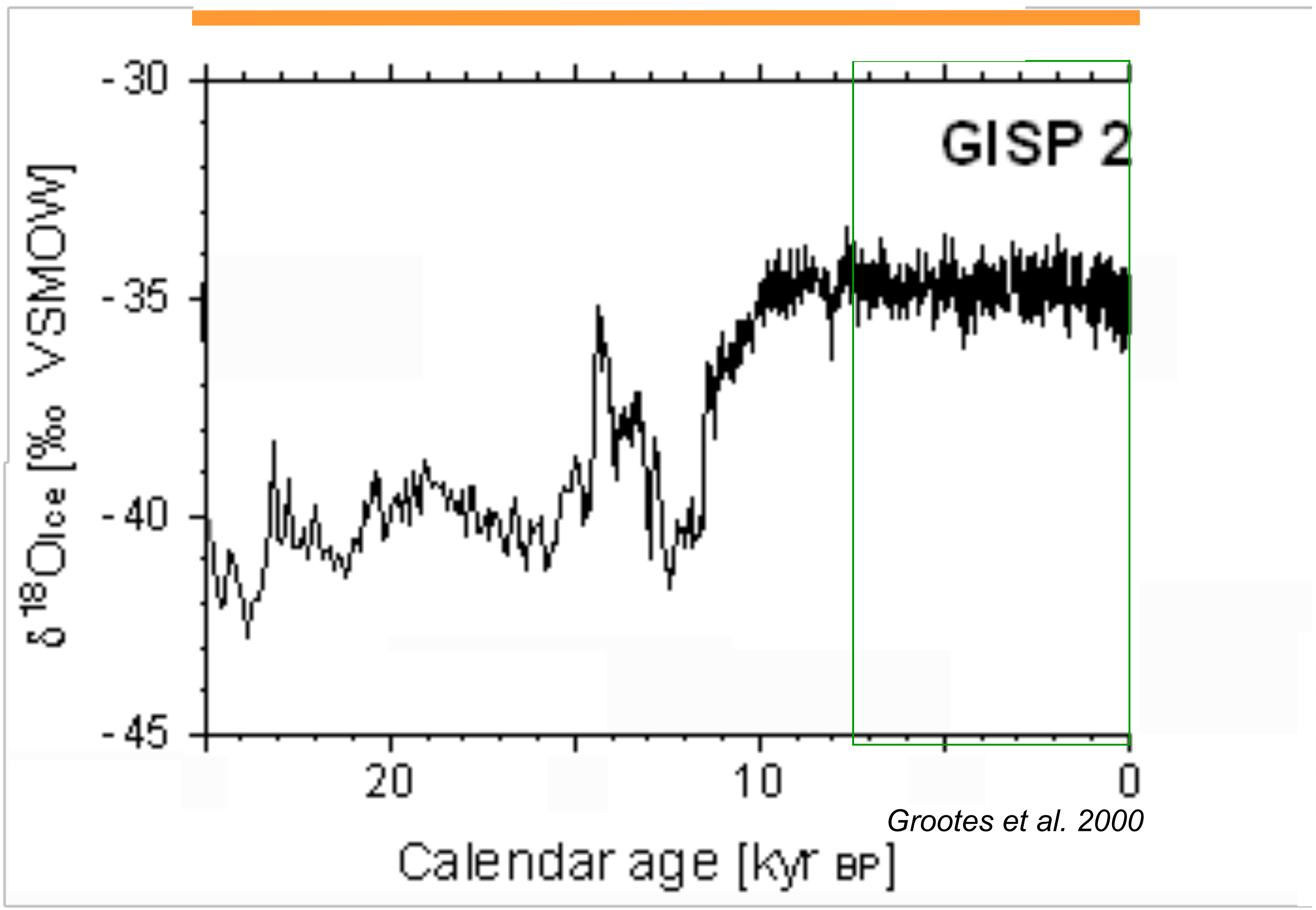




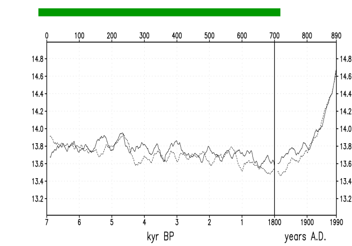
Further back in time?

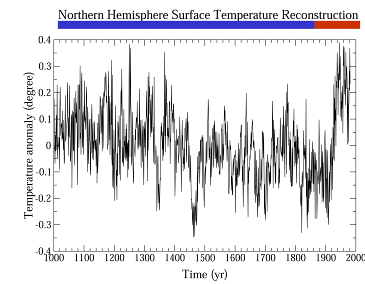
Climate Trends at different Timescales

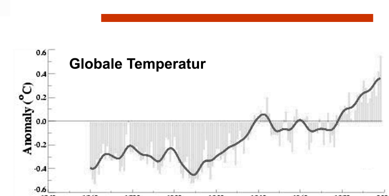
Deglaciation – Greenland ice core



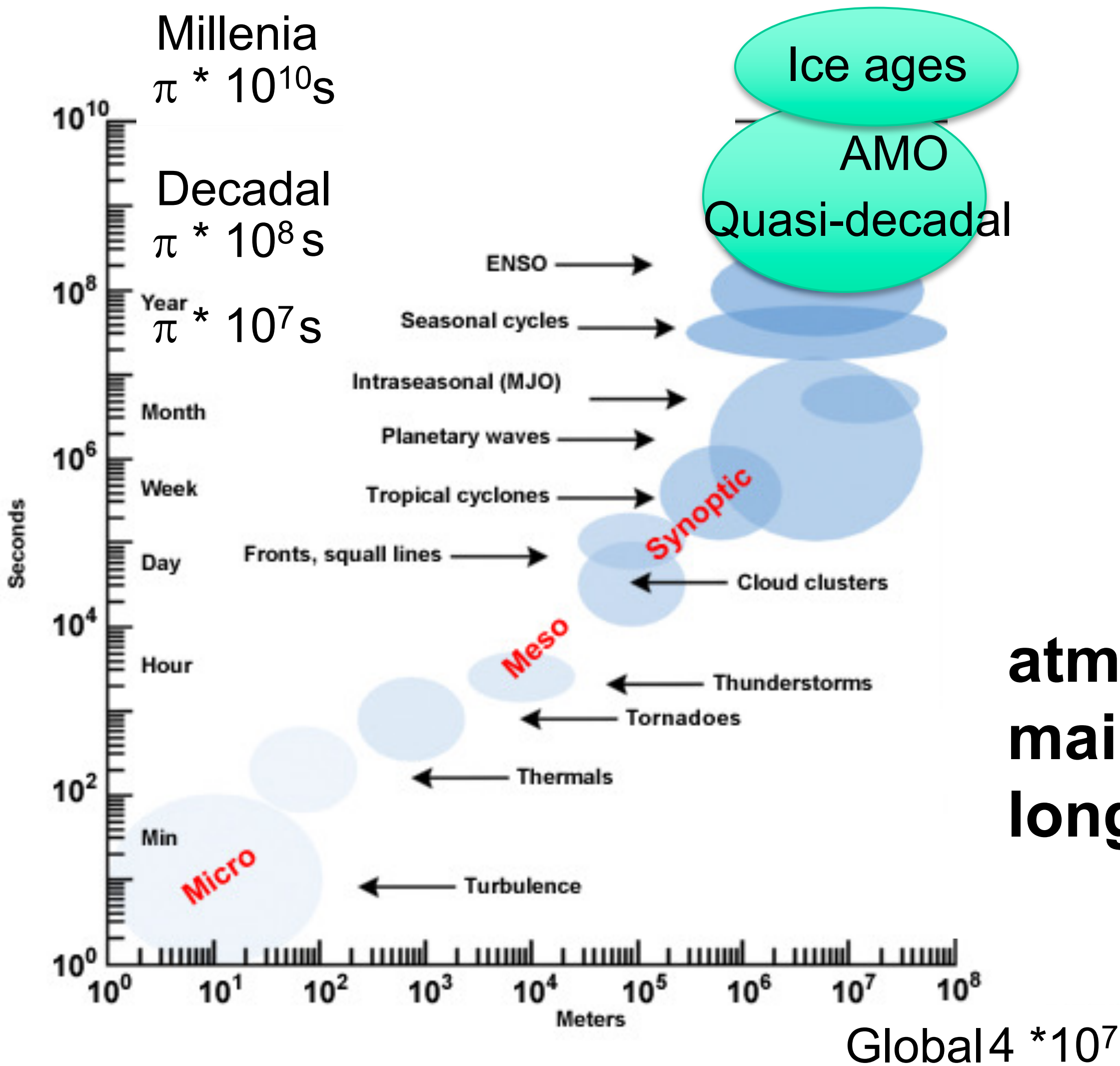
Grootes et al. 2000







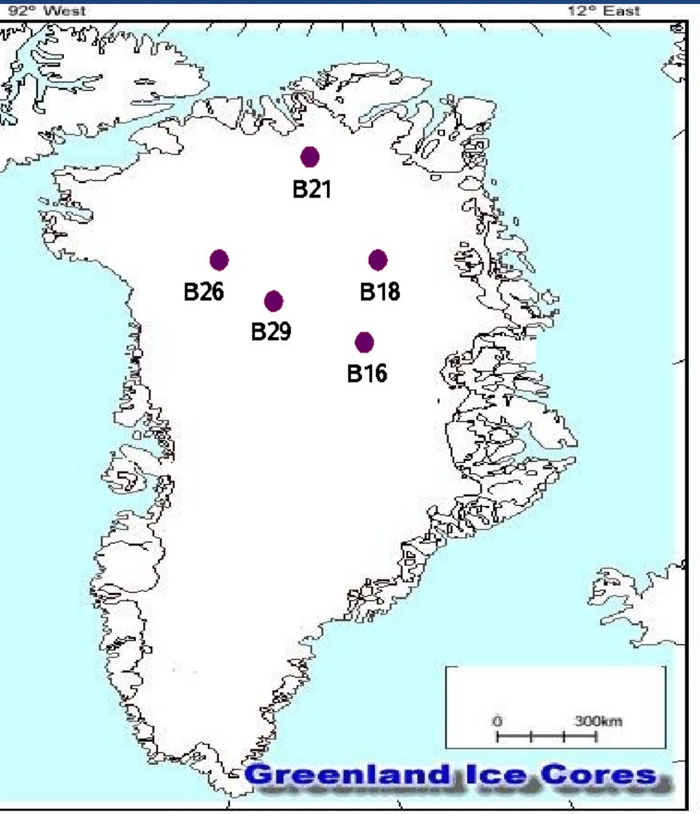
Spatio-Temporal Scales



Spatial || temporal Scales

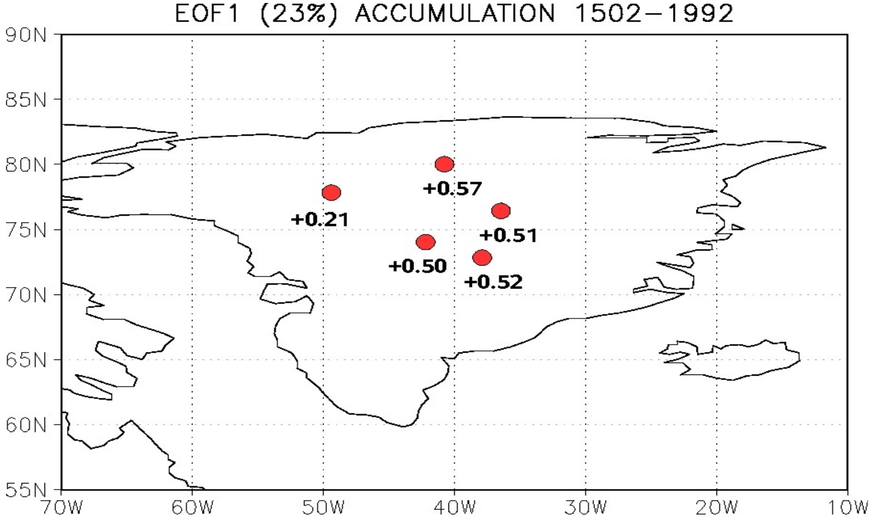
atmosphere & ocean cannot maintain large gradients on long time scales

Shallow ice cores

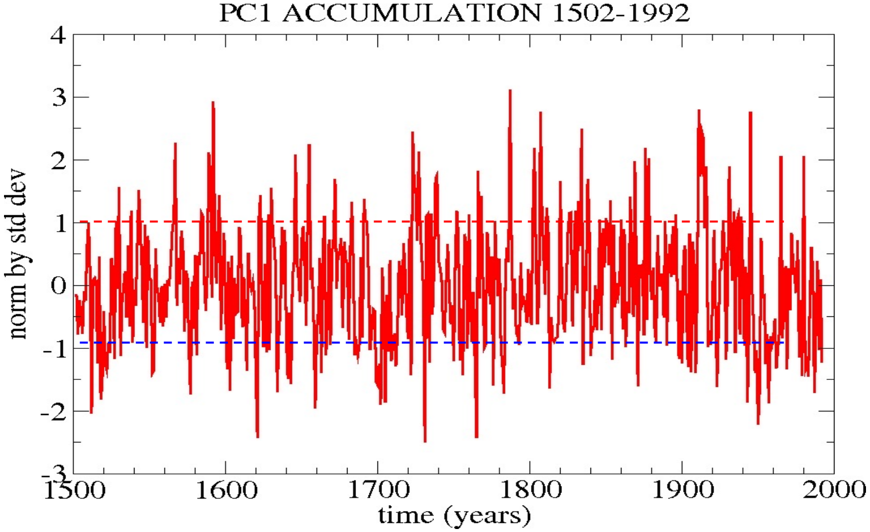


Atmospheric Blocking Circulation

Greenland Shallow Ice Core Positions



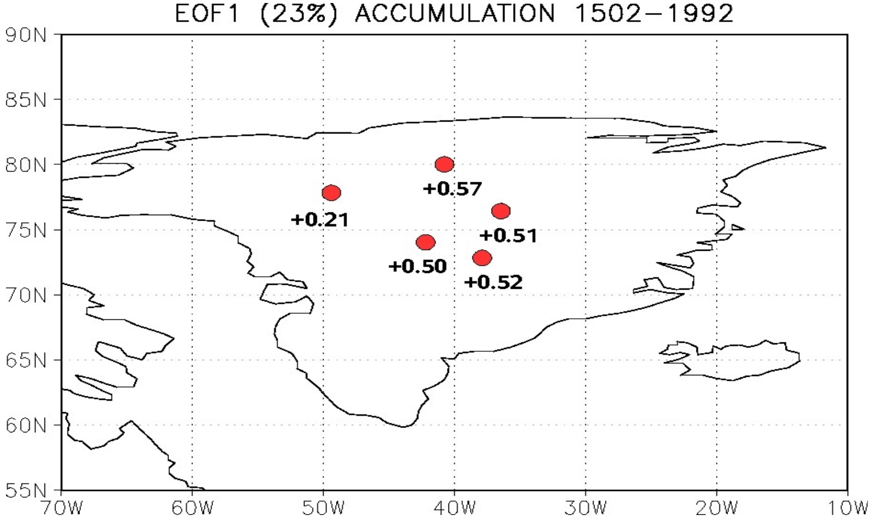
Variability of Accumulation Rate



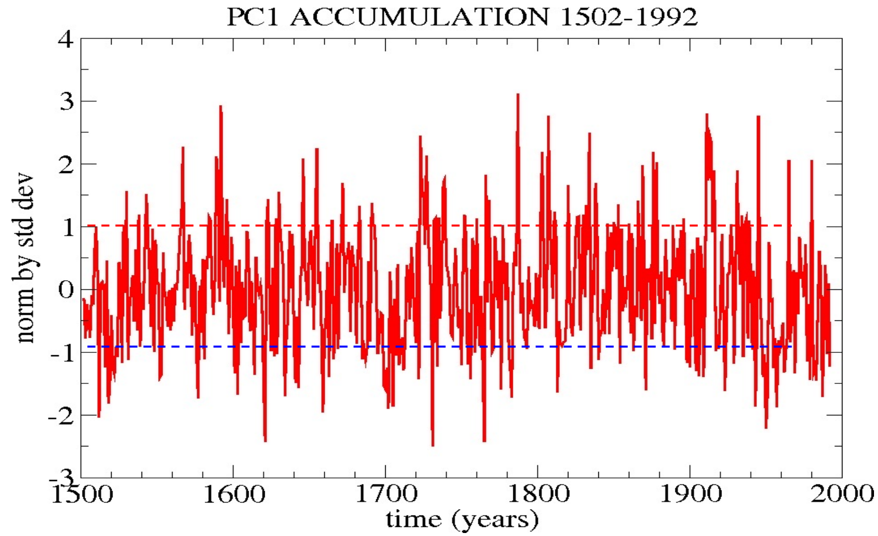
Rimbu and Lohmann 2009

Atmospheric Blocking Circulation

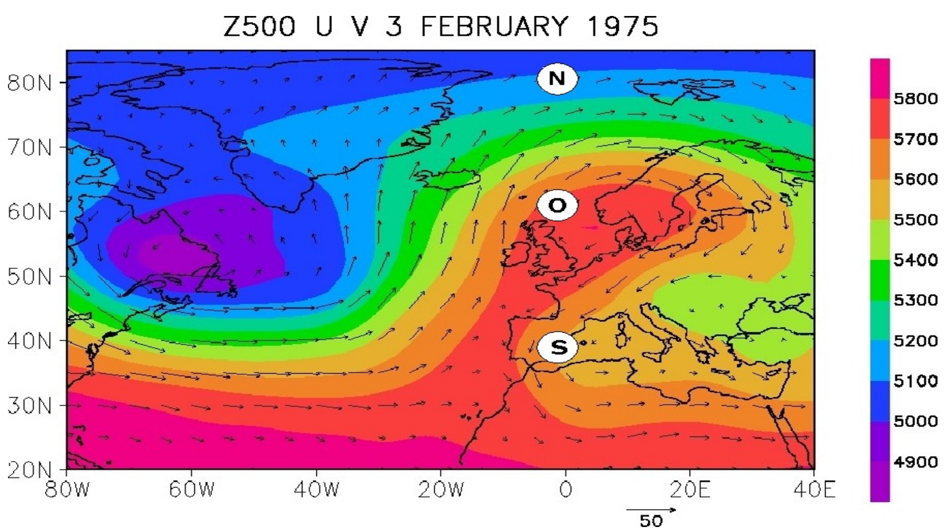
Greenland Shallow Ice Core Positions



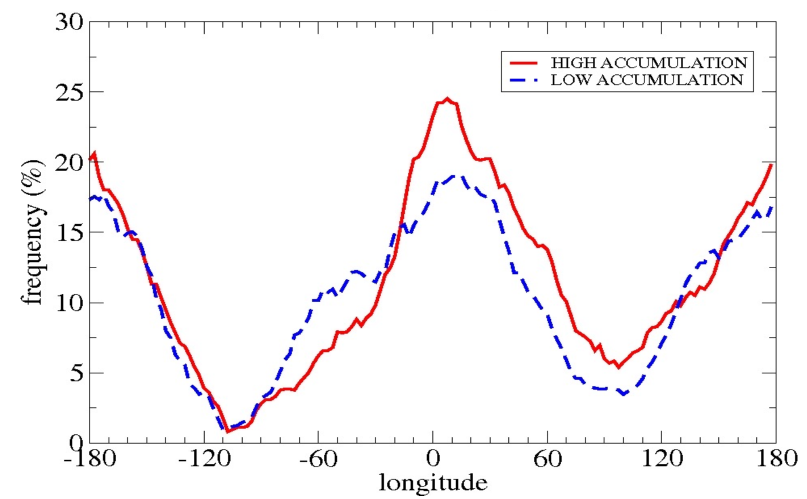
Variability of Accumulation Rate



Synoptic Scale Blocking Situation

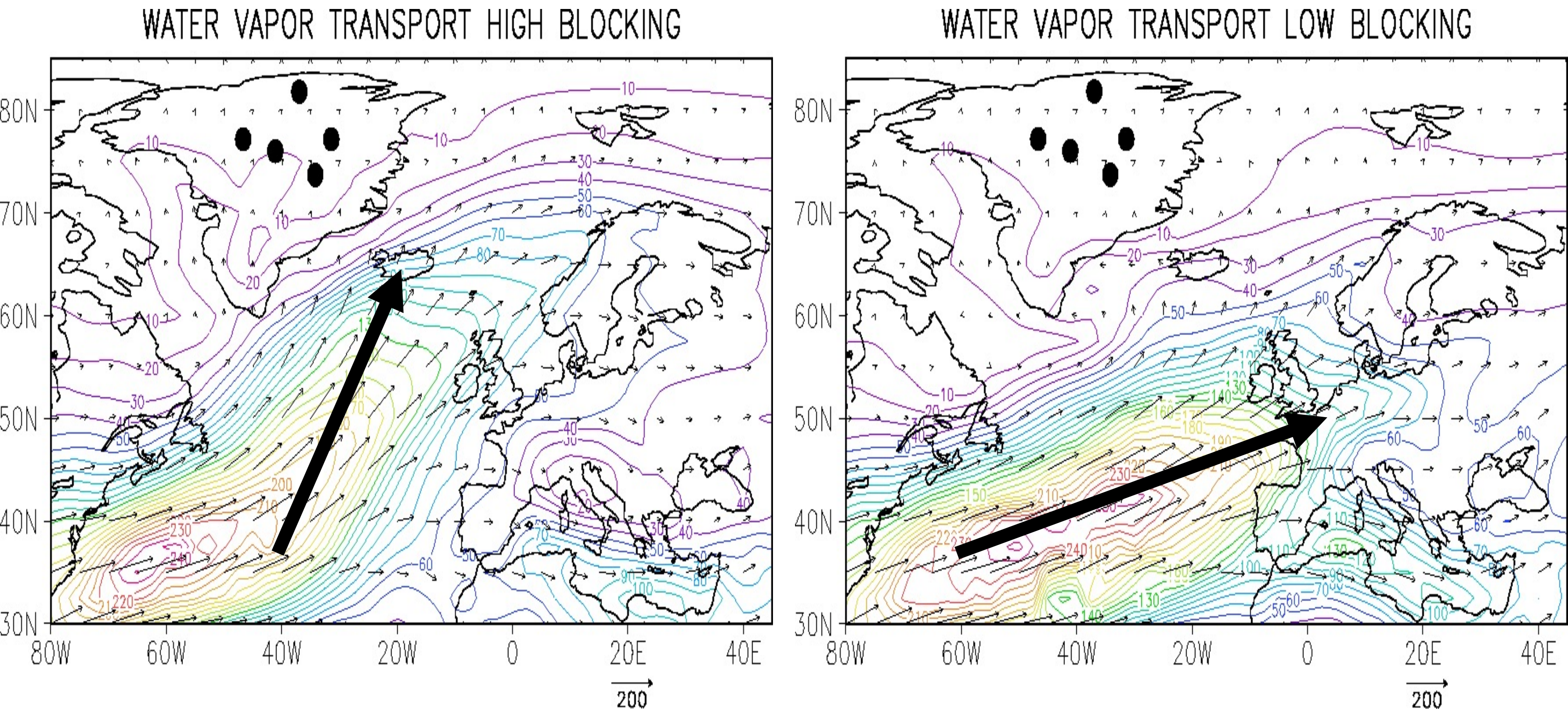


Blocking Frequency for 1948-1992



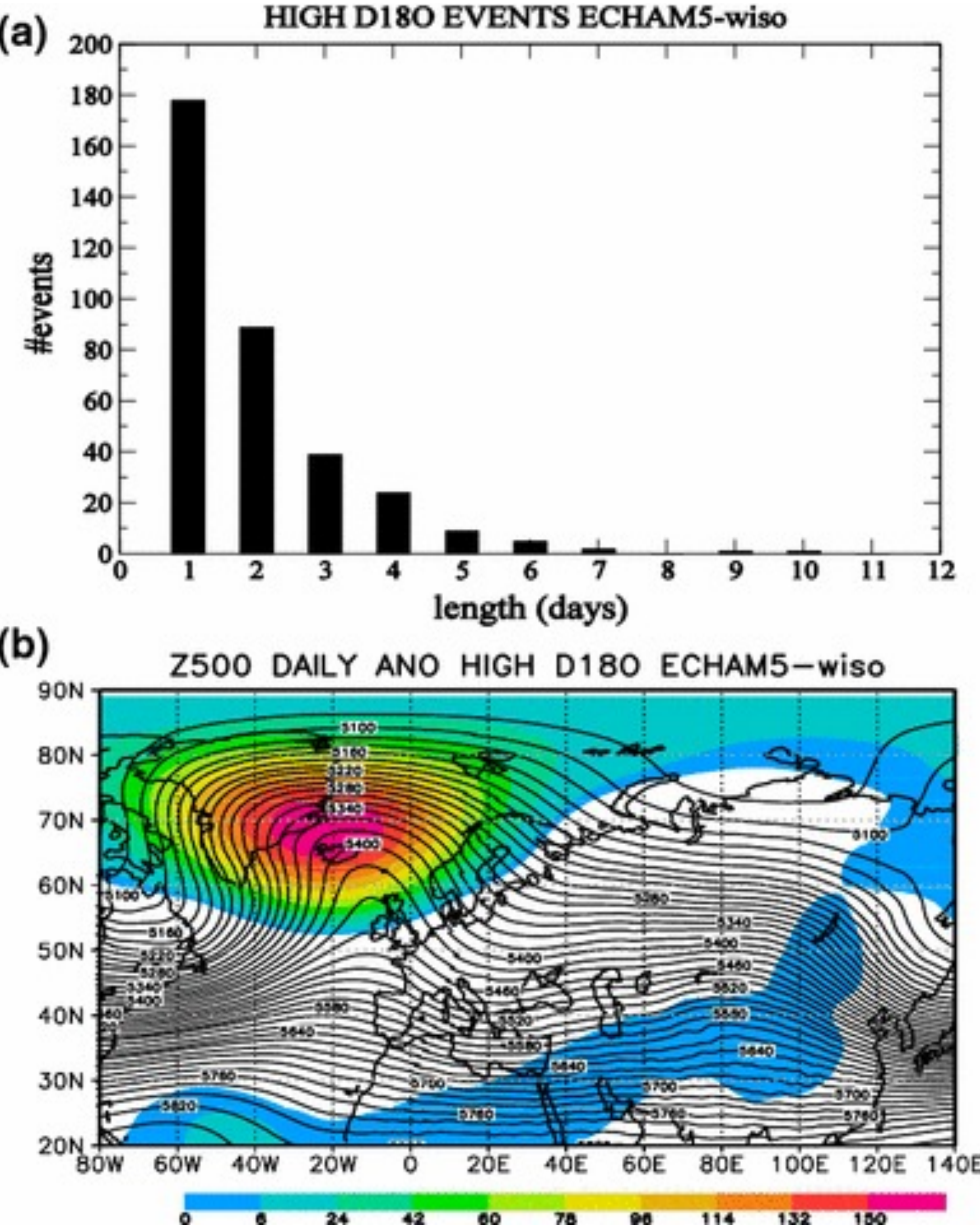
Rimbu and Lohmann 2009

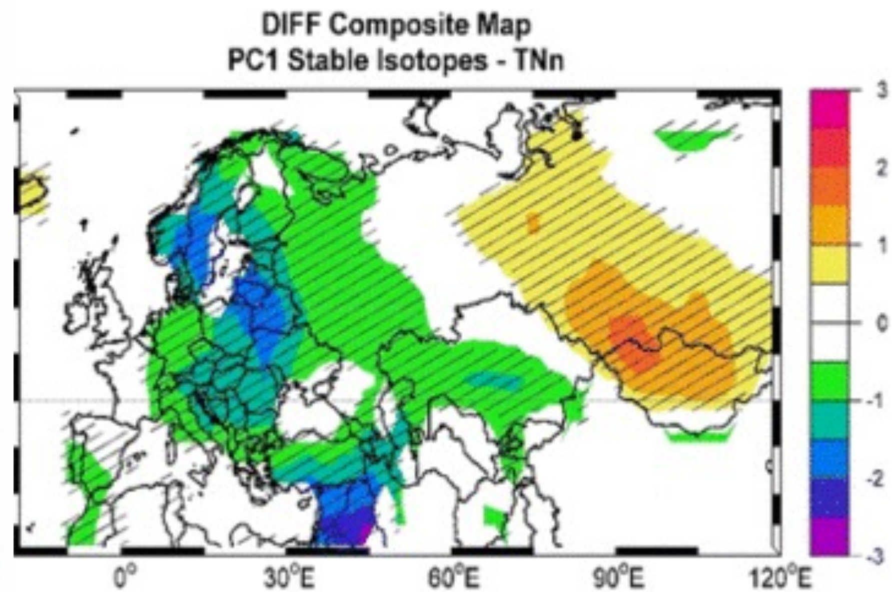
WATER VAPOR TRANSPORT



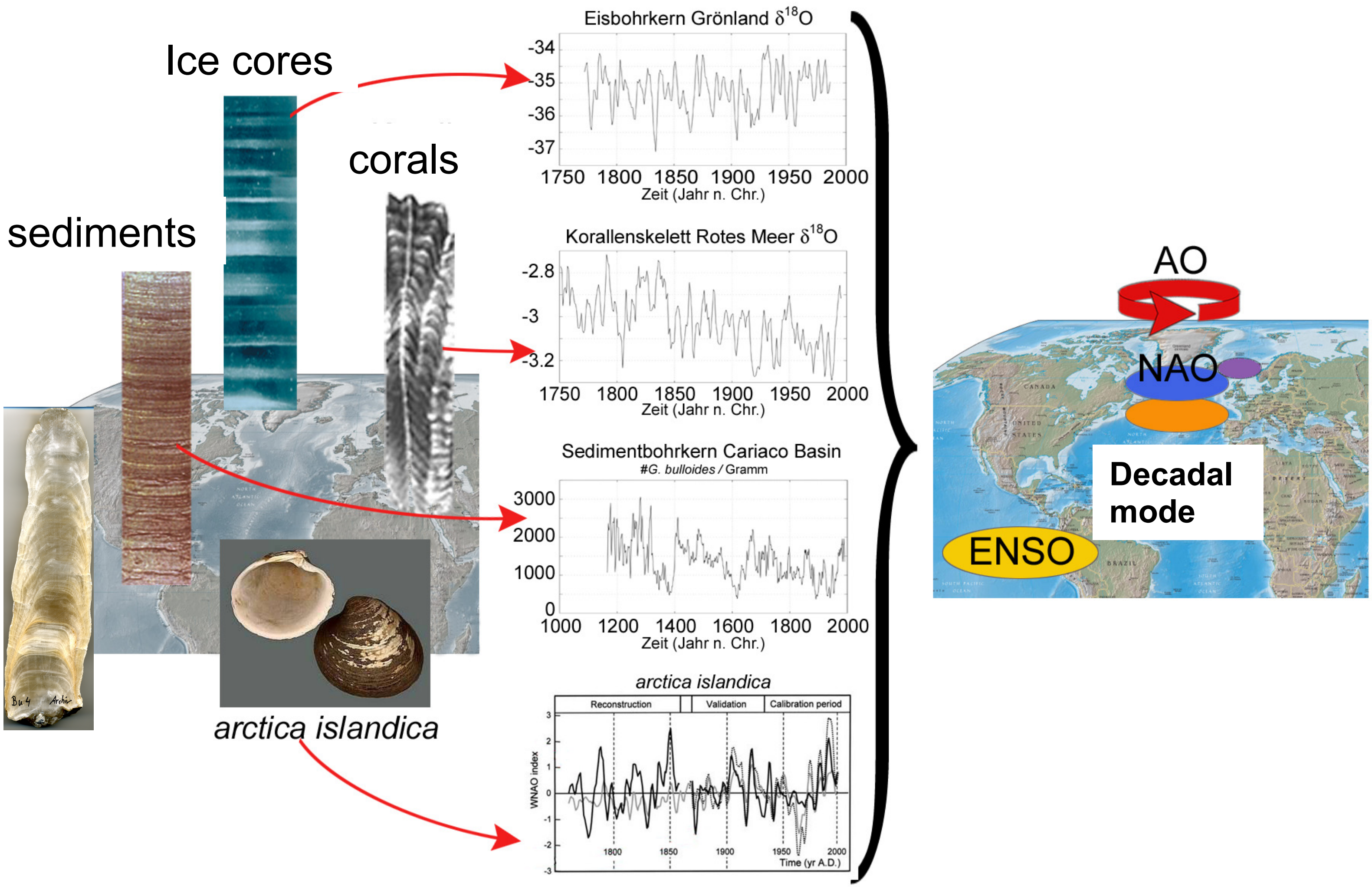
Enhanced moisture transport during high blocking activity

minimum value of daily minimum temperature (TNn)





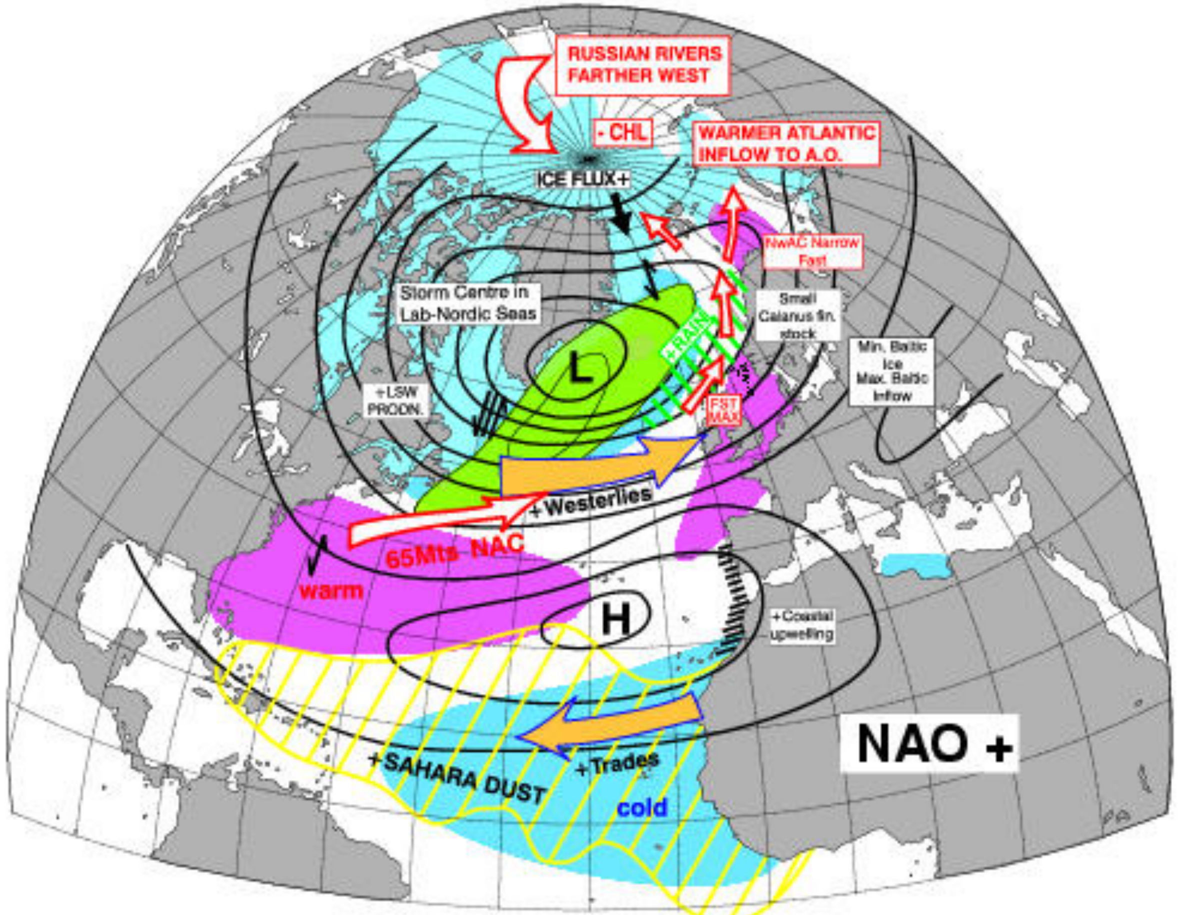
Upscaling concept



Climate archives **Climate variabiliy**

Lohmann, 2007

The Phases of the North Atlantic Oscillation



During the high phase of the NAO westerlies in the North Atlantic are enhanced, resulting in mild and wet winter conditions over Northern Europe. (Courtesy of CEFAS, UK)

Statistics

covariance is a measure of how much two random variables change together

Covariance (cross, auto)

$$\gamma(\Delta) = \mathrm{E}\left((\,x\,(t) - \bar{x}\,)\,(\,y\,(t+\Delta) - \bar{y}\,) \right)$$

e.g. coral   e.g. meteorol. data

$$\mathrm{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - E(X))(y_i - E(Y)).$$

Correlation (cross, auto)

$$\rho_{xy} = \frac{\gamma(\Delta)}{\text{normalized}}$$

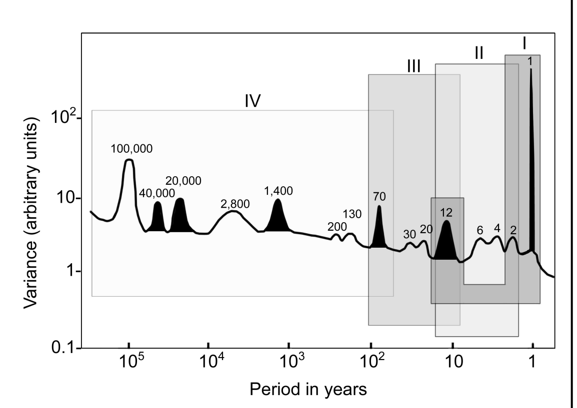
measures the tendency of x (t) and y (t) to covary, between -1 and 1

Spectrum (cross, auto)

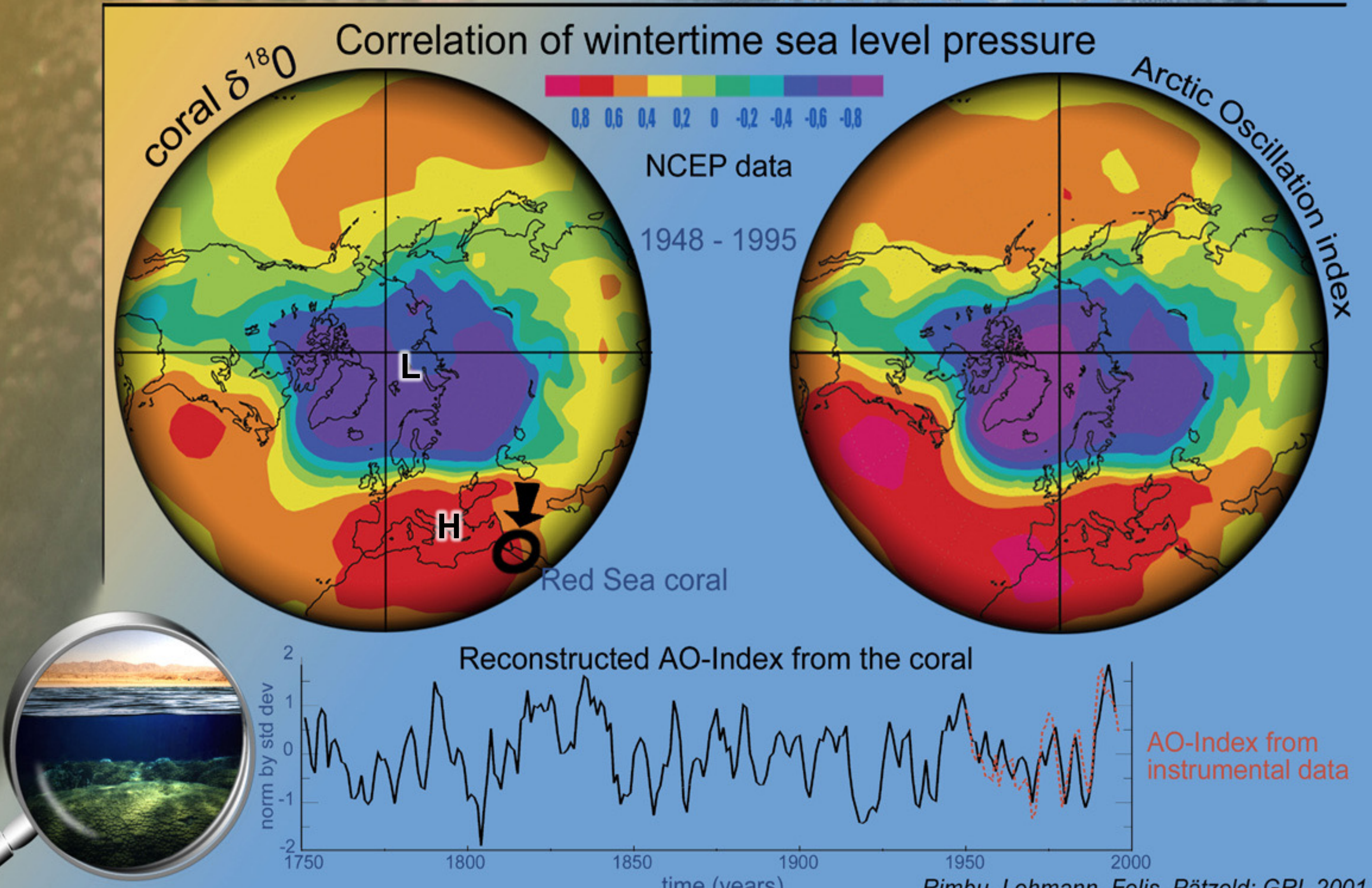
(spectral density)

$$\Gamma(\omega) = \sum_{\Delta=-\infty}^{\infty} \gamma\,(\Delta)\, e^{-2\pi i\Delta}$$

measures variance

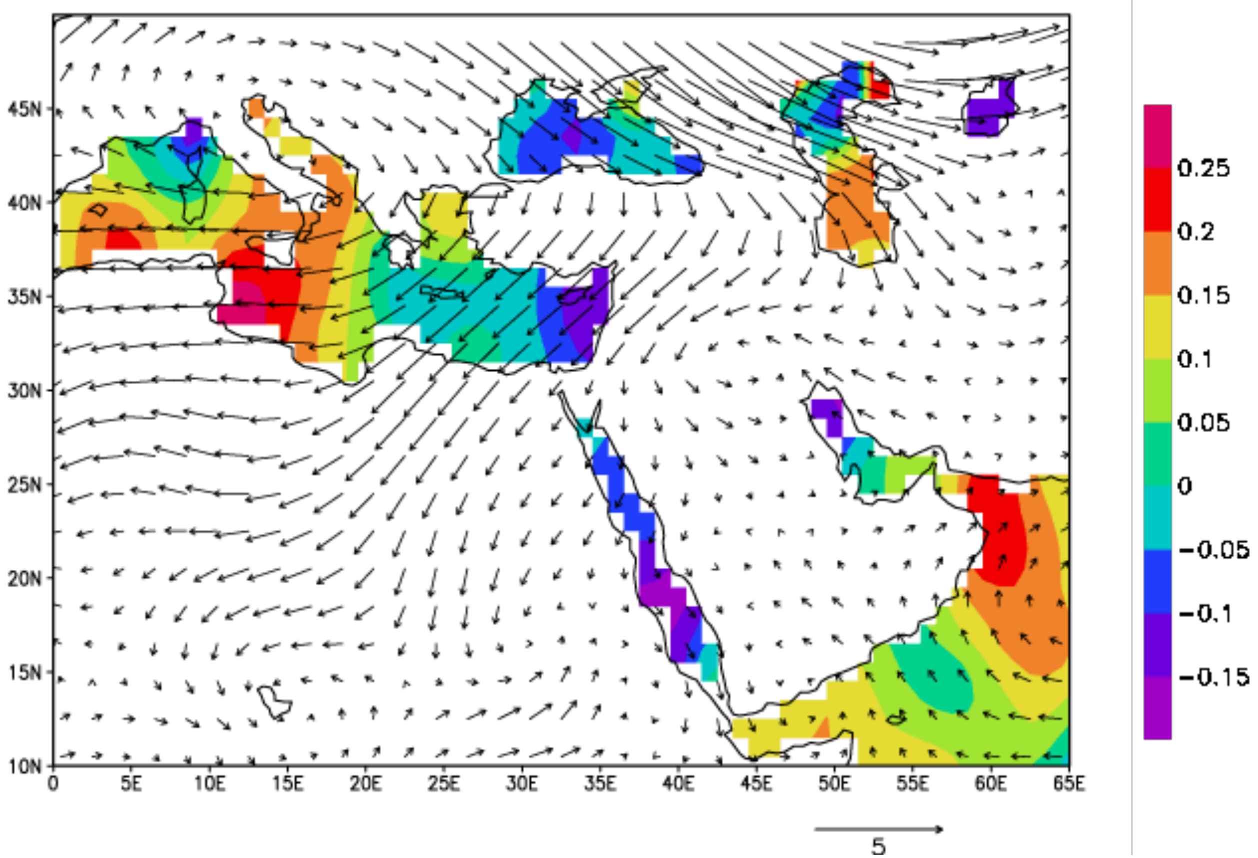


ARCTIC OSCILLATION SIGNATURE IN A RED SEA CORAL



Rimbu, Lohmann, Felis, Pätzold; GRL 2001

ARCTIC OSCILLATION SIGNATURE IN A RED SEA CORAL



Composite Map of SST [°C] and 925 hPa wind [m/s]
for 1948 -1995, January - February

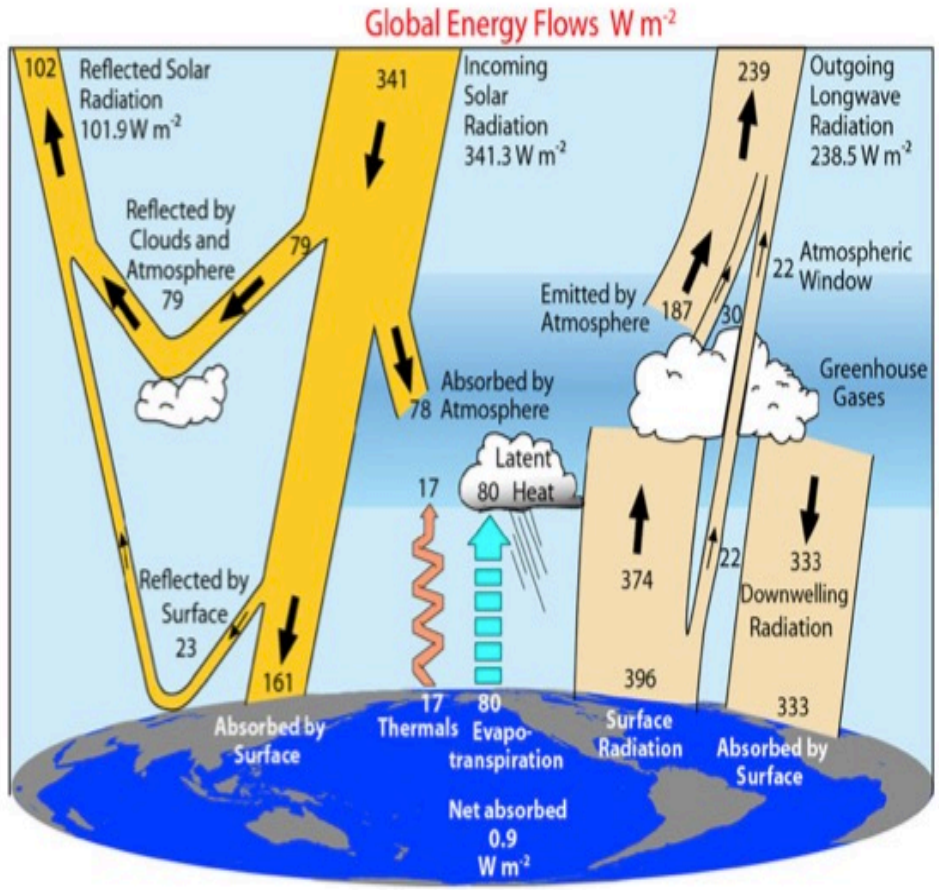
mechanistic understanding

boring for the hundredth time

Energy Budget

CHANGE IN STORAGE = IN – OUT

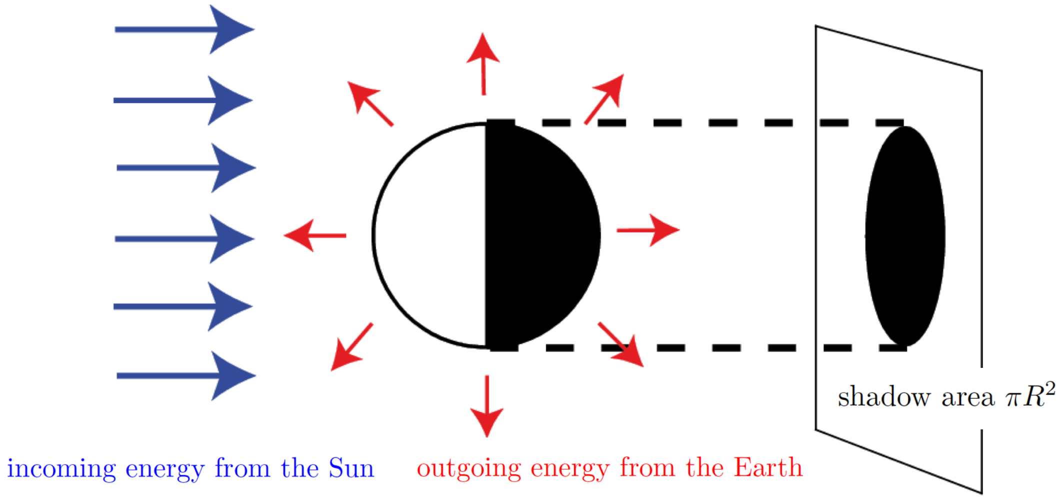
- many papers discuss an imbalance in this equation, which results in missing energy



(Trenberth & Fasullo, 2012)

Energy balance model

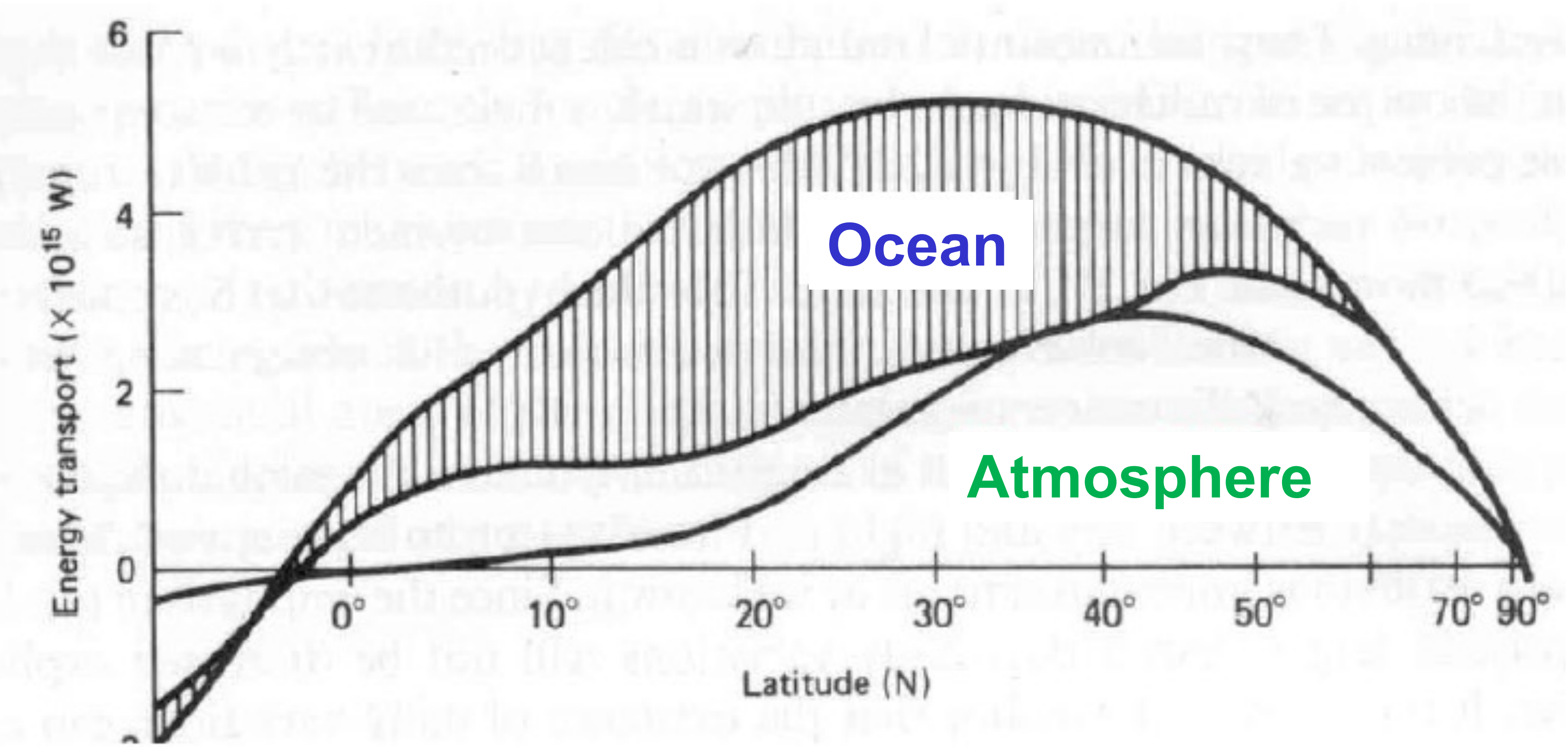
$$(1-\alpha)S\pi R^2 = 4\pi R^2 \epsilon\sigma T^4$$



$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

boring for the hundredth time, but …

Northward Heat Transport

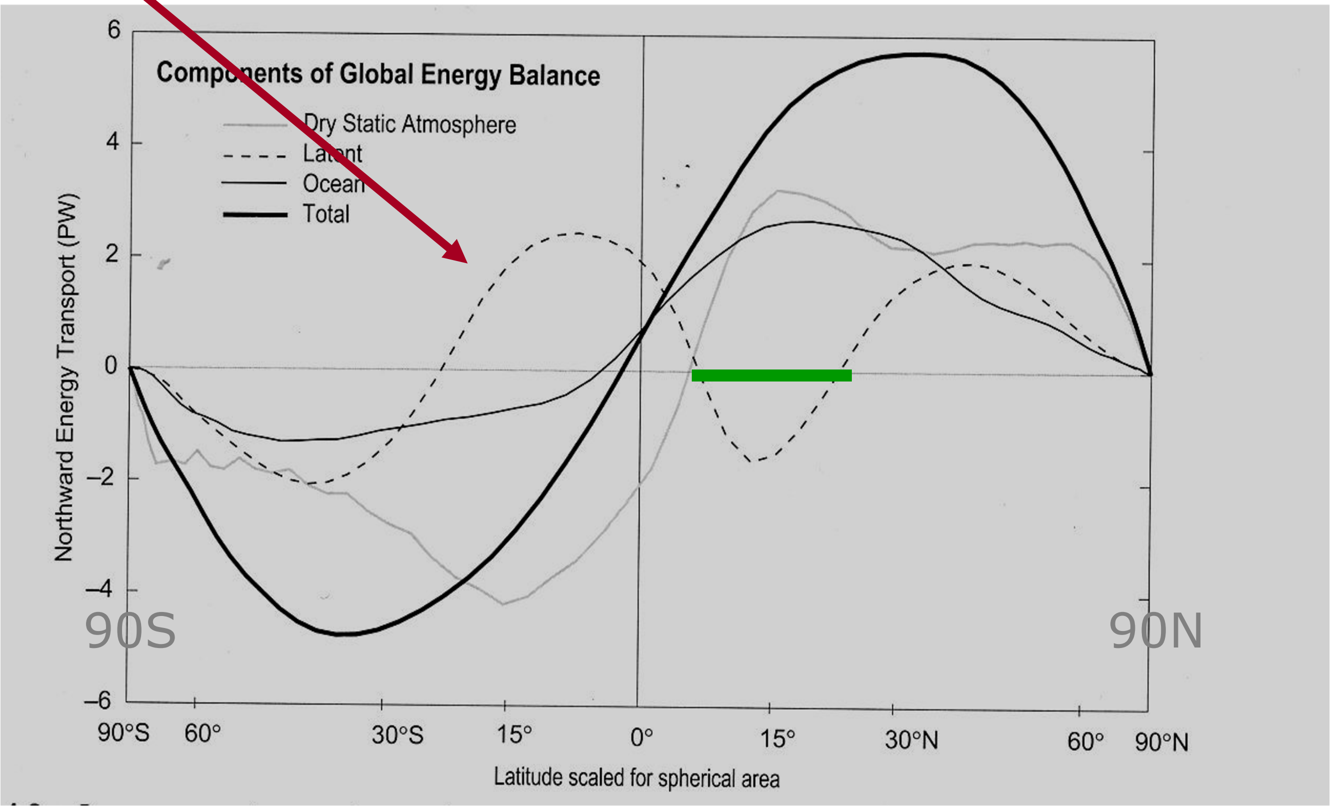


nach Von der Haar & Ort; Quelle: Gill

Global meridional heat transport divides roughly equally into 3 modes:

1. atmosphere (dry static energy)
2. ocean (sensible heat)
3. water vapor/latent heat transport

The three modes of poleward transport are comparable in amplitude, and distinct in character (sensible heat flux divergence focused in tropics, latent heat flux divergence focus in the subtropics)



(residual method, TOA radiation
1985-89 and ECMWF/NMC atmos obs)

$$C_p \partial_t T = \nabla \cdot HT + (1-\alpha) S(\varphi, t) - \epsilon \sigma T^4$$

$$HT = -k \nabla T$$

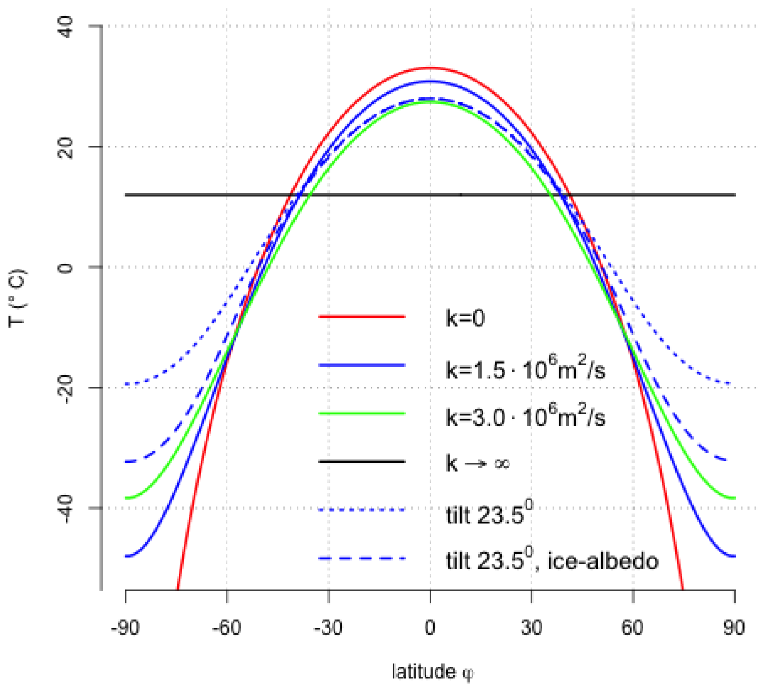


Figure 4. Equilibrium temperature of (15) using different diffusion coefficients. The blue lines use $1.5 \cdot 10^6 m^2/s$ with no tilt (solid line), a tilt of 23.5° (dotted line), and as the dashed line a tilt of 23.5° and ice-albedo feedback using the respresentation of Sellers (1969). Except for the dashed line, the global mean values are identical to the value calculated in (12). Units are °C.

In the exercise, long-wave radiation as A + BT

Practical Jan 11, 2022

Exercise

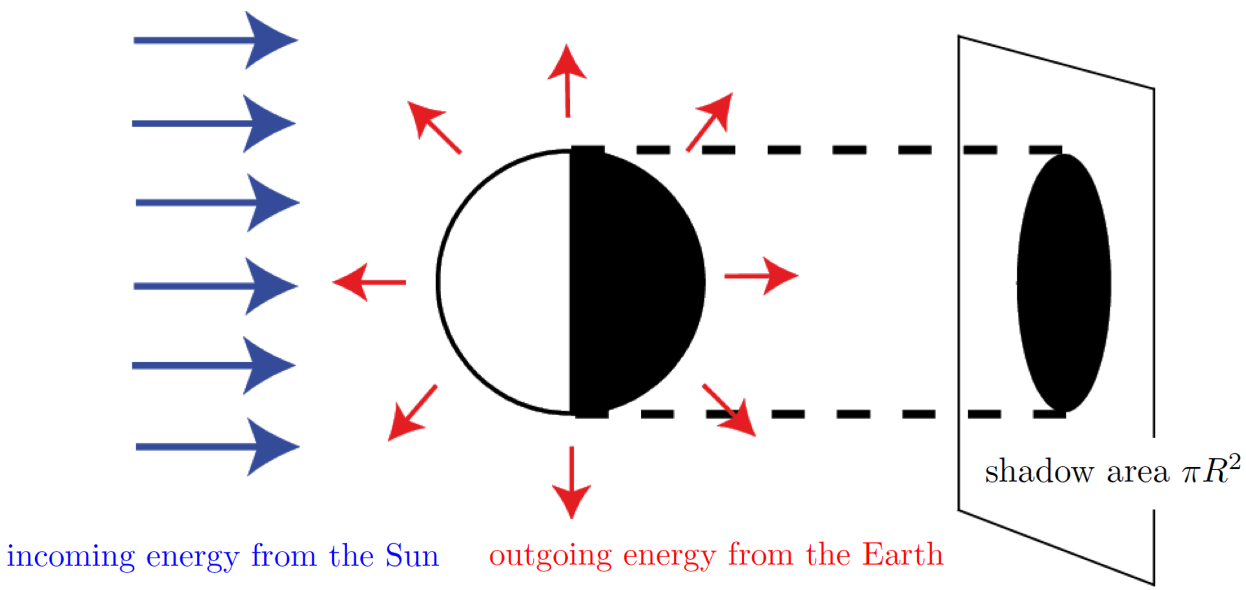
EBM analysis

- https://1drv.ms/u/s!AnZSDMNwdkDMgbx6sr3gVubSIqlYVw?e=MacPeK

Energy balance model

$$(1-\alpha)S\pi R^2 = 4\pi R^2 \epsilon\sigma T^4$$

$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$



Simple models are helpful for understanding of

- Critical parameters
- Concepts of climate

Incoming radiation

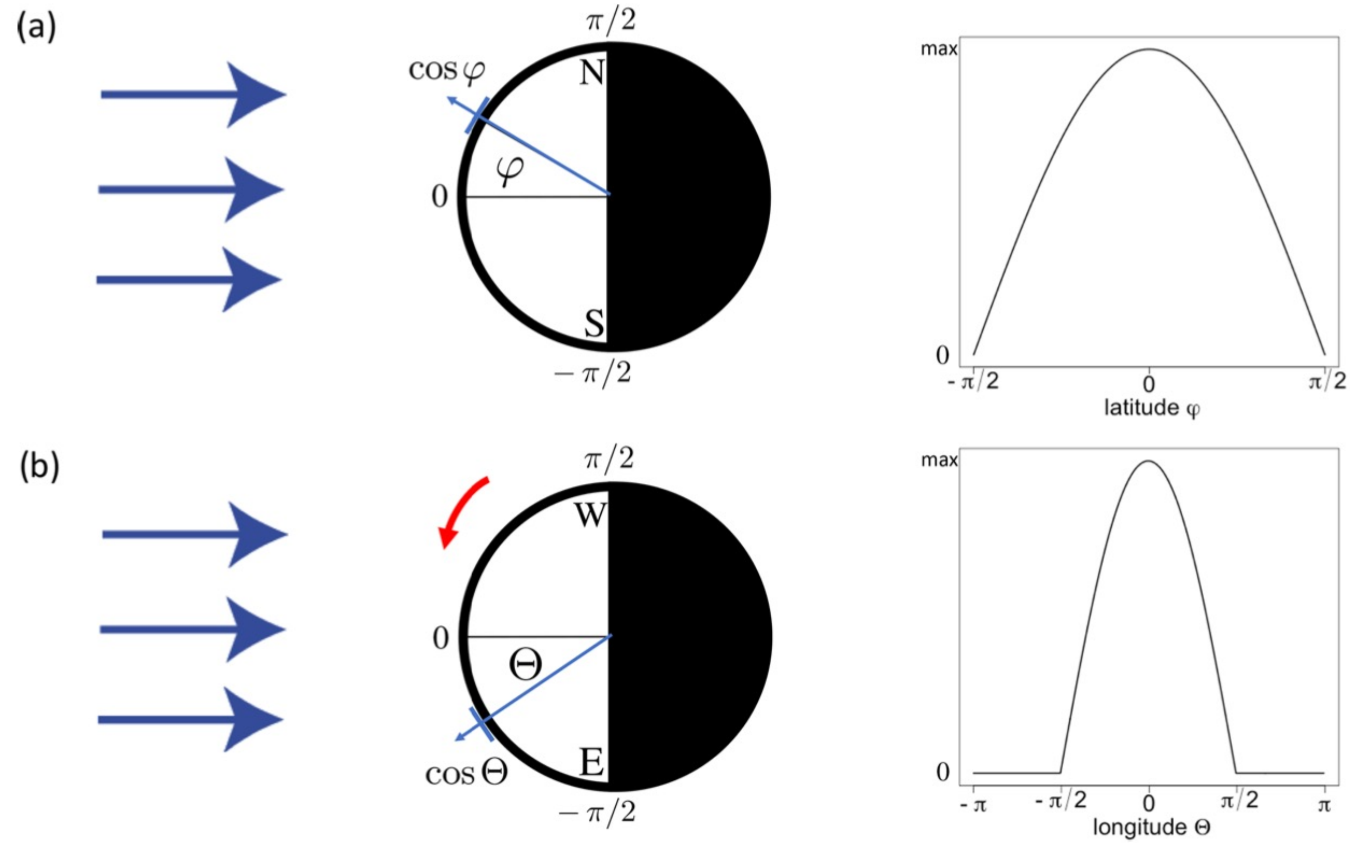


Figure 2. Latitudinal **(a)** and longitudinal **(b)** dependence of the incoming shortwave radiation. On the right-hand side, the insolation as a function of latitude φ and longitude Θ with maximum insolation $(1-\alpha)S$ is shown. See the text for the details.

What we really want is the mean of the temperature $\overline{T}$.
fourth root of (4):

$$T = \sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}} \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$$

$$T(\varphi) = \frac{\sqrt{2}}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\Theta)^{1/4} d\Theta}_{\sqrt{\pi}\Gamma(5/8)/\Gamma(9/8)} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} (\cos\varphi)^{1/4}$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}} \frac{\Gamma(5/8)}{\Gamma(9/8)}}_{\approx 0.608} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} (\cos\varphi)^{1/4}$$

When we integrate this over the latitudes, we obtain

$$\overline{T} = \frac{1}{2}\int\limits_{-\pi/2}^{\pi/2} T(\varphi)\cos\varphi\, d\varphi$$

$$= \frac{1}{2}\frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\int\limits_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}$$

$$= \underbrace{\frac{1}{2}\frac{1}{\sqrt{2}}\frac{\Gamma(5/8)}{\Gamma(13/8)}}_{\frac{\sqrt{2}}{4}\,\frac{8}{5}=0.4\sqrt{2}} \cdot \underline{\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}}$$

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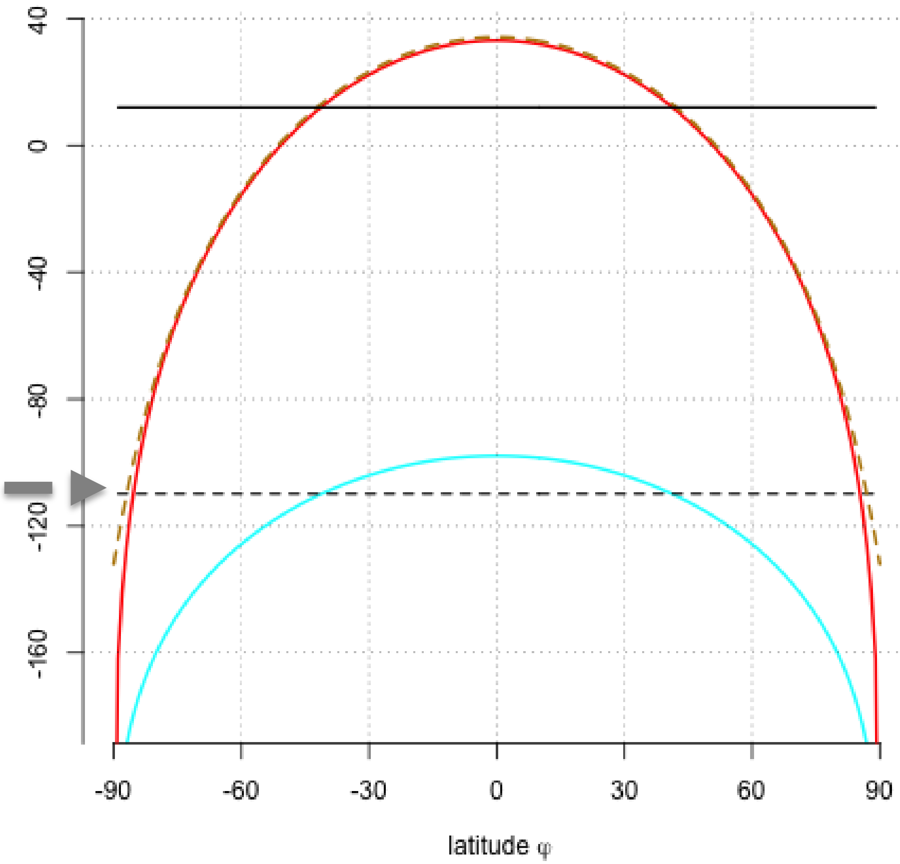
$$\overline{T} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} T(\varphi) \cos\varphi \, d\varphi$$

$$= \frac{1}{2} \frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}$$

$$= \underbrace{\frac{1}{2} \frac{1}{\sqrt{2}} \frac{\Gamma(5/8)}{\Gamma(13/8)}}_{\frac{\sqrt{2}}{4}\,\frac{8}{5}=0.4\sqrt{2}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad . \qquad (8)$$



https://esd.copernicus.org/articles/11/1195/2020/esd-11-1195-2020.pdf

Heat capacity term

The energy balance shall take the heat capacity into account:

$$C_p\,\partial_t T \;=\; (1-\alpha) S \cos\varphi \cos\Theta \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \;-\; \epsilon\sigma T^4 \tag{9}$$

The energy balance (9) is integrated over the longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int_0^{2\pi} T(t)\,d\Theta \quad \text{with} \quad \tilde{T}^4 \approx \frac{1}{2\pi}\int_0^{2\pi} T^4\,d\Theta$$

and therefore

$$\begin{aligned} C_p\,\partial_{\tilde{t}}\tilde{T} \;&=\; (1-\alpha) S \cos\varphi \cdot \frac{1}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} \cos\Theta\,d\Theta}_{2} - \epsilon\sigma\tilde{T}^4 \\ &=\; (1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \end{aligned} \tag{10}$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \tag{11}$$

shown in Fig. 2 as the read line

The new solution

The energy balance shall take the heat capacit

$$C_p\,\partial_t T \;=\; (1-\alpha) S \cos\varphi \cos\Theta$$

The ene

longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int_0^{2\pi} T(t)\,d\Theta \qquad \text{with}$$

and therefore

$$C_p\,\partial_{\tilde{t}}\tilde{T} \;=\; (1-\alpha) S\cos\varphi \cdot \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \mathrm{c}$$

$$= (1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \qquad (10)$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \qquad (11)$$

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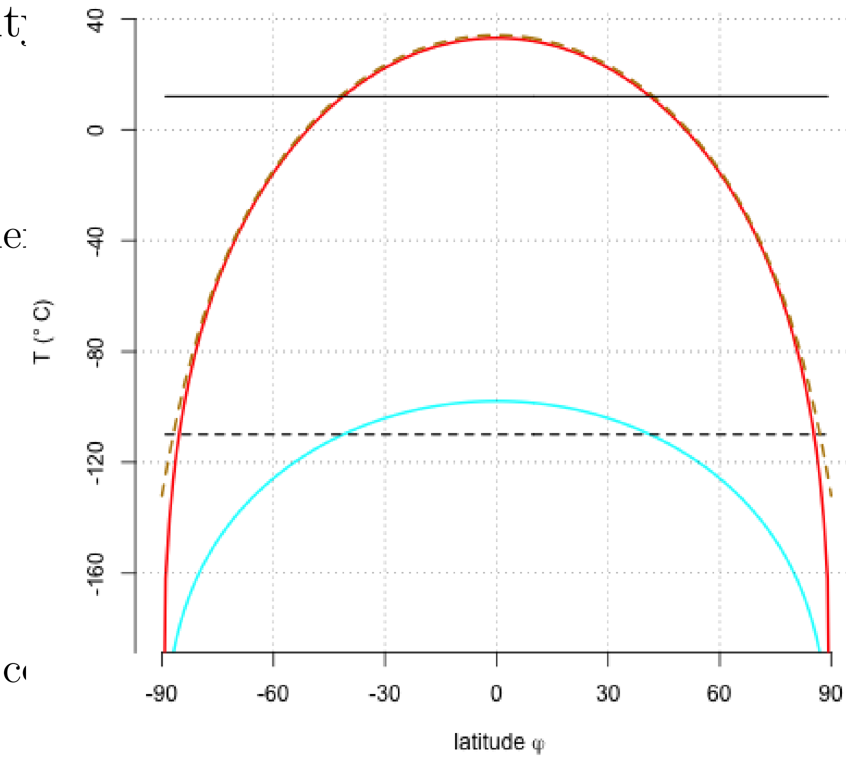

Figure 2. Latitudinal temperatures of the EBM with zero heat capacity (7) in cyan (its mean as a dashed line), the global approach (3) as solid black line, and the zonal and time averaging (11) in red. The dashed brownish curve shows the numerical solution by taking the zonal mean of (9).

$$\overline{\tilde{T}} = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad \frac{1}{2} \underbrace{\int\limits_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{1.862}$$

$$= \sqrt[4]{\frac{4}{\pi}} \frac{1.862}{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.989 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \tag{12}$$

Therefore, $\overline{\tilde{T}} = 285 \approx 288$ K, very similar as in (1). A numerical solution of (9) is shown as the brownish dashed line in Fig. 2 where the diurnal cycle has been taken into account and $C_p = C_p^a$ has been chosen as the atmospheric heat capacity

$$C_p^a = c_p p_s/g = 1004\, JK^{-1}kg^{-1} \cdot 10^5 Pa/(9.81 ms^{-2}) = 1.02 \cdot 10^7 JK^{-1}m^{-2}$$

which is the specific heat at constant pressure c_p times the total mass p_s/g. p_s is the surface pressure and g the gravity. The temperature $\overline{T}$ is 286 K, again close to 288 K.