CHAPTER

6

The equations of fluid motion

To proceed further with our discussion of the circulation of the atmosphere, and later the ocean, we must develop some of the underlying theory governing the motion of a fluid on the spinning Earth. A differentially heated, stratified fluid on a rotating planet cannot move in arbitrary paths. Indeed, there are strong constraints on its motion imparted by the angular momentum of the spinning Earth. These constraints are profoundly important in shaping the pattern of atmosphere and ocean circulation and their ability to transport properties around the globe. The laws governing the evolution of both fluids are the same and so our theoretical discussion will not be specific to either atmosphere or ocean, but can and will be applied to both. Because the properties of rotating fluids are often counterintuitive and sometimes difficult to grasp, alongside our theoretical development we will describe and carry out laboratory experiments with a tank of water on a rotating table (Fig. 6.1). Many of the laboratory

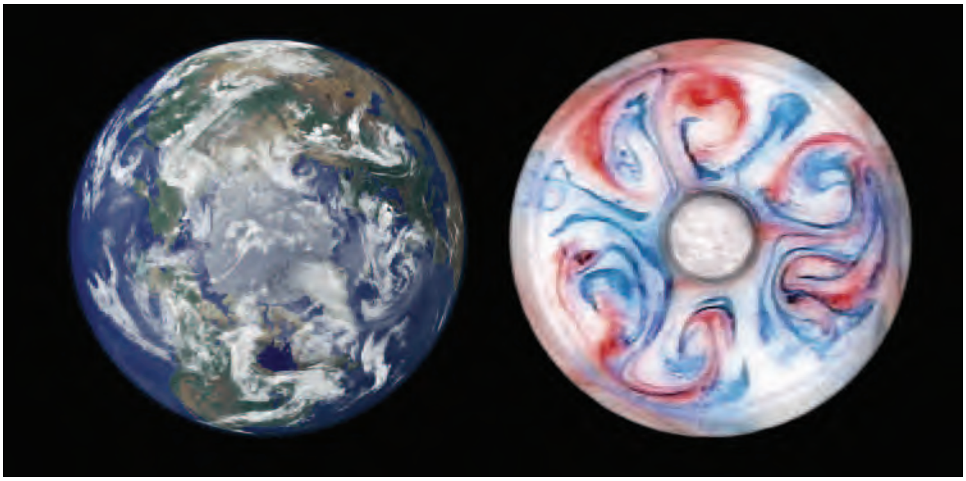

FIGURE 6.1. Throughout our text, running in parallel with a theoretical development of the subject, we study the constraints on a differentially heated, stratified fluid on a rotating planet (left), by using laboratory analogues to illustrate the fundamental processes at work (right). A complete list of the laboratory experiments can be found in Appendix A.4.

experiments we use are simplified versions of ''classics'' of geophysical fluid dynamics. They are listed in Appendix A.4. Furthermore we have chosen relatively simple experiments that, in the main, do not require sophisticated apparatus. We encourage you to ''have a go'' or view the attendant movie loops that record the experiments carried out in preparation of our text.

We now begin a more formal development of the equations that govern the evolution of a fluid. A brief summary of the associated mathematical concepts, definitions, and notation we employ can be found in Appendix A.2.

6.1. DIFFERENTIATION FOLLOWING THE MOTION

When we apply the laws of motion and thermodynamics to a fluid to derive the equations that govern its motion, we must remember that these laws apply to material elements of fluid that are usually mobile. We must learn, therefore, how to express the rate of change of a property of a fluid element, *following that element as it moves along*, rather than at a fixed point in space. It is useful to consider the following simple example.

Consider again the situation sketched in Fig. 4.13 in which a wind blows over a hill. The hill produces a pattern of waves in its lee. If the air is sufficiently saturated in water vapor, the vapor often condenses out to form a cloud at the ''ridges'' of the waves as described in Section 4.4 and seen in Figs. 4.14 and 4.15.

Let us suppose that a steady state is set up so the pattern of cloud does not change in time. If $C = C(x, y, z, t)$ is the cloud amount, where (x, y) are horizontal coordinates, z is the vertical coordinate, and t is time, then

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed point}\\ \text{in space}}} = 0,$$

in which we keep at a fixed point in space, but at which, because the air is moving, there are constantly changing fluid parcels. The derivative $\left(\frac{\partial}{\partial t}\right)_{\text{fixed point}}$ is called the *Eulerian derivative* after Euler.[1]

But C is not constant *following along a particular parcel*; as the parcel moves upwards into the ridges of the wave, it cools, water condenses out, a cloud forms, and so C increases (recall GFD Lab 1, Section 1.3.3); as the parcel moves down into the troughs it warms, the water goes back in to the gaseous phase, the cloud disappears and C decreases. Thus

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed}\\ \text{particle}}} \neq 0,$$

even though the wave-pattern is fixed in space and constant in time.

So, how do we mathematically express "differentiation following the motion"? To follow particles in a continuum, a special type of differentiation is required. Arbitrarily small variations of $C(x, y, z, t)$, a function of position and time, are given to the first order by

$$\delta C = \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial x}\delta x + \frac{\partial C}{\partial y}\delta y + \frac{\partial C}{\partial z}\delta z,$$

where the partial derivatives $\partial/\partial t$ etc. are understood to imply that the other variables are kept fixed during the differentiation. The fluid velocity is the rate of change of position of the fluid element, following that element along. The variation of a property C *following an element of fluid* is thus derived by setting $\delta x = u\delta t$, $\delta y = v\delta t$, $\delta z = w\delta t$, where u is the speed in the x-direction, v is the speed in the y-direction, and w is the speed in the z-direction, thus

$$(\delta C)_{\substack{\text{fixed}\\ \text{particle}}} = \left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}\right)\delta t,$$

where (u, v, w) is the velocity of the material element, which by definition is the fluid velocity. Dividing by δt and in the limit of small variations we see that

$$\left(\frac{\partial C}{\partial t}\right)_{\substack{\text{fixed}\\ \text{particle}}} = \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \frac{DC}{Dt},$$

in which we use the symbol $\frac{D}{Dt}$ to identify the rate of change following the motion

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \equiv \frac{\partial}{\partial t} + \mathbf{u}.\nabla. \tag{6-1}$$

Here $\mathbf{u} = (u, v, w)$ is the velocity vector, and $\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ is the gradient operator. D/Dt is called the Lagrangian derivative (after Lagrange; 1736–1813) (it is also called variously the *substantial*, the *total*, or the *material* derivative). Its physical meaning is *time rate of change of some characteristic of a particular element of fluid* (which in general is changing its position). By contrast, as introduced above, the Eulerian derivative $\partial/\partial t$ expresses the rate of change of some characteristic at a *fixed point* in space (but with constantly changing fluid element because the fluid is moving).

[1] 
Leonhard Euler (1707–1783). Euler made vast contributions to mathematics in the areas of analytic geometry, trigonometry, calculus and number theory. He also studied continuum mechanics, lunar theory, elasticity, acoustics, the wave theory of light, and hydraulics, and laid the foundation of analytical mechanics. In the 1750s Euler published a number of major works setting up the main formulas of fluid mechanics, the continuity equation, and the Euler equations for the motion of an inviscid, incompressible fluid.

Some writers use the symbol d/dt for the Lagrangian derivative, but this is better reserved for the ordinary derivative of a function of one variable, the sense it is usually used in mathematics. Thus for example the rate of change of the radius of a rain drop would be written dr/dt, with the identity of the drop understood to be fixed. In the same context D/Dt could refer to the motion of individual particles of water circulating within the drop itself. Another example is the vertical velocity, defined as $w = Dz/Dt$; if one sits in an air parcel and follows it around, w is the rate at which one's height changes.[2]

The term $\mathbf{u}.\nabla$ in Eq. 6-1 represents *advection* and is the mathematical representation of the ability of a fluid to carry its properties with it as it moves. For example, the effects of advection are evident to us every day. In the northern hemisphere, southerly winds (from the south) tend to be warm and moist because the air carries with it properties typical of tropical latitudes; northerly winds tend to be cold and dry because they advect properties typical of polar latitudes.

We will now use the Lagrangian derivative to help us apply the laws of mechanics and thermodynamics to a fluid.

6.2. EQUATION OF MOTION FOR A NONROTATING FLUID

The state of the atmosphere or ocean at any time is defined by five key variables:

$$\mathbf{u} = (u, v, w);\ p \text{ and } T,$$

(six if we include specific humidity in the atmosphere, or salinity in the ocean). Note that by using the equation of state, Eq. 1-1, we can infer ρ from p and T. To ''tie'' these variables down we need five independent equations. They are:

1. the laws of motion applied to a fluid parcel, yielding three independent

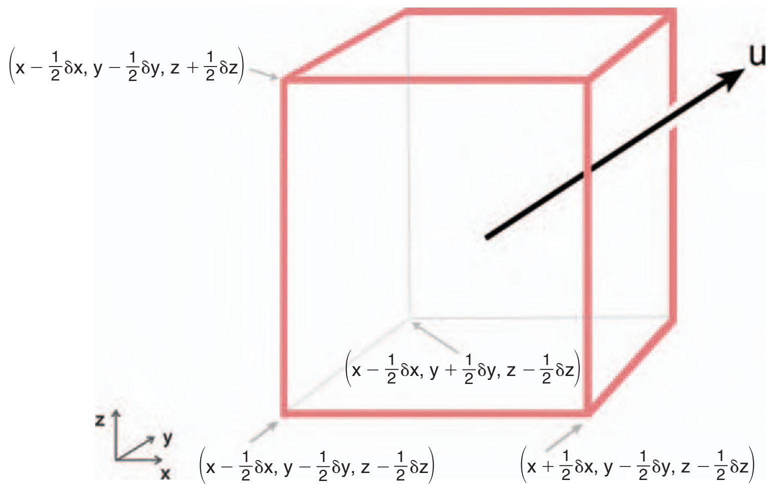

FIGURE 6.2. An elementary fluid parcel, conveniently chosen to be a cube of sides $\delta x, \delta y, \delta z$, centered on (x, y, z). The parcel is moving with velocity $\mathbf{u}$.

[2]Meteorologists like working in pressure coordinates in which p is used as a vertical coordinate rather than z. In this coordinate an equivalent definition of ''vertical velocity'' is:

$$\omega = \frac{Dp}{Dt},$$

the rate at which pressure changes as the air parcel moves around. Since pressure varies much more quickly in the vertical than in the horizontal, this is still, for all practical purposes, a measure of vertical velocity, but expressed in units of hPa s^{-1}. Note also that upward motion has negative ω.

equations in each of the three orthogonal directions

2. conservation of mass
3. the law of thermodynamics, a statement of the thermodynamic state in which the motion takes place.

These equations, five in all, together with appropriate boundary conditions, are sufficient to determine the evolution of the fluid.

6.2.1. Forces on a fluid parcel

We will now consider the forces on an elementary fluid parcel, of infinitesimal dimensions $(\delta x,\ \delta y,\ \delta z)$ in the three coordinate directions, centered on (x, y, z) (see Fig. 6.2).

Since the mass of the parcel is $\delta M = \rho\ \delta x\ \delta y\ \delta z$, then, when subjected to a net force $\mathbf{F}$, Newton's Law of Motion for the parcel is

$$\rho\ \delta x\ \delta y\ \delta z \frac{D\mathbf{u}}{Dt} = \mathbf{F}, \qquad (6\text{-}2)$$

where $\mathbf{u}$ is the parcel's velocity. As discussed earlier we must apply Eq. 6-2 to the same material mass of fluid, which means we must follow the same parcel around. Therefore, the time derivative in Eq. 6-2 is the total derivative, defined in Eq. 6-1, which in this case is

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + u\frac{\partial \mathbf{u}}{\partial x} + v\frac{\partial \mathbf{u}}{\partial y} + w\frac{\partial \mathbf{u}}{\partial z} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u}.$$

Gravity

The effect of gravity acting on the parcel in Fig. 6.2 is straightforward: the gravitational force is $g\ \delta M$, and is directed downward,

$$\mathbf{F}_{gravity} = -g\rho\widehat{\mathbf{z}}\ \delta x\ \delta y\ \delta z, \qquad (6\text{-}3)$$

where $\widehat{\mathbf{z}}$ is the unit vector in the upward direction and g is assumed constant.

Pressure gradient

Another force acting on a fluid parcel is the pressure force within the fluid. Consider Fig. 6.3. On each face of our parcel there is a force (directed inward) acting on the parcel equal to the pressure on that face multiplied by the area of the face. On face A, for example, the force is

$$F(A) = p(x - \frac{\delta x}{2}, y, z)\ \delta y\ \delta z,$$

directed in the positive x-direction. Note that we have used the value of p at the midpoint of the face, which is valid for small δy, δz. On face B, there is an x-directed force

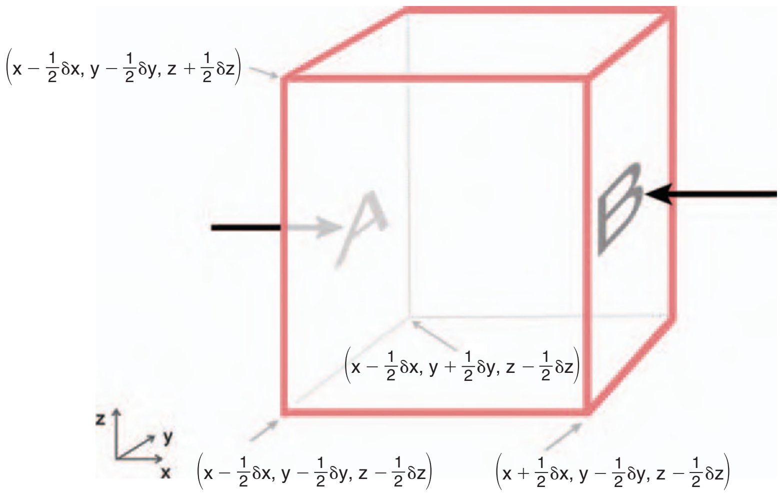

FIGURE 6.3. Pressure gradient forces acting on the fluid parcel. The pressure of the surrounding fluid applies a force to the right on face A and to the left on face B.

$$F(B) = -p(x + \frac{\delta x}{2}, y, z)\ \delta y\ \delta z,$$

which is negative (toward the left). Since these are the only pressure forces acting in the x-direction, the net x-component of the pressure force is

$$F_x = \left[p(x - \frac{\delta x}{2}, y, z) - p(x + \frac{\delta x}{2}, y, z)\right]\ \delta y\ \delta z.$$

If we perform a Taylor expansion (see Appendix A.2.1) about the midpoint of the parcel, we have

$$p(x + \frac{\delta x}{2}, y, z) = p(x, y, z) + \frac{\delta x}{2}\left(\frac{\partial p}{\partial x}\right),$$
$$p(x - \frac{\delta x}{2}, y, z) = p(x, y, z) - \frac{\delta x}{2}\left(\frac{\partial p}{\partial x}\right),$$

where the pressure gradient is evaluated at the midpoint of the parcel, and where we have neglected the small terms of $O(\delta x^2)$ and higher. Therefore the x-component of the pressure force is

$$F_x = -\frac{\partial p}{\partial x}\ \delta x\ \delta y\ \delta z.$$

It is straightforward to apply the same procedure to the faces perpendicular to the y- and z-directions, to show that these components are

$$F_y = -\frac{\partial p}{\partial y}\ \delta x\ \delta y\ \delta z,$$
$$F_z = -\frac{\partial p}{\partial z}\ \delta x\ \delta y\ \delta z.$$

In total, therefore, the net pressure force is given by the vector

$$\begin{aligned} \mathbf{F}_{pressure} &= \left(F_x, F_y, F_z\right) \\ &= -\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right)\ \delta x\ \delta y\ \delta z \\ &= -\nabla p\ \delta x\ \delta y\ \delta z. \end{aligned} \tag{6-4}$$

Note that the net force depends only on the *gradient* of pressure, ∇p; clearly, a uniform pressure applied to all faces of the parcel would not introduce any net force.

Friction

For typical atmospheric and oceanic flows, frictional effects are negligible except close to boundaries where the fluid rubs over the Earth's surface. The atmospheric boundary layer—which is typically a few hundred meters to 1 km or so deep—is exceedingly complicated. For one thing, the surface is not smooth; there are mountains, trees, and other irregularities that increase the exchange of momentum between the air and the ground. (This is the main reason why frictional effects are greater over land than over ocean.) For another, the boundary layer is usually *turbulent*, containing many small-scale and often vigorous eddies; these eddies can act somewhat like mobile molecules and diffuse momentum more effectively than molecular viscosity. The same can be said of oceanic boundary layers, which are subject, for example, to the stirring by turbulence generated by the action of the wind, as will be discussed in Section 10.1. At this stage, we will not attempt to describe such effects quantitatively but instead write the consequent frictional force on a fluid parcel as

$$\mathbf{F}_{fric} = \rho\ \mathcal{F}\ \delta x\ \delta y\ \delta z, \tag{6-5}$$

where, for convenience, $\mathcal{F}$ is the frictional force *per unit mass*. For the moment we will not need a detailed theory of this term. Explicit forms for $\mathcal{F}$ will be discussed and employed in Sections 7.4.2 and 10.1.

6.2.2. The equations of motion

Putting all this together, Eq. 6-2 gives us

$$\rho\ \delta x\ \delta y\ \delta z \frac{D\mathbf{u}}{Dt} = \mathbf{F}_{gravity} + \mathbf{F}_{pressure} + \mathbf{F}_{fric}.$$

Substituting from Eqs. 6-3, 6-4, and 6-5, and rearranging slightly, we obtain

$$\frac{D\mathbf{u}}{Dt} + \frac{1}{\rho}\nabla p + g\hat{\mathbf{z}} = \mathcal{F}. \tag{6-6}$$

This is our equation of motion for a fluid parcel.

Note that because of our use of vector notation, Eq. 6-6 seems rather simple. However, when written out in component form, as below, it becomes somewhat intimidating, even in Cartesian coordinates:

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} &= \mathcal{F}_x \quad (a)\\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial y} &= \mathcal{F}_y \quad (b)\\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g &= \mathcal{F}_z \,. \quad (c)
\end{aligned} \tag{6-7}$$

Fortunately we will often be able to make a number of simplifications. One such simplification, for example, is that, as discussed in Section 3.2, large-scale flow in the atmosphere and ocean is almost always close to hydrostatic balance, allowing Eq. 6-7c to be radically simplified as follows.

6.2.3. Hydrostatic balance

From the vertical equation of motion, Eq. 6-7c, we can see that if friction and the vertical acceleration Dw/Dt are negligible, we obtain

$$\frac{\partial p}{\partial z} = -\rho g, \tag{6-8}$$

thus recovering the equation of hydrostatic balance, Eq. 3-3. For large-scale atmospheric and oceanic systems in which the vertical motions are weak, the hydrostatic equation is almost always accurate, though it may break down in vigorous systems of smaller horizontal scale such as convection.[3]

6.3. CONSERVATION OF MASS

In addition to Newton's laws there is a further constraint on the fluid motion: *conservation of mass*. Consider a fixed *fluid volume* as illustrated in Fig. 6.4. The volume has dimensions $(\delta x, \delta y, \delta z)$. The mass of the fluid occupying this volume, $\rho\, \delta x\, \delta y\, \delta z$, may change with time if ρ does so. However, mass continuity tells us that this can only

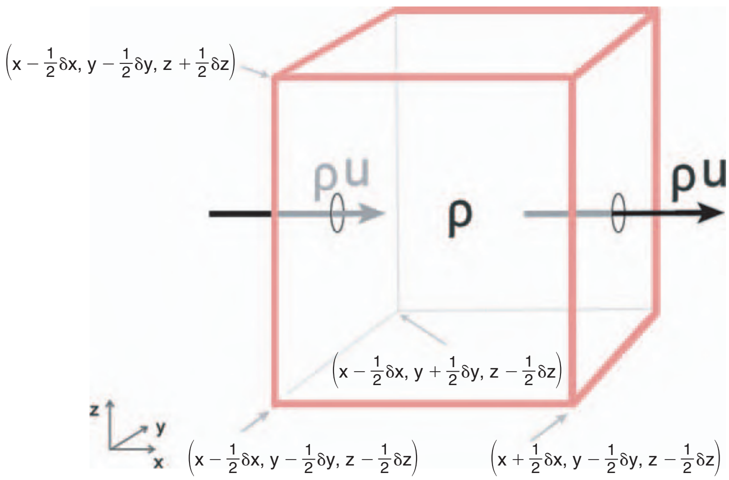

FIGURE 6.4. The mass of fluid contained in the fixed volume, $\rho\delta x\, \delta y\, \delta z$, can be changed by fluxes of mass out of and into the volume, as marked by the arrows.

[3]It might appear from Eq. 6-7c that $|Dw/Dt| << g$ is a sufficient condition for the neglect of the acceleration term. This indeed is almost always satisfied. However, for hydrostatic balance to hold to sufficient accuracy to be useful, the condition is actually $|Dw/Dt| << g\Delta\rho/\rho$, where $\Delta\rho$ is a typical density variation on a pressure surface. Even in quite extreme conditions this more restrictive condition turns out to be very well satisfied.

occur if there is a flux of mass into (or out of) the volume, meaning that

$$\frac{\partial}{\partial t}\left(\rho\ \delta x\,\delta y\,\delta z\right) = \frac{\partial \rho}{\partial t}\ \delta x\ \delta y\ \delta z$$
$$= \left(\text{net mass flux into the volume}\right).$$

Now the volume flux in the x-direction per unit time into the left face in Fig. 6.4 is $u\left(x - 1/2\,\delta x, y, z\right)\delta y\ \delta z$, so the corresponding mass flux is $[\rho u]\left(x - 1/2\,\delta x, y, z\right)\delta y\ \delta z$, where $[\rho u]$ is evaluated at the left face. The flux out through the right face is $[\rho u]\left(x + 1/2\,\delta x, y, z\right)\delta y\ \delta z$; therefore the net mass import in the x-direction into the volume is (again employing a Taylor expansion)

$$-\frac{\partial}{\partial x}\left(\rho u\right)\ \delta x\ \delta y\ \delta z.$$

Similarly the rate of net import of mass in the y-direction is

$$-\frac{\partial}{\partial y}\left(\rho v\right)\ \delta x\ \delta y\ \delta z,$$

and in the z-direction is

$$-\frac{\partial}{\partial z}\left(\rho w\right)\ \delta x\ \delta y\ \delta z.$$

Therefore the net mass flux into the volume is $-\nabla\cdot\left(\rho\mathbf{u}\right)\delta x\ \delta y\ \delta z$. Thus our *equation of continuity* becomes

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\mathbf{u}\right) = 0. \qquad (6\text{-}9)$$

This has the general form of a physical conservation law:

$$\frac{\partial\,\text{Concentration}}{\partial t} + \nabla\cdot\left(\text{flux}\right) = 0$$

in the absence of sources and sinks.

Using the *total derivative* D/Dt, Eq. 6-1, and noting that $\nabla\cdot\left(\rho\mathbf{u}\right) = \rho\nabla\cdot\mathbf{u} + \mathbf{u}\cdot\nabla\rho$ (see the vector identities listed in Appendix A.2.2) we may therefore rewrite Eq. 6-9 in the alternative, and often very useful, form:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0. \qquad (6\text{-}10)$$

6.3.1. Incompressible flow

For incompressible flow (e.g., for a liquid such as water in our laboratory tank or in the ocean), the following simplified approximate form of the continuity equation almost always suffices:

$$\nabla\cdot\mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \qquad (6\text{-}11)$$

Indeed this is the definition of incompressible flow: it is *nondivergent*—no bubbles allowed! Note that in any real fluid, Eq. 6-11 is never *exactly* obeyed. Moreover, despite Eq. 6-10, use of the incompressibility condition should not be understood as implying that $\frac{D\rho}{Dt} = 0$. On the contrary, the density of a parcel of water can be changed by internal heating and/or conduction (see, for example, Section 11.1). Although these density changes may be large enough to affect the buoyancy of the fluid parcel, they are too small to affect the mass budget. For example, the thermal expansion coefficient of water is typically $2\times10^{-4}\,\mathrm{K}^{-1}$, and so the volume of a parcel of water changes by only 0.02% per degree of temperature change.

6.3.2. Compressible flow

A compressible fluid, such as air, is nowhere close to being nondivergent—ρ changes markedly as fluid parcels expand and contract. This is inconvenient in the analysis of atmospheric dynamics. However it turns out that, provided the hydrostatic assumption is valid (as it nearly always is), one can get around this inconvenience by adopting pressure coordinates. In pressure coordinates, $\left(x, y, p\right)$, the elemental fixed "volume" is $\delta x\ \delta y\ \delta p$. Since $z = z\left(x, y, p\right)$, the vertical dimension of the elemental volume (in geometric coordinates) is $\delta z = \partial z/\partial p\ \delta p$, and so its mass is δM given by

$$\begin{aligned}\delta M &= \rho\ \delta x\ \delta y\ \delta z\\ &= \rho\left(\frac{\partial p}{\partial z}\right)^{-1}\ \delta x\ \delta y\ \delta p\\ &= -\frac{1}{g}\ \delta x\ \delta y\ \delta p,\end{aligned}$$

where we have used hydrostatic balance, Eq. 3-3. So the mass of an elemental fixed volume *in pressure coordinates* cannot change! In effect, comparing the top and bottom line of the previous equation, the equivalent of "density" in pressure coordinates—the mass per unit "volume"—is $1/g$, a constant. Hence, in the pressure-coordinate version of the continuity equation, there is no term representing rate of change of density; it is simply

$$\nabla_p \cdot \mathbf{u}_p = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0, \qquad \text{(6-12)}$$

where the subscript p reminds us that we are in pressure coordinates. The greater simplicity of this form of the continuity equation, as compared to Eqs. 6-9 or 6-10, is one of the reasons why pressure coordinates are favored in meteorology.

6.4. THERMODYNAMIC EQUATION

The equation governing the evolution of temperature can be derived from the first law of thermodynamics applied to a moving parcel of fluid. Dividing Eq. 4-12 by δt and letting $\delta t \longrightarrow 0$ we find:

$$\frac{DQ}{Dt} = c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}. \qquad \text{(6-13)}$$

DQ/Dt is known as the *diabatic heating rate* per unit mass. In the atmosphere, this is mostly due to latent heating and cooling (from condensation and evaporation of H_2O) and radiative heating and cooling (due to absorption and emission of radiation). If the heating rate is zero then $DT/Dt = \frac{1}{\rho c_p} Dp/Dt$, and, as discussed in Section 4.3.1, the temperature of a parcel will decrease in ascent (as it moves to lower pressure) and increase in descent (as it moves to higher pressure). Of course this is why we introduced potential temperature in Section 4.3.2; in adiabatic motion, θ is conserved. Written in terms of θ, Eq. 6-13 becomes

$$\frac{D\theta}{Dt} = \left(\frac{p}{p_0}\right)^{-\kappa} \frac{\dot{Q}}{c_p}, \qquad \text{(6-14)}$$

where $\dot{Q}$ (with a dot over the top) is a shorthand for $\frac{DQ}{Dt}$. Here θ is given by Eq. 4-17, the factor $\left(\frac{p}{p_0}\right)^{-\kappa}$ converts from T to θ, and $\frac{\dot{Q}}{c_p}$ is the diabatic heating in units of $\mathrm{K\,s^{-1}}$. The analogous equations that govern the evolution of temperature and salinity in the ocean will be discussed in Chapter 11.

6.5. INTEGRATION, BOUNDARY CONDITIONS, AND RESTRICTIONS IN APPLICATION

The three equations in 6–7, together with 6–11 or 6–12, and 6–14 are our five equations in five unknowns. Together with initial conditions and boundary conditions, they are sufficient to determine the evolution of the flow.

Before going on, we make some remarks about restrictions in the application of our governing equations. The equations themselves apply very accurately to the detailed motion. In practice, however, variables are always averages over large volumes. We can only tentatively suppose that the equations are applicable to the average motion, such as the wind integrated over a 100-km square box. Indeed, the assumption that the equations do apply to average motion is often incorrect. This fact is associated with the representation of turbulent scales, both small scale and large scale. The treatment of turbulent motions remains one of the major challenges in dynamical meteorology and oceanography. Finally, our governing equations have been derived relative to a 'fixed' coordinate system. As we now go on to discuss, this is not really a restriction, but is usually an inconvenience.