1. **Ocean thermohaline circulation** *(3 points)*

   Consider a geostrophic flow (u, v)

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \tag{1}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \quad . \tag{2}$$

   The meridional overturning stream function $\Phi(y, z)$ is defined via

$$\Phi(y,z) = \int_0^z \frac{\partial \Phi}{\partial \tilde{z}}\, d\tilde{z} \tag{3}$$

$$\frac{\partial \Phi}{\partial \tilde{z}} = \int_{x_e}^{x_w} v(x, y, \tilde{z})\, dx \quad \text{(zonally integrated transport),} \tag{4}$$

   where x_e and x_w are the eastward and westward boundaries in the ocean basin (think e.g. of the Atlantic Ocean). Units of Φ are $m^3 s^{-1}$. At the surface $\Phi(y, 0) = 0$.

   Calculate $\Phi(y, z)$ as a function of density ρ at the basin boundaries!

   Use

$$v(x, y, z) = \frac{1}{\rho_0 f}\frac{\partial p}{\partial x}$$

   and the hydrostatic approximation

$$\frac{\partial p}{\partial z} = -g\rho$$

2. **Wind-driven ocean circulation** *(8 points)*

   The **Sverdrup transport** V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \tag{5}$$

   where τ_x and τ_y are the components of the wind stress.

   The **Ekman transports** V_E, U_E describe the dynamics in the upper mixed layer:

$$fV_E = -\tau_x/\rho_0 \quad , \quad fU_E = \tau_y/\rho_0 \tag{6}$$

   where $U_E = \int_{-E}^0 u dz$ and $V_E = \int_{-E}^0 v dz$ are the depth-integrated velocities in the thin friction-dominated Ekman layer at the sea surface.

Ekman vertical velocity w_E: Using the continuity equation, the divergence of the Ekman transports leads to a vertical velocity w_E at the bottom of the Ekman layer:

$$w_E = -\int_{-E}^{0} \frac{\partial w}{\partial z} dz = \frac{\partial}{\partial x} U_E + \frac{\partial}{\partial y} V_E = \frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0\, f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0\, f}\right) \quad . \tag{7}$$

a) Assume that the windstress is only zonal with

$$\tau_x = -\tau_0 \cos(\pi y/B) \qquad \text{for an ocean basin with } \; 0 < x < L,\; 0 < y < B. \tag{8}$$

Calculate the Sverdrup transport, Ekman transports, and Ekman pumping velocity for this special case.

b) Make a schematic diagram of the windstress, Sverdrup transport, Ekman transports, and Ekman pumping velocity.

c) Using a), at what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B = 5000$ km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

d) Using the values in b), calculate the maximum of w_E for constant f.

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Hanna Knahl (hanna.knahl@awi.de), Alexander Thorneloe (alexander.thorn@awi.de).*