1. **Non-dimensional vorticity dynamics including wind stress** *(3 points)*

The Sverdrup transport V for the depth-integrated flow is calculated by

$$\rho_0 \beta V = \frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x \qquad (1)$$

where τ_x and τ_y are the components of the wind stress.

a) Show that (1) is a special case of the vorticity equation

$$\frac{D}{Dt}(\zeta + f) = A_H \nabla^2 \zeta + \frac{1}{\rho E}\left(\frac{\partial}{\partial x}\tau_y - \frac{\partial}{\partial y}\tau_x\right) \quad . \qquad (2)$$

E is the Ekman layer, see Table 1.

b) Derive the the non-dimensional version of (2). Include the Reynolds number $Re = UL/A_H$, Rossby number $Ro = U/(f_0 L)$, and the wind stress strength number $\alpha = \tau_0 L/(\rho_0 E U^2)$.

c) Estimate the order of magnitude of the characteristic numbers for the ocean ! Use Table 1.

	Quantity	Ocean
horizontal velocity	U	$1.6 \cdot 10^{-2}\, ms^{-1}$
horizontal length	L	$10^6\, m$
vertical length	E	$10^2\, m$
wind stress	τ_0	$1.5 \cdot 10^{-1}\, Pa$
Coriolis parameter at 45°N	$f_0 = 2\Omega \sin \varphi_0$	$10^{-4}\, s^{-1}$
density	ρ_0	$10^3\, kgm^{-3}$
viscosity (turbulent)	A_H	$10^2 - 10^4\, m^2 s^{-1}$

Table 1: Table shows the typical scales in the ocean system for the exercise.

2. **Rossby, gravity, and Kelvin waves** *(5 points)*

Start with the shallow water equations

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \tag{3}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \tag{4}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{5}$$

with H=const. as mean depth and η as surface anomaly.

a) With the elimination of the fast gravity waves in equation (5)

$$\frac{\partial \eta}{\partial t} = 0$$

derive the dispersion relation for divergence-free Rossby waves! Ansatz: Introduce a streamfunction for u,v: $\Psi \sim \exp(ikx + ily - i\omega t)$

b) With the assumption of $f = f_0 = 0$ derive the dispersion relation for gravity waves! The restoring force is related to gravity. Ansatz: take one of the equations (3,4,5) and derive the solution.

c) Kelvin waves:
What is the dispersion relation for Kelvin waves?
Make a sketch of the coastally trapped Kelvin wave on the Northern Hemisphere ocean basin.
Make a sketch of the equatorial trapped Kelvin waves.

d) Explain the difference between dispersive and non-dispersive waves!
You could use the $\omega(k)$ formula for Rossby and Kelvin waves.

The following questions are related to the box model (2 points each): These are extra points!

In the regions of deep water formation in the North Atlantic, relatively small amounts of fresh water added to the surface can stabilize the water column to the extent that convection can be prevented from occurring. Such interruption decreases the poleward oceanic mass transport Φ. Furthermore, this perturbation of the meridional transport can be amplified by positive feedbacks: a weaker northward salt transport brings less dense water to high latitudes, which further reduces the high-latitude density. Discuss the case where the initial conditions in salinity at different latitudes is changed.

Show this scenario in the box model by adding freshwater to the high-latitude northern box!

3. **Calculate the ocean heat transport in the model and compare it with the following estimate!**

$$
\begin{aligned}
H &= \int_{bottom}^{top} \rho_0 v T dz && (6)\\
&= -c_p \int_{bottom}^{top} \frac{\partial \Phi}{\partial z} \quad T dz && (7)\\
&= c_p \int_{bottom}^{top} \Phi \quad \frac{\partial T}{\partial z} dz && (8)\\
&= c_p \int_{T(bottom)}^{T(top)} \quad \Phi dT && (9)
\end{aligned}
$$

where $\Phi = \rho_0 \Phi_{MOC}$ with Φ_{MOC} being the volume transport. Therefore, the heat transport can be estimated in terms of the mass transport in temperature layers:

$$H = c_p \underbrace{(T(top) - T(bottom))}_{15K} \quad \underbrace{\Phi_{max}}_{20 \cdot 10^9 kg/s} \qquad (10)$$

which is about 1.2 $PW (PW = 10^{15} W)$.

4. Comment on the scenario of climate change as shown in the cinema movie The Day After Tomorrow: link to the website or go to the trailer.

5. Which feedbacks are acting for global warming? You can change the long wave radiation. A doubling of pCO_2 is equivalent to an additional forcing of 4 Wm^{-2}. For this you have to modify the net radiation balance through reduction in the outgoing longwave radiation (parameter γ). Additional radiative forcing may come from increased tracer gas concentrations in the atmosphere. Please evaluate the hydrological cycle and atmospheric heat transports! What is the change in the overturning rate?

6. Change the ocean heat capacity by a factor of 10 and describe the change in the response to warming induced by 90% of the longwave radiation.

7. The initial values of the model represent averages for present-day climate conditions. Can you derive a glacial climate? The glacial climate was 3 K colder in the tropics.

Notes on submission form of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. The answers to the questions can be send until the due date (12:00) to Fernanda Matos (Fernanda.Matos@awi.de), Ahmadreza Masoum (Ahmadreza.Masoum@awi.de).*