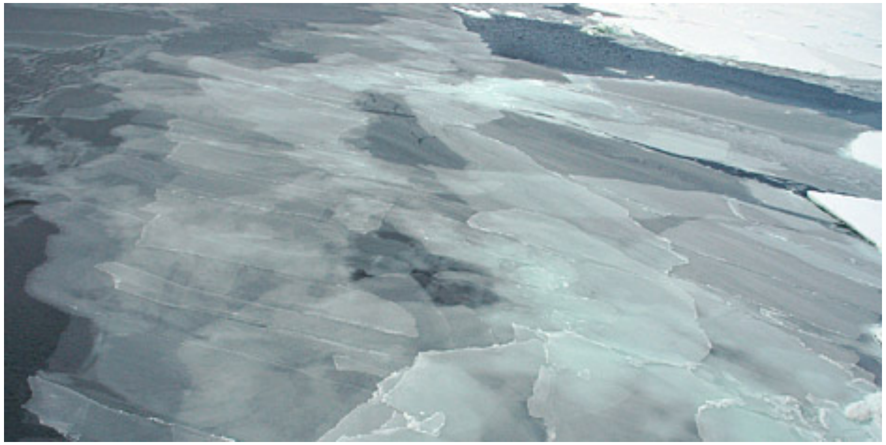
Gerrit Lohmann
MES, 23.06.2021

EBM & Sea ice

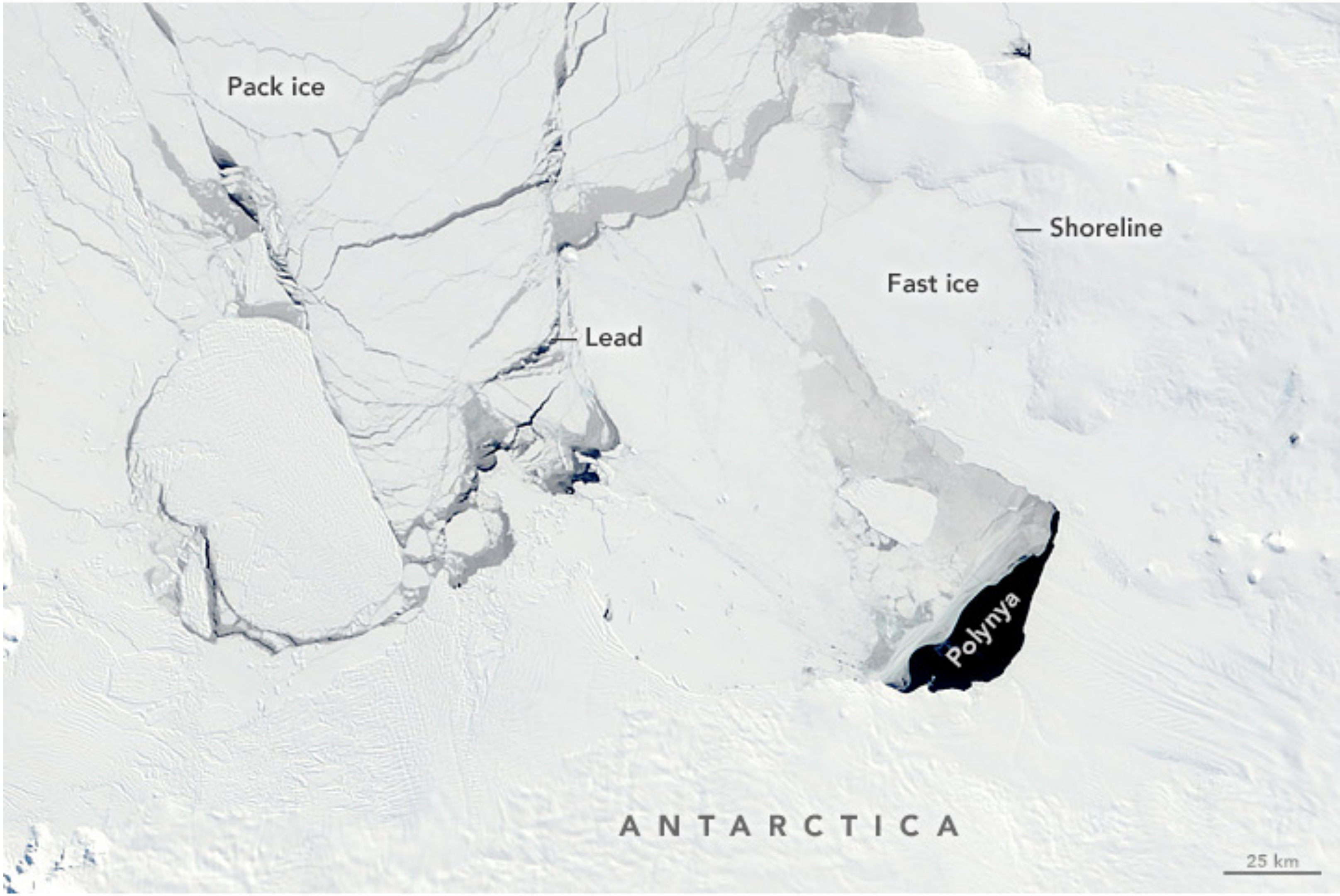
Sea ice

Sea ice is frozen seawater that floats on the ocean surface. It forms in both the Arctic and the Antarctic in each hemisphere's winter; it retreats in the summer, but does not completely disappear. This floating ice has a profound influence on the polar environment, influencing ocean circulation, weather, and regional climate.

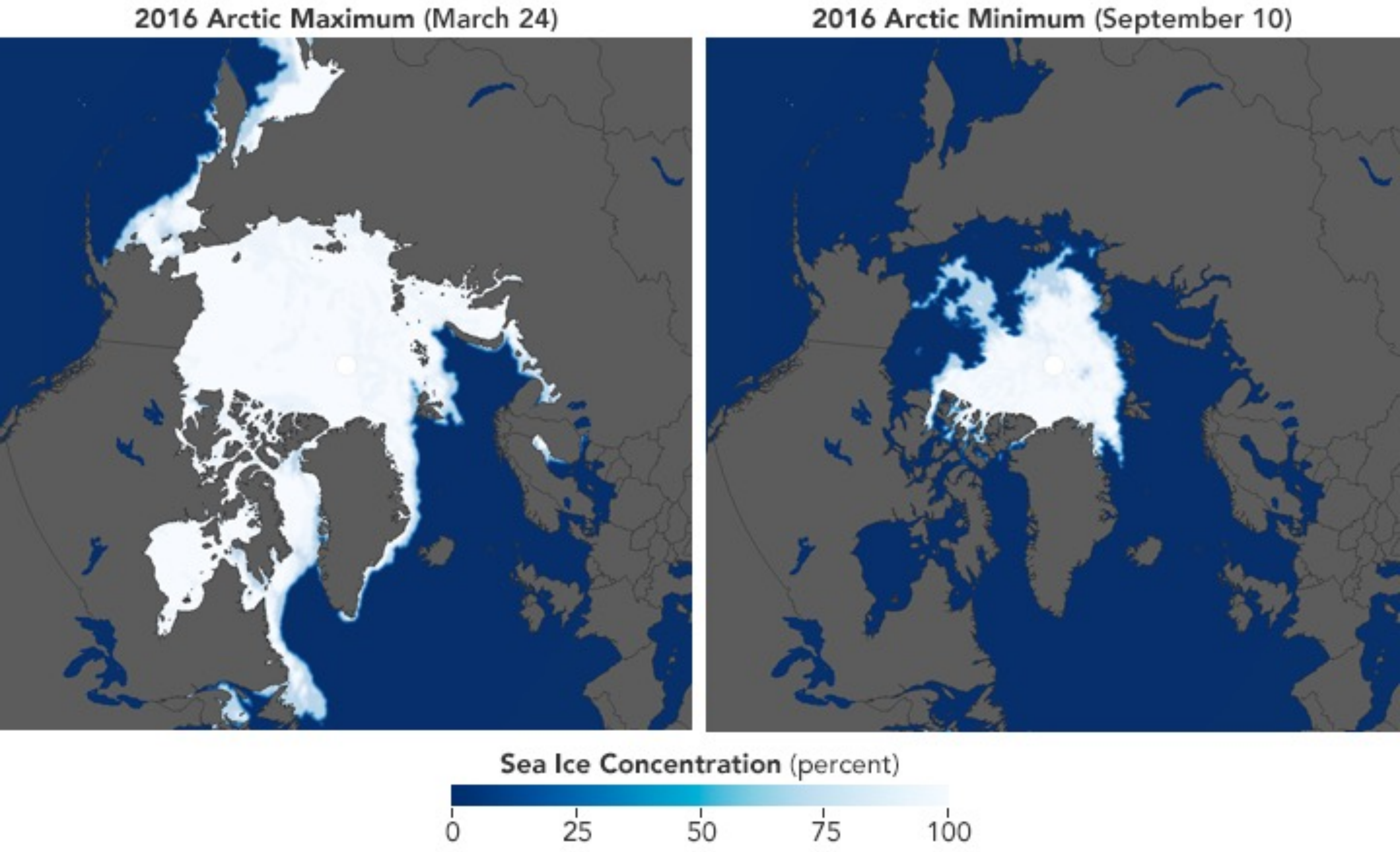



Sea ice begins as thin sheets of smooth nilas in calm water (top) or disks of pancake ice in choppy water (top right).
Individual pieces pile up to form rafts and eventually solidify (lower left). Over time, large sheets of ice collide, forming thick pressure ridges along the margins (lower right).

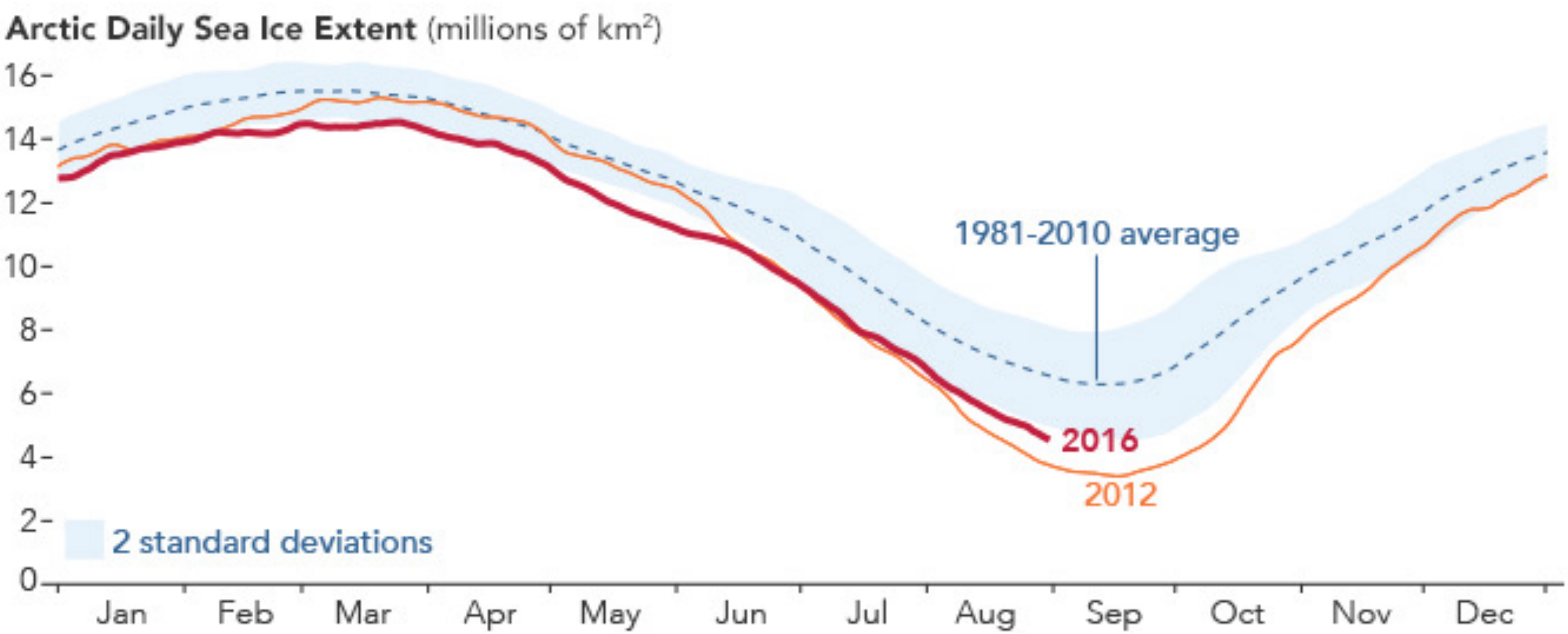

Sea ice



Arctic sea ice



Seasonal cycle



Sea ice dynamics

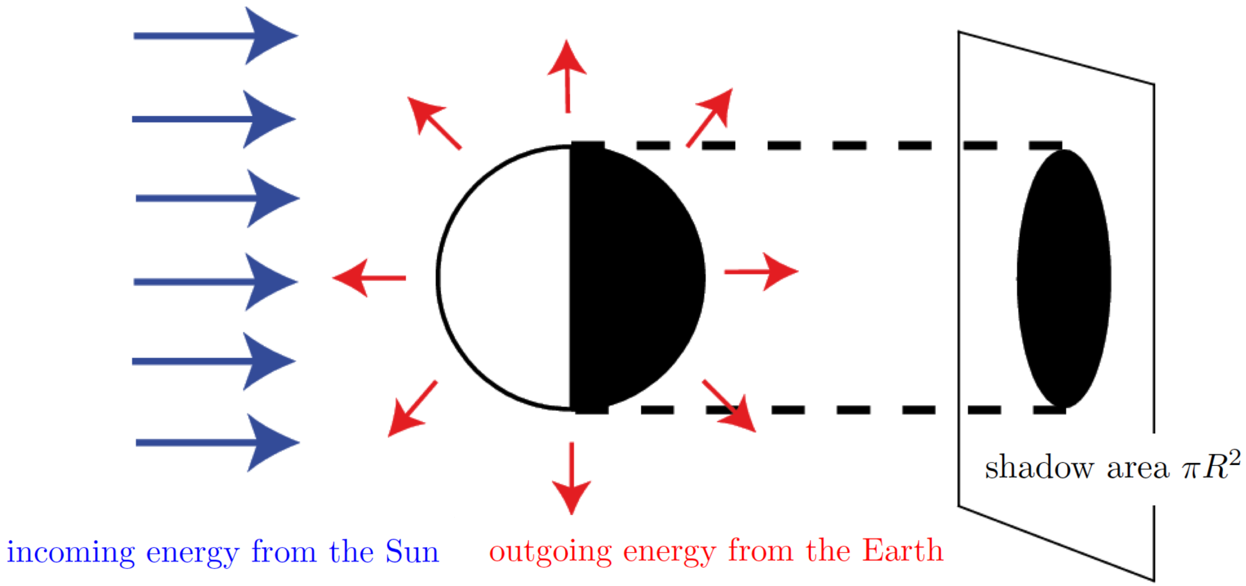
https://fesom.de/media/video/

- https://youtu.be/Im-v6w5_NFw

Energy balance model

$$(1-\alpha)S\pi R^2 = 4\pi R^2 \epsilon\sigma T^4$$

$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$



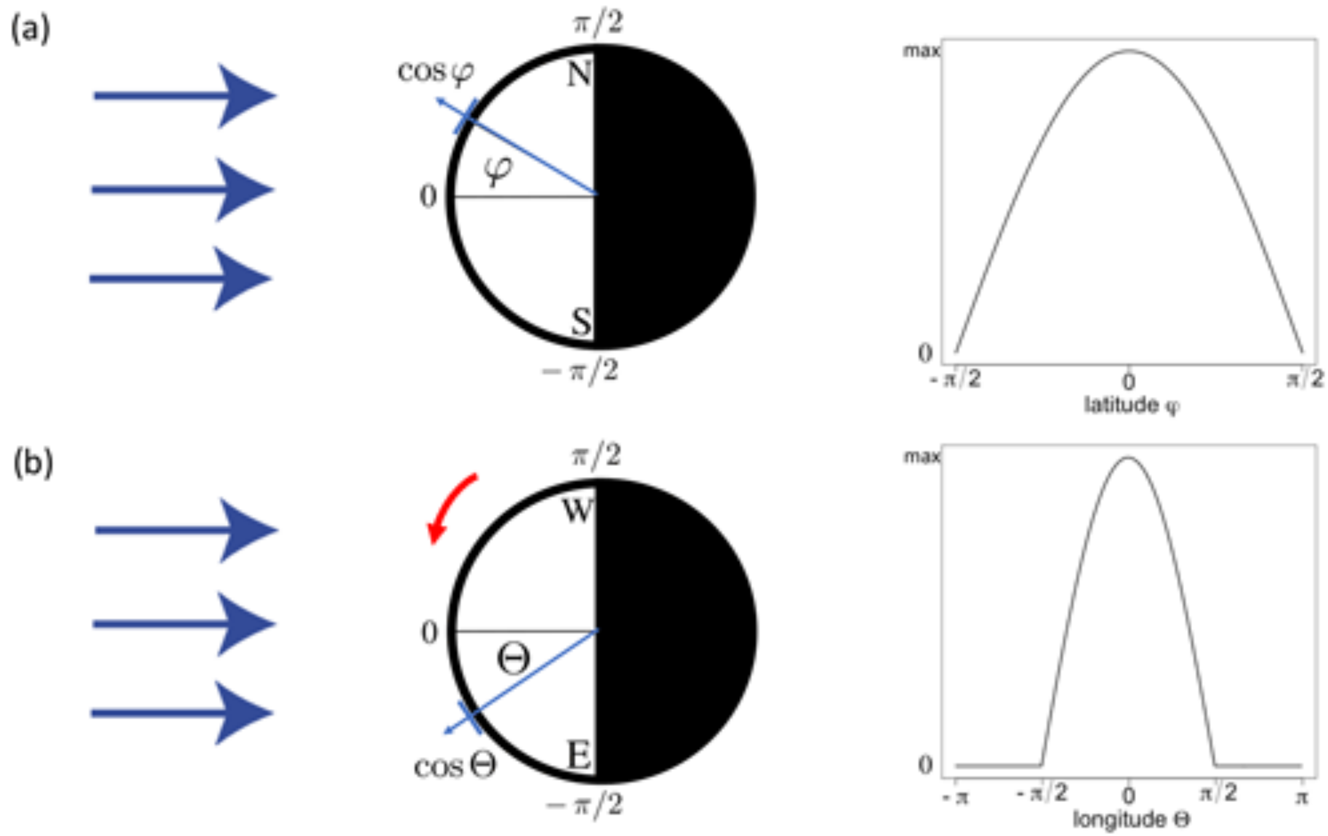
Heat capacity

Heat transport

GCM experiment

Simple models are helpful for understanding of
- Critical parameters
- Concepts of climate

EBM


(a)
$\pi/2$
$\cos\varphi$
N
φ
0
S
$-\pi/2$
max
0
$-\pi/2$
0
$\pi/2$
latitude φ
(b)
$\pi/2$
W
0
Θ
$\cos\Theta$
E
$-\pi/2$
max
0
$-\pi$
$-\pi/2$
0
$\pi/2$
π
longitude Θ

What we really want to know ...

What we really want is the mean of the temperature $\overline{T}$. Therefore, we take the fourth root of (4):

$$T = \sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}} \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \quad . \tag{6}$$

If we calculate the zonal mean of (6) by integration at the latitudinal cycles we have

$$\begin{aligned} T(\varphi) &= \frac{1}{2\pi}\int_{-\pi/2}^{-\pi/2}\sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}}d\Theta \\ &= \frac{\sqrt{2}}{2\pi}\underbrace{\int_{-\pi/2}^{\pi/2}(\cos\Theta)^{1/4}d\Theta}_{2.700} \quad \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \\ &= 0.608\cdot\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \end{aligned} \tag{7}$$

as a function on latitude (Fig. 2).

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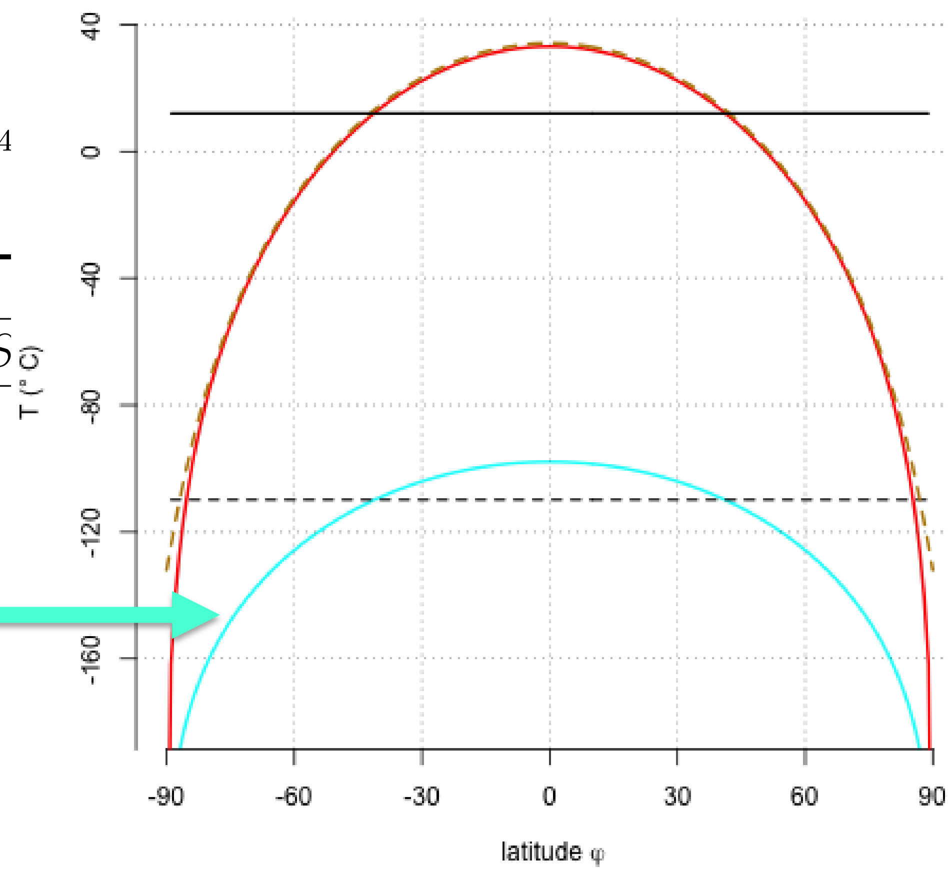
$$= \frac{\sqrt{2}}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\Theta)^{1/4}}_{2.700}$$

$$= 0.608 \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

as a function on latitude (Fig. 2).

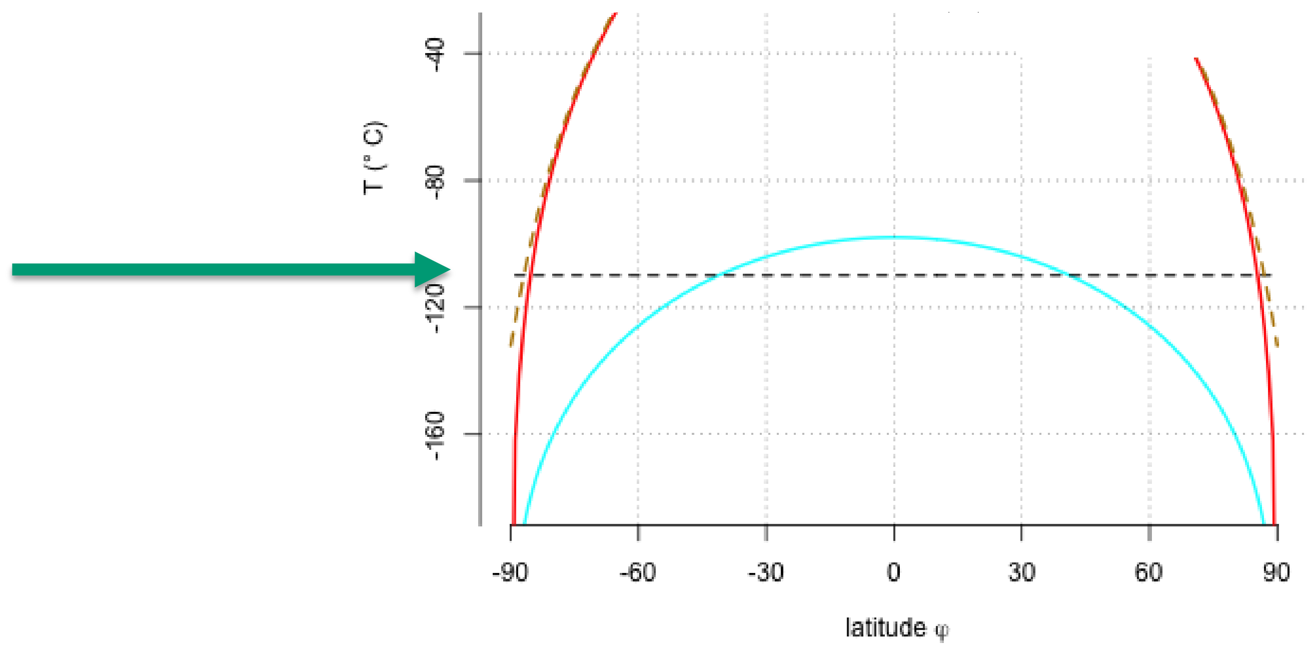


When we integrate this over the latitudes, we obtain

$$\overline{T} = \frac{1}{2} \int_{-\pi/2}^{\pi/2} T(\varphi)\, \cos\varphi \, d\varphi$$

$$= \frac{0.608}{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad \underbrace{\int_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{1.862}$$

$$= 0.4\sqrt{2}\, \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.566\, \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \tag{8}$$



What is the difference ?

Let us have a closer look onto (1). The local radiative equilibrium of the Earth is

$$\epsilon\sigma T^4 = (1-\alpha) S \cos\varphi \cos\Theta \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \tag{4}$$

where φ and Θ are the latitude and longitute, respectively. Integration over the Earth surface is

$$\int_{-\pi/2}^{\pi/2} \left(\int_0^{2\pi} \epsilon\sigma T^4 \, R \cos\varphi d\Theta \right) R d\varphi \quad = (1-\alpha) S \int_{-\pi/2}^{\pi/2} R\cos^2\varphi d\varphi \cdot \int_{-\pi/2}^{\pi/2} R \cos\Theta d\Theta$$

$$\epsilon\sigma R^2 \frac{4\pi}{4\pi} \int^{\pi/2} \left(\int^{2\pi} T^4 \, \cos\varphi d\Theta \right) d\varphi = (1-\alpha) S R^2 \int^{\pi/2} \cos^2\varphi \, d\varphi \cdot \int^{\pi/2} \cos\Theta \, d\Theta$$

$$\epsilon\sigma 4\pi \overline{T^4} \quad = (1-\alpha) S \; \pi \tag{5}$$

giving a similar formula as (3) with the definition for the average $\overline{T^4}$.

Heat capacity term

The energy balance shall take the heat capacity into account:

$$C_p\,\partial_t T \;=\; (1-\alpha)S\cos\varphi\cos\Theta \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \;-\; \epsilon\sigma T^4 \tag{9}$$

The energy balance (9) is integrated over the longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int_0^{2\pi} T(t)\,d\Theta \qquad \text{with} \qquad \tilde{T}^4 \approx \frac{1}{2\pi}\int_0^{2\pi} T^4\,d\Theta$$

and therefore

$$\begin{aligned} C_p\,\partial_{\tilde{t}}\tilde{T} \;&=\; (1-\alpha)S\cos\varphi\cdot\frac{1}{2\pi}\underbrace{\int_{-\pi/2}^{\pi/2}\cos\Theta\,d\Theta}_{2} - \epsilon\sigma\tilde{T}^4 \\ &=\; (1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \end{aligned} \tag{10}$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}}\cdot\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \tag{11}$$

shown in Fig. 2 as the read line

The new solution

The energy balance shall take the heat capacit

$$C_p\,\partial_t T \;=\; (1-\alpha)S\cos\varphi\cos\Theta$$

The ene

longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi}\int_0^{2\pi} T(t)\,d\Theta \qquad \text{with}$$

and therefore

$$C_p\,\partial_{\tilde{t}}\tilde{T} \;=\; (1-\alpha)S\cos\varphi\cdot\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \mathrm{c}$$

$$=\;(1-\alpha)\frac{S}{\pi}\cos\varphi \;-\; \epsilon\sigma\tilde{T}^4 \qquad (10)$$

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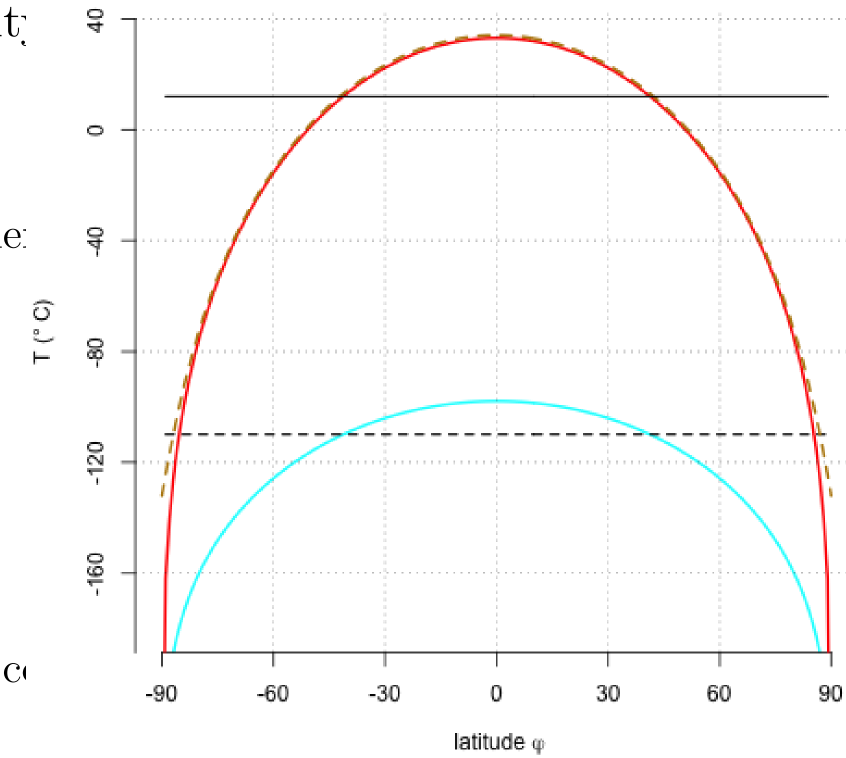

Figure 2. Latitudinal temperatures of the EBM with zero heat capacity (7) in cyan (its mean as a dashed line), the global approach (3) as solid black line, and the zonal and time averaging (11) in red. The dashed brownish curve shows the numerical solution by taking the zonal mean of (9).

$$
\begin{aligned}
\overline{\tilde{T}} &= \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad \underbrace{\frac{1}{2}\int\limits_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{1.862} \\
&= \sqrt[4]{\frac{4}{\pi}\frac{1.862}{2}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.989 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}
\end{aligned}
\tag{12}
$$

Therefore, $\overline{\tilde{T}} = 285 \approx 288$ K, very similar as in (1). A numerical solution of (9) is shown as the brownish dashed line in Fig. 2 where the diurnal cycle has been taken into account and $C_p = C_p^a$ has been chosen as the atmospheric heat capacity

$$
C_p^a = c_p p_s/g = 1004\, JK^{-1}kg^{-1} \cdot 10^5 Pa/(9.81 ms^{-2}) = 1.02 \cdot 10^7 JK^{-1}m^{-2}
$$

which is the specific heat at constant pressure c_p times the total mass p_s/g. p_s is the surface pressure and g the gravity. The temperature $\overline{T}$ is 286 K, again close to 288 K.

Heat capacity C_p/C_p^a

The effect of heat capacity is systematically analyzed in Fig. 3. The temperatures are relative insensitive for a wide range of C_p. We find a severe drop in temperatures for heat capacities below 0.01 of the atmospheric heat capacity C_p^a.

2

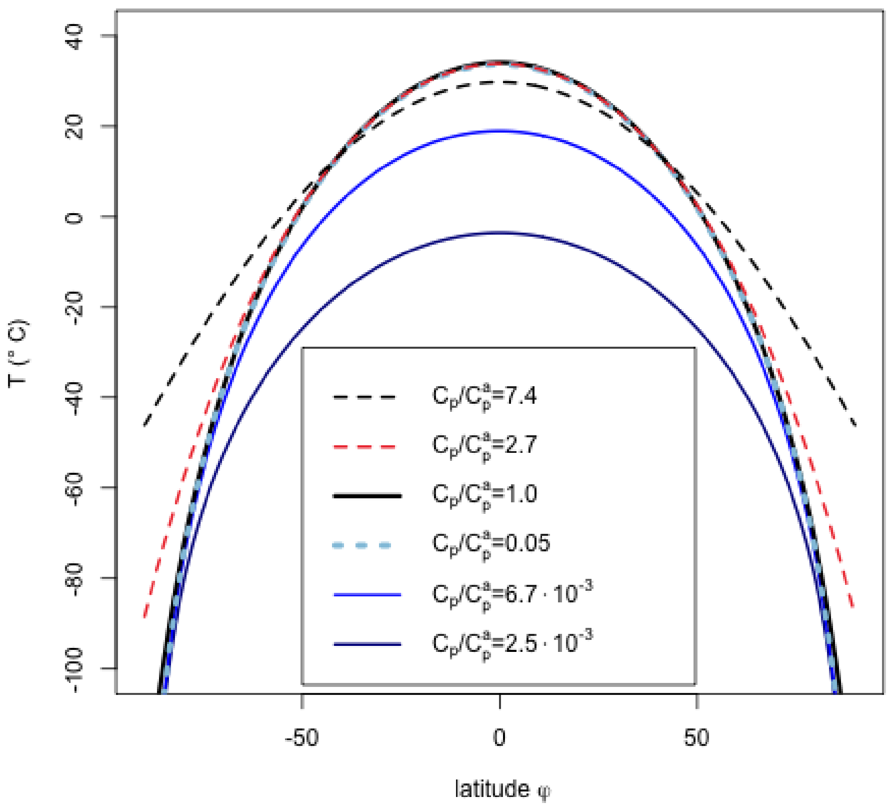

Figure 3. Temperature depending on C_p when solving (9) numerically. The reference heat capacity is the atmospheric heat capacity $C_p^a = 1.02 \cdot 10^7 JK^{-1}m^{-2}$. The climate is insensitive to changes in heat capacity $C_p \in [0.05 \cdot C_p^a, 2.0 \cdot C_p^a]$.

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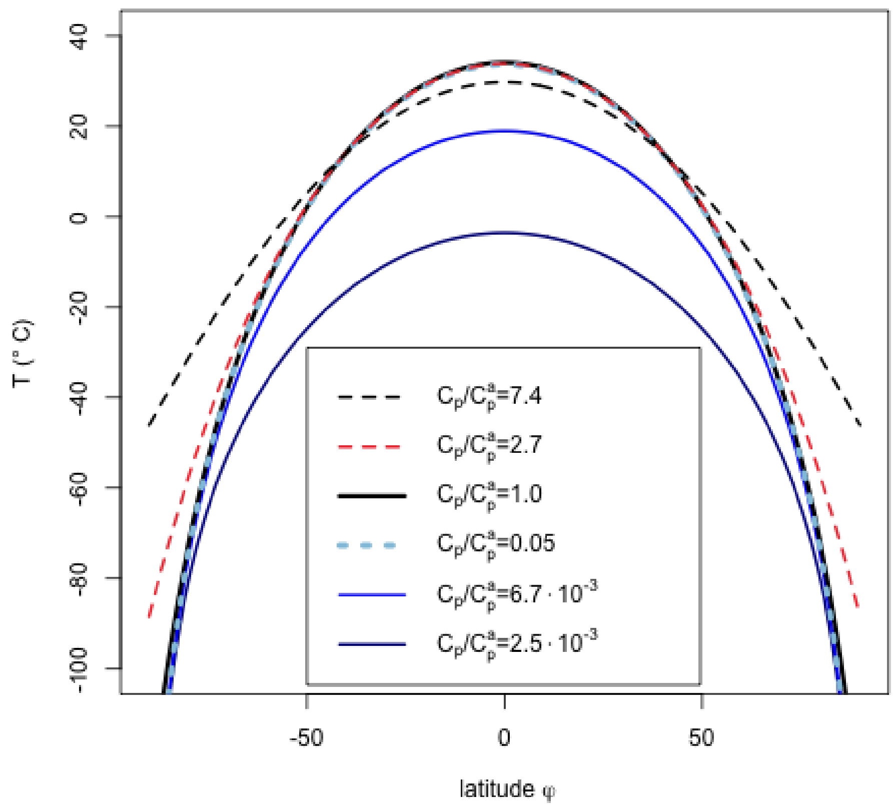

2

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Additionally: rotation rate -> 4.5° colder for 240 h

Planetary boundary layer

Mixed layer depth

A diffusive heat flux goes down the gradient of temperature and the convergence of this heat flux drives a ocean temperature tendency:

$$C_p^o \partial_t T = -\partial_z (k^o \partial_z T) \tag{13}$$

where $k_v = k^o / C_p^o$ is the oceanic vertical eddy diffusivity in $m^2\, s^{-1}$, and C_p^o the oceanic heat capacity relevant on the specific time scale. The vertical eddy diffusivity k_v 10^{-5} $10^{-4}\, m^2\, s^{-1}$ depending on depth and region.

A scale analysis of (13) yields a characteristic depth scale h_T through

$$\frac{\Delta T}{\Delta t} = k_v \frac{\Delta T}{h_T^2} \quad \longrightarrow h_T = \sqrt{k_v\ \Delta t} \tag{14}$$

For the diurnal cycle h_T is less that half a meter and the heat capacity generally less than that of the atmosphere.

heat transport across a latitude is $HT = -k\nabla T$. One can solve the EBM

$$C_p\,\partial_t T \;=\; \nabla\cdot HT \,+\, (1-\alpha)\frac{S}{\pi}\cos\varphi \,-\, \epsilon\sigma T^4 \quad . \tag{15}$$

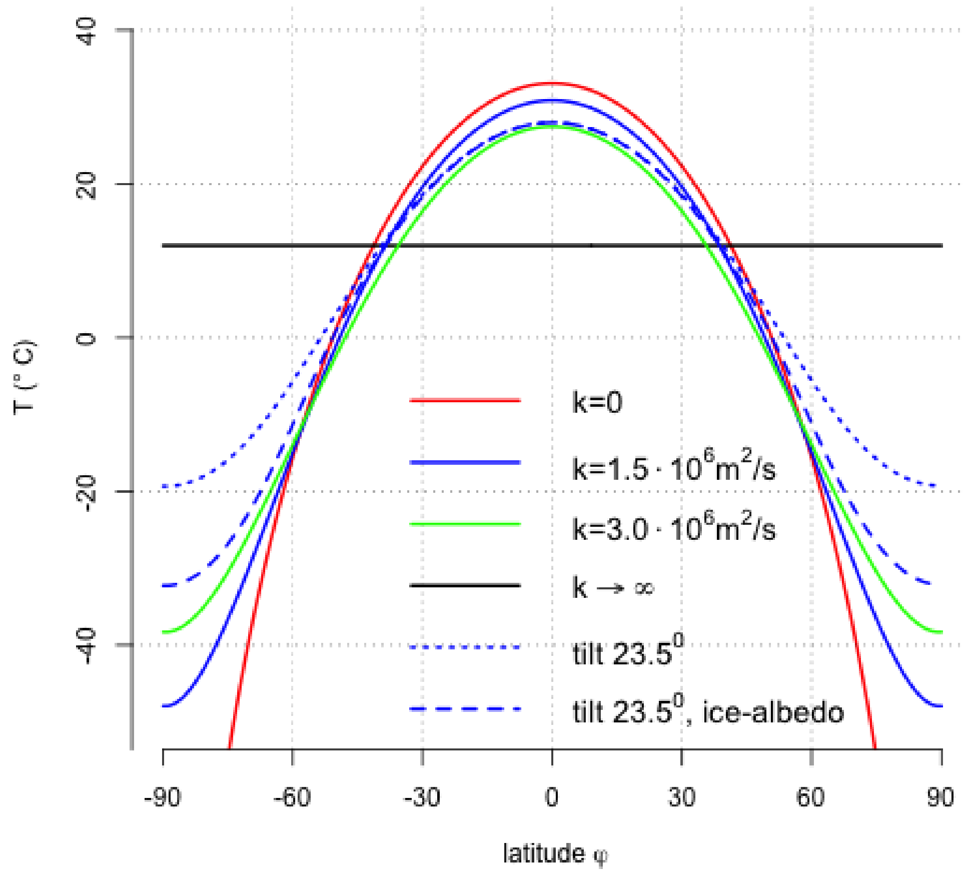


Figure 4. Equilibrium temperature of (15) using different diffusion coefficients. The blue lines use $1.5\cdot 10^6 m^2/s$ with no tilt (solid line), a tilt of 23.5° (dotted line), and as the dashed line a tilt of 23.5° and ice-albedo feedback using the respresentation of Sellers (1969). Except for the dashed line, the global mean values are identical to the value calculated in (12). Units are °C.

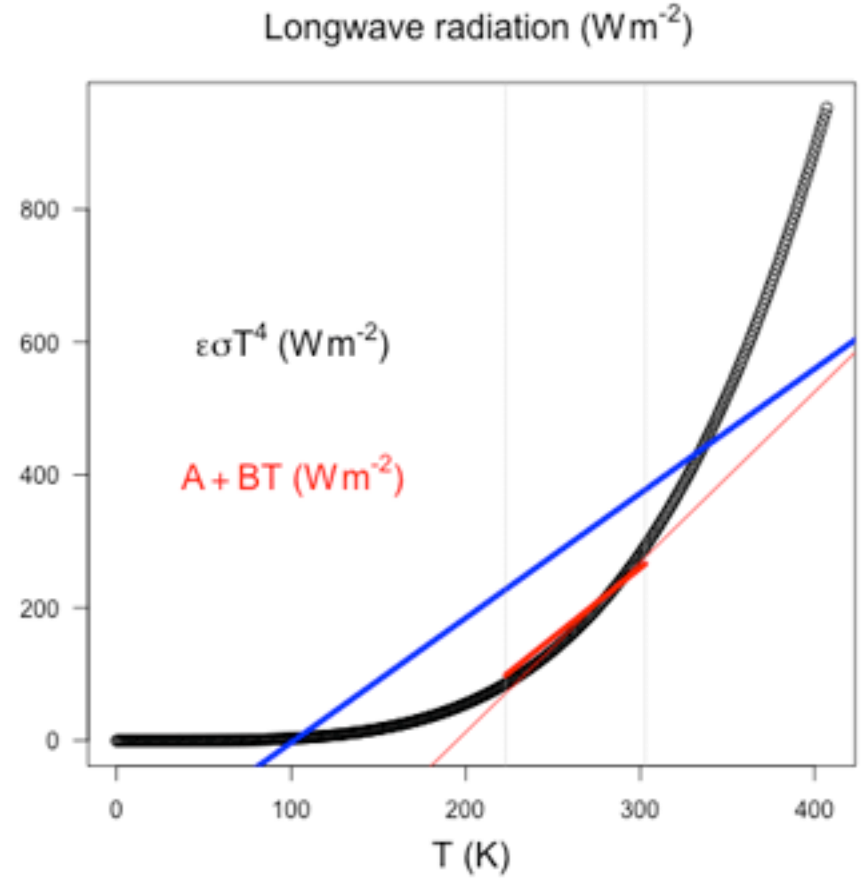
EBM revisited

EBMs crucial tool in climate research, especially because they describe the processes essential for the genesis of the global climate (confirmed by complex climate models)

EBMs conceptual tools, due to the essentials ”scientific understanding” : radiation balance on the ground and the absorption in the atmosphere are the essential factors, independent of the size of the Earth and the thermal characteristics, but depends on the albedo, emissivity and solar constant.

Here: length of the day, effective heat capacity

Ironically, the global mean in the revised EBM is very close to the original proposed value



Black dots show $\epsilon\sigma T^4$. The blue line shows the linear fit for the range of temperatures 0 to 407 K. The thin red line shows the linear fit for temperatures between −50 to 30 °C (range is shown as vertical dotted lines). The thick red line shows Budyko's (1969) linearization with A=203.3 W m^{-2} and B=2.09 W m^{-2} °C^{-1}.

The regression coefficients for the blue line are A=321.4 W m^{-2} and B=1.88 W m^{-2} °C^{-1}, whereas for the thin red line they are A=199.5 W m^{-2} and B=2.56 W m^{-2} °C^{-1}.

EBM with sea ice

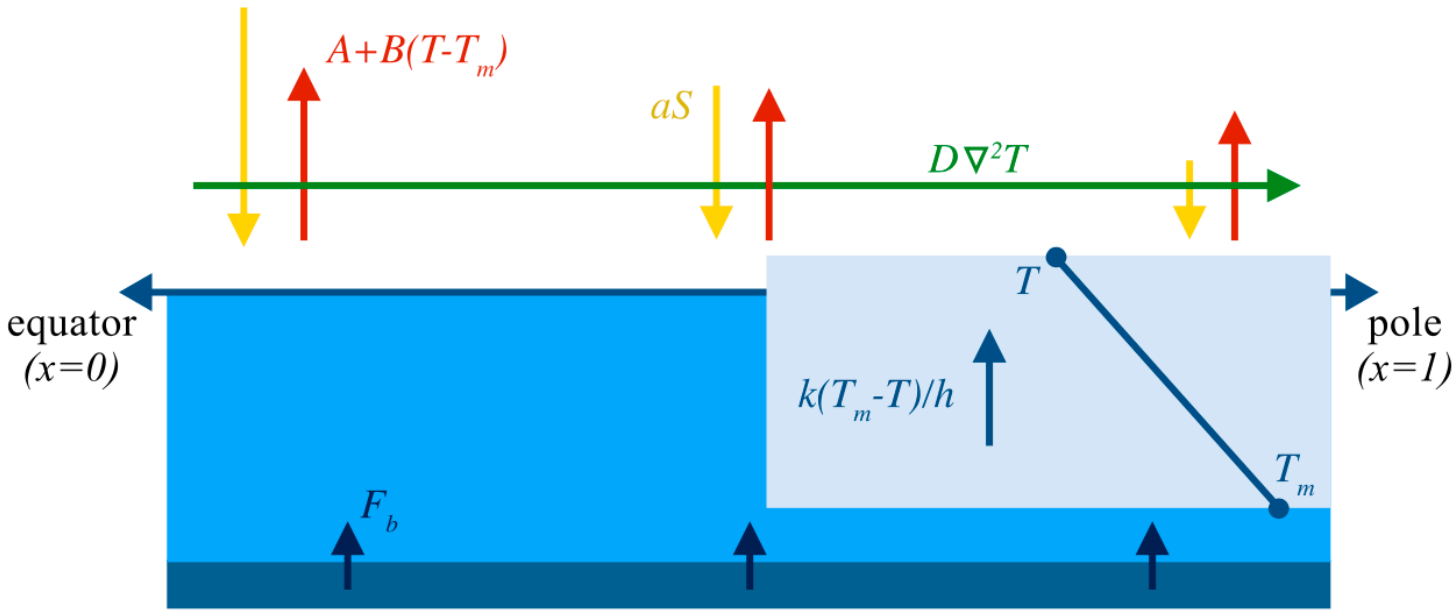


FIG. 1. Schematic of the global model of climate and sea ice described in section 2, showing the fluxes included in the model: insolation (yellow), OLR (red), horizontal heat transport (green), ocean heating (dark blue), and vertical heat flux through the ice (blue). The temperature of the ice is given by T at the surface and T_m at the base.

How Climate Model Complexity Influences Sea Ice Stability
TILL J. W. WAGNER AND IAN EISENMAN
JOURNAL OF CLIMATE
VOLUME 28

The equations

$$\frac{\partial E}{\partial t} = \underbrace{aS}_{\text{solar}} - \underbrace{L}_{\text{OLR}} + \underbrace{D\nabla^2 T}_{\text{transport}} + \underbrace{F_b}_{\substack{\text{ocean}\\ \text{heating}}} + \underbrace{F}_{\text{forcing}}$$

$$E \equiv \begin{cases} -L_f h, & E < 0 \quad \text{(sea ice)}, \\ c_w(T - T_m), & E \geq 0 \quad \text{(open water)}, \end{cases}$$

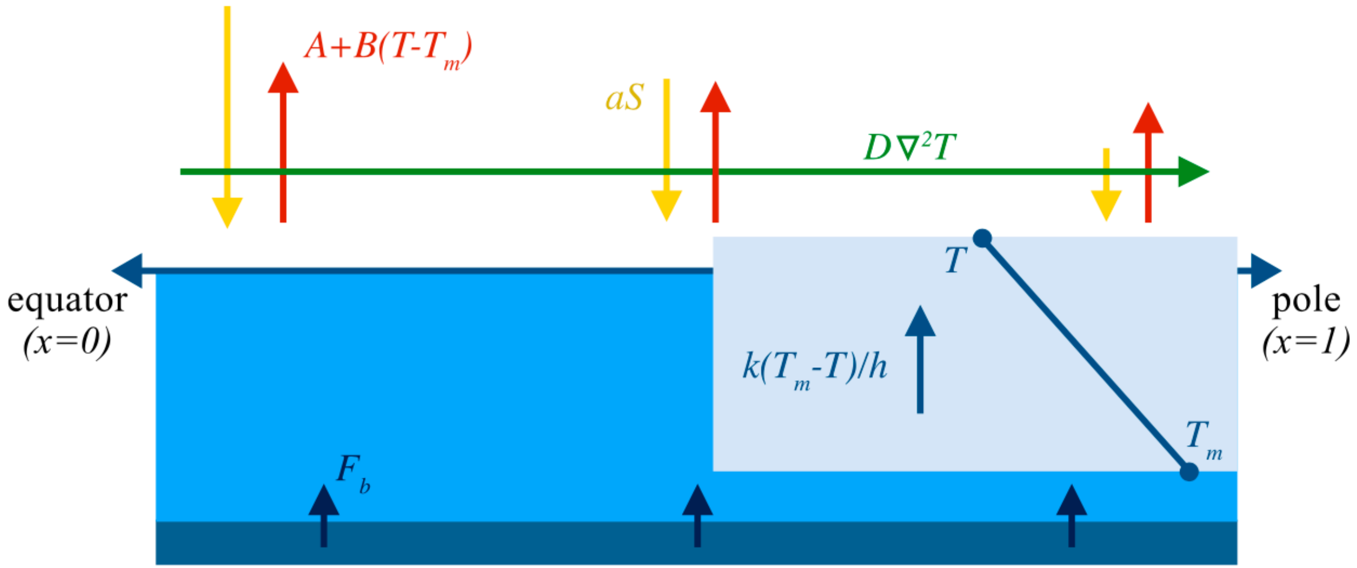
$$S(t,x) = S_0 - S_1 x \cos\omega t - S_2 x^2$$

$$a(x,E) = \begin{cases} a_0 - a_2 x^2, & E > 0 \quad \text{(open water)}, \\ a_i, & E < 0 \quad \text{(ice)}, \end{cases}$$

$$D\nabla^2 T = D\frac{\partial}{\partial x}\left[(1 - x^2)\frac{\partial T}{\partial x}\right]$$

$$L = A + B(T - T_m)$$

EBM with sea ice



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$$k(T_m - T_0)/h = -aS + A + B(T_0 - T_m) - D\nabla^2 T - F$$

$$T = \begin{cases} T_m + E/c_w, & E > 0 \qquad\qquad\qquad \text{(open water)}, \\ T_m, & E < 0, \quad T_0 > T_m \quad \text{(melting ice)}, \\ T_0, & E < 0, \quad T_0 < T_m \quad \text{(freezing ice)}. \end{cases}$$

output

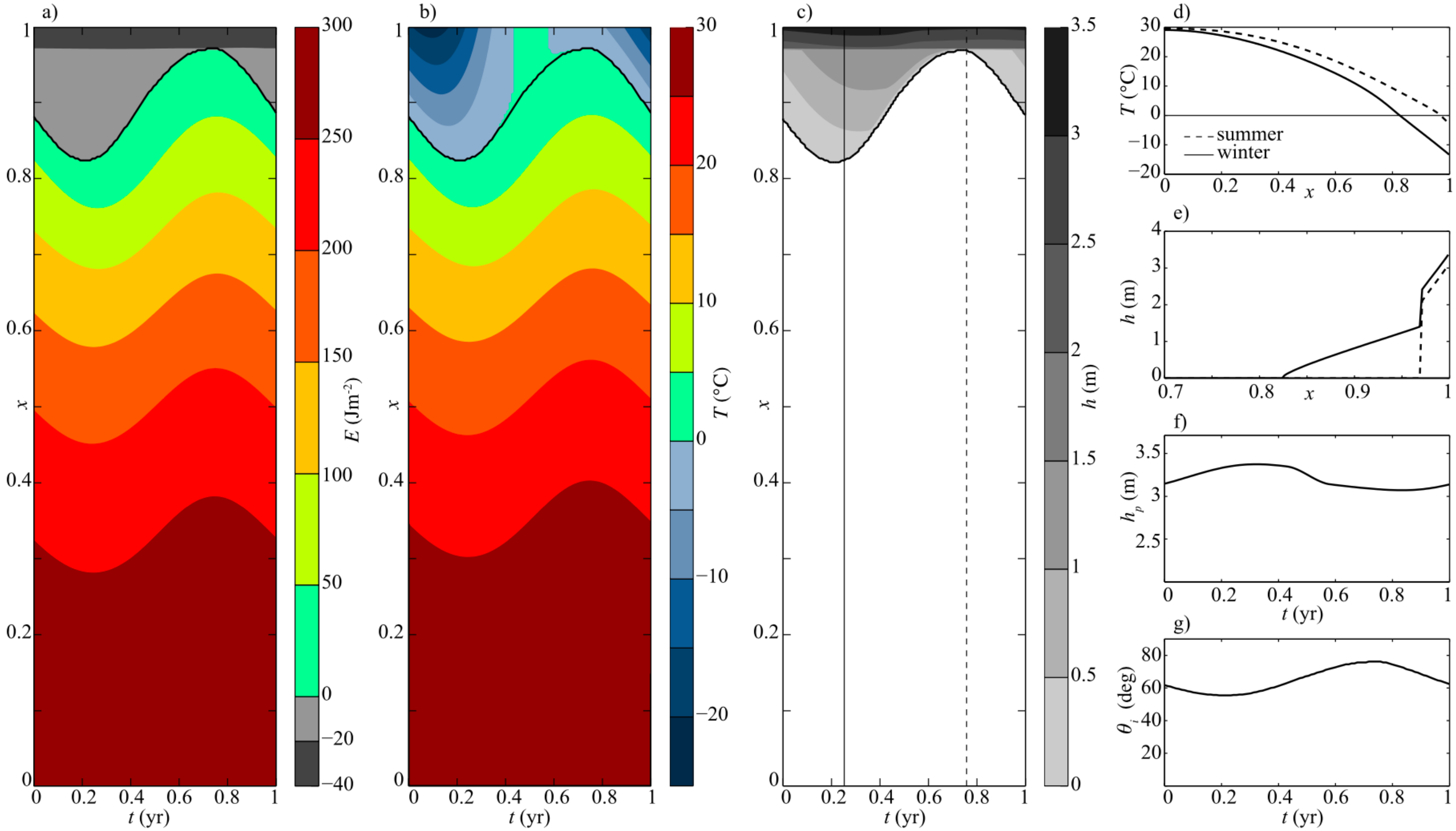

FIG. 2. Simulated climate in the default parameter regime. Contour plot of the seasonal cycle of (a) surface enthalpy $E(x, t)$, (b) surface temperature $T(x, t)$, and (c) sea ice thickness $h(x, t)$. The black curve in (a)–(c) indicates the ice edge. (d) Surface temperature T in summer and winter, corresponding to dashed and solid vertical lines in (c). (e) Ice thickness h in summer and winter where $x > 0.7$. (f) Seasonal cycle of ice thickness at the pole h_p. (g) Seasonal cycle of the latitude of the sea ice edge θ_i.

Exercise 1: EBM

D∗=Wm−2K−1 Diffusivity for heat transport: 0.6

A=Wm−2 OLR:193

B=Wm−2K−1 OLR temperature dependence: 2.1

cw=Wyrm−2K−1 Ocean mixed layer heat capacity: 9.8

S0=Wm−2 Insolation at equator: 420

S2=Wm−2 Insolation spatial dependence: 240

A0 Ice-free coalbedo at equator: 0.7

A2 Ice-free coalbedo spatial dependence: 0.1

Ai Coalbedo when there is sea ice: 0.4

Wm−2 Radiative forcing: 0

γ Gamma: 1

1) What will happen if the CO_2 content in the atmosphere is doubled? Radiative forcing= 4 Wm-2
2) What will happen if the factor γ is 1% higher/lower in the long-wave radiation?
3) Describe the effect if the diffusivity is enhanced by a factor of 2!
4) The coalbedo of sea ice can vary between 0.3 and 0.4. Describe the effect when varying the value!
5) Write down the numerical scheme (time stepping etc. from the source code) !
6) Show the evolution at one specific latitude and discuss it!

https://paleodyn.uni-bremen.de/study/MES/ebm/

Upper model

Exercise 2: Aquaplanet EBM with seasonal cycle

1) What will happen if the CO_2 content in the atmosphere is doubled? Radiative forcing= 4 W/m^2= lowering of A
2) Discuss the sea ice evolution during the year !
3) Reduce and enhance the sea ice thermal conductivity by 20% mimiking more /less snow on top of sea ice !
4) Write down the numerical scheme (time stepping) !

https://paleodyn.uni-bremen.de/study/MES/ebm/
Lower model