



# EBM & Sea ice

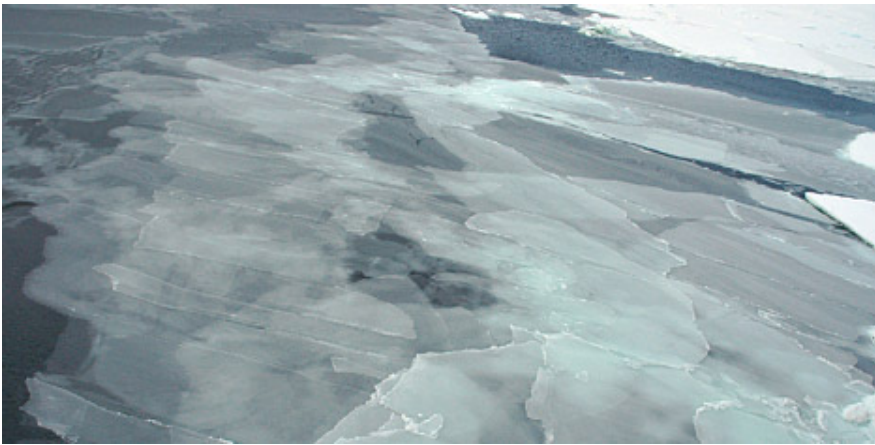




# Sea ice

**Sea ice is frozen seawater that floats on the ocean surface.**

It forms in both the Arctic and the Antarctic in each hemisphere's winter; it retreats in the summer, but does not completely disappear. This floating ice has a profound influence on the polar environment, influencing ocean circulation, weather, and regional climate.

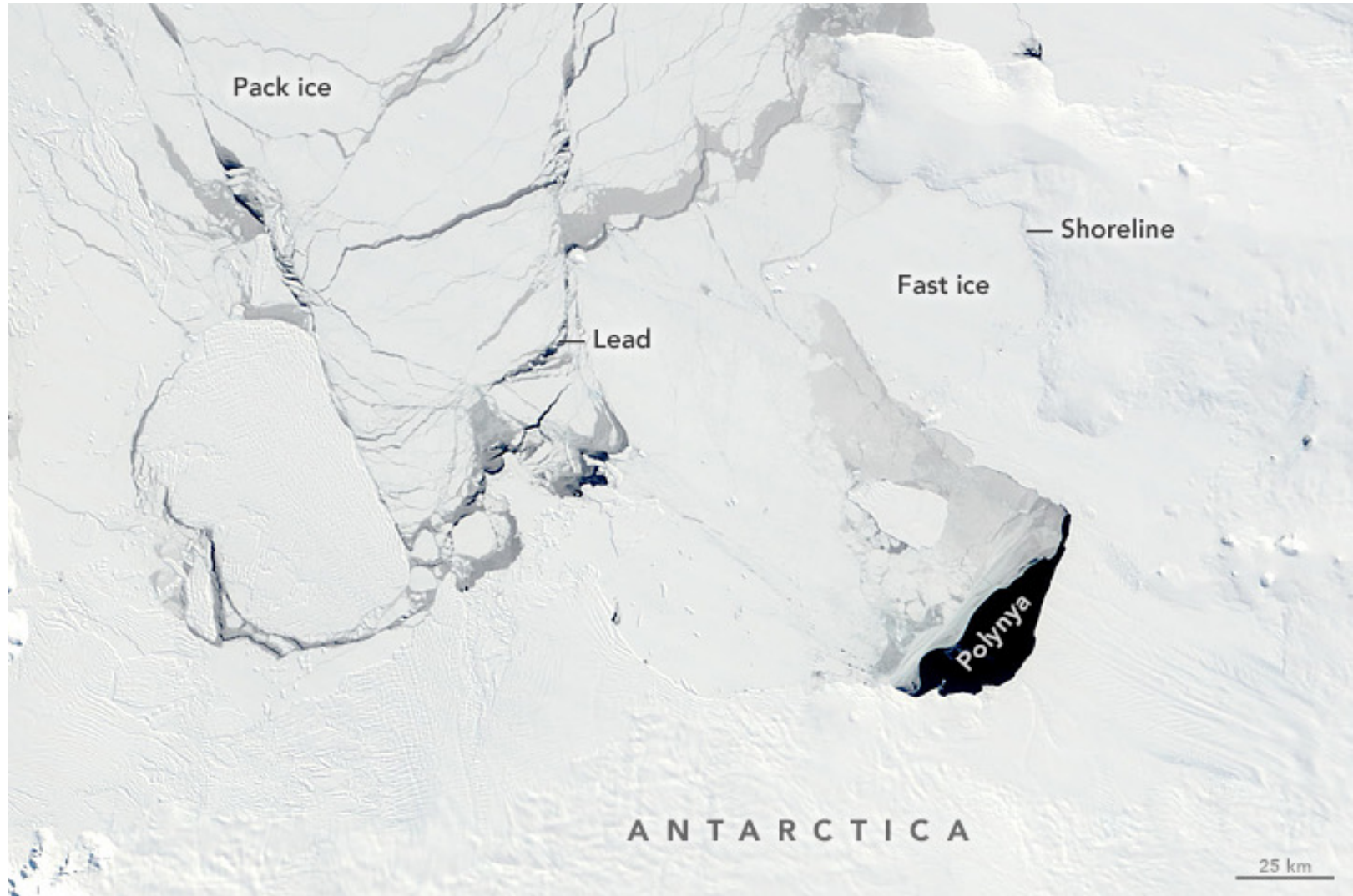


Sea ice begins as thin sheets of smooth nilas in calm water (top) or disks of pancake ice in choppy water (top right).

Individual pieces pile up to form rafts and eventually solidify (lower left). Over time, large sheets of ice collide, forming thick pressure ridges along the margins (lower right).



# Sea ice



# Arctic sea ice



2016 Arctic Maximum (March 24)



2016 Arctic Minimum (September 10)

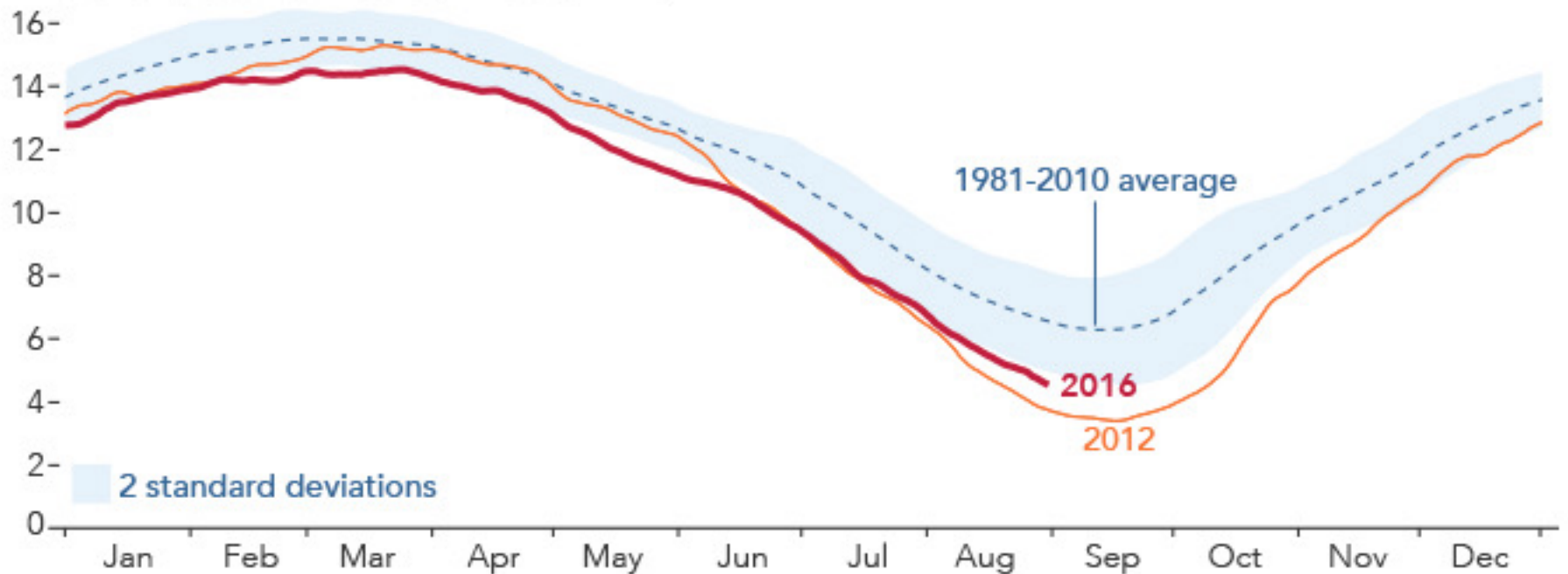


Sea Ice Concentration (percent)



# Seasonal cycle

Arctic Daily Sea Ice Extent (millions of km<sup>2</sup>)




# Sea ice dynamics

<https://fesom.de/media/video/>

- [https://youtu.be/Im-v6w5\\_NFw](https://youtu.be/Im-v6w5_NFw)



~25 m

 Feltham DL. 2008.  
Annu. Rev. Fluid Mech. 40:91–112



# Basics

- *Sea ice* is frozen salt water
- *Icebergs* are frozen fresh water, calved off of glaciers.
- Biologically important to the polar fauna
- Formed at roughly  $-2^{\circ}$  C
- Salt is forced out during freezing, so the ice is actually fresh (5 psu)
- This results in an increased salinity in the water.
- Forms a boundary layer between the (relatively) warm polar waters and the colder atmosphere



# Heat Flux in the Poles

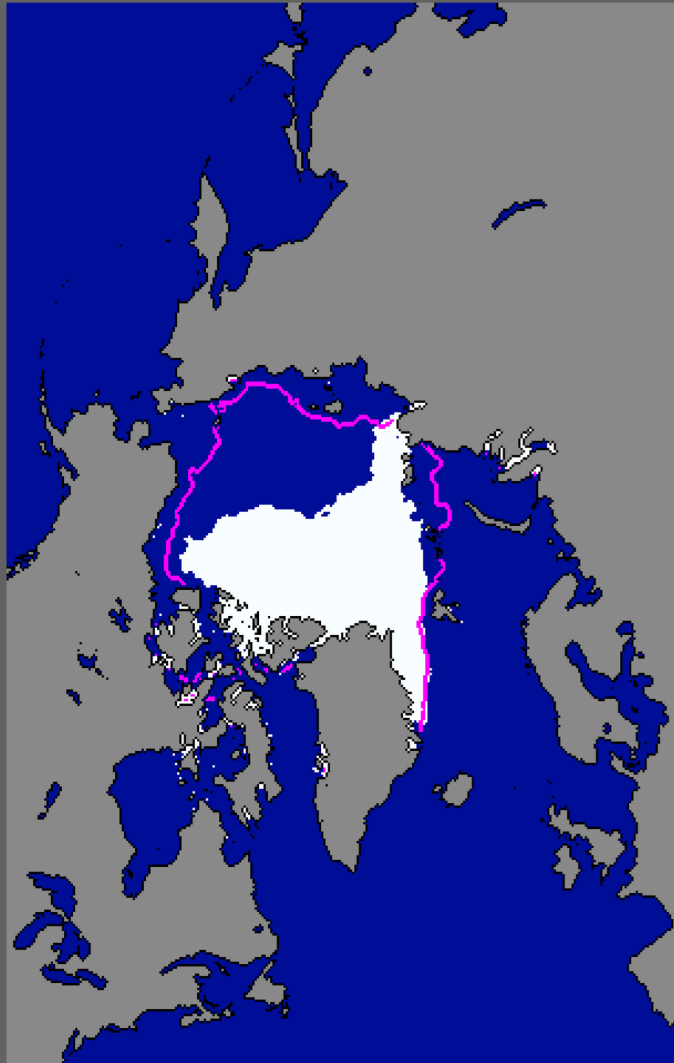
- Ice has one of the highest albedos
- Snow-covered polar ice can be up to 95%
- Nearly all absorption goes to melting ice, not heating water
- Precipitation greatly outweighs evaporation ( $Q_e$ )
- Conduction primary method of oceanic heat loss ( $Q_h$ )
- With insulation, restricts ice thickness to about 3-5 m
- Small open areas or *polynyas* that shift through seasons



Melting pack ice, Pribilof Islands, March 2007

# Albedo and Radiation

Current Ice Extent  
09/16/2007

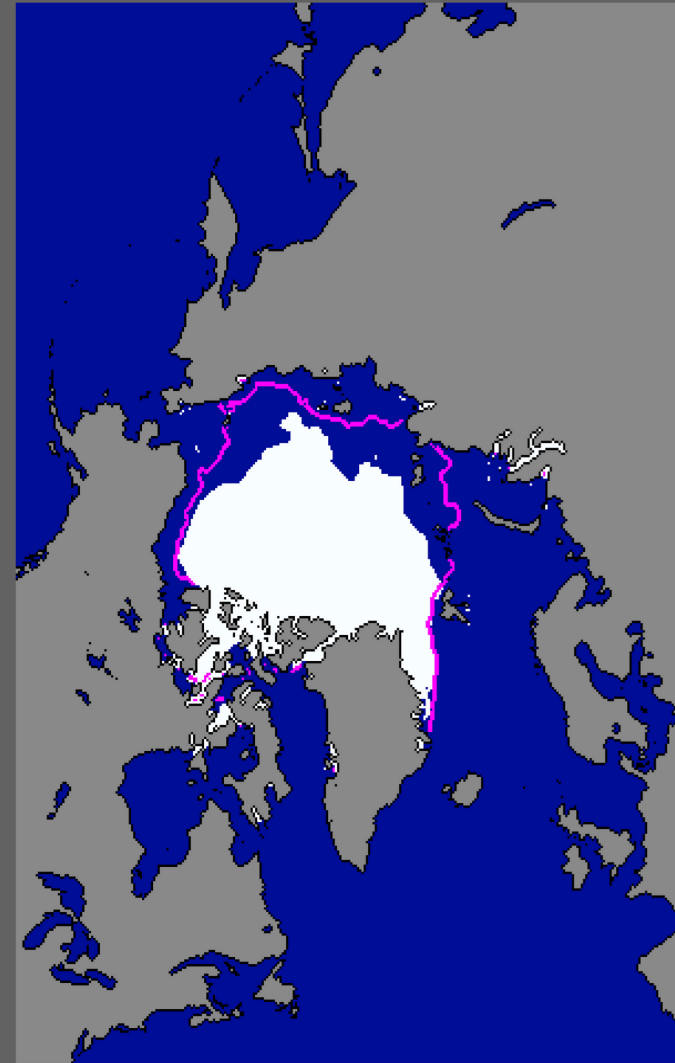


National Snow and Ice Data Center, Boulder, CO

median  
ice edge

Total extent = 4.1 million sq km

Current Ice Extent  
09/21/2005



National Snow and Ice Data Center, Boulder, CO

median  
ice edge

Total extent = 5.3 million sq km

# Possible Marine Effects of Oceanic Warming

(or “Doomsday Predictions 101”)

## BIOLOGICAL:

- Ice-dependent species lose critical habitat: “charismatic megafauna”
- Ranges are skewed to higher latitudes. Non-native species in new areas.
- Increased primary production

## ECONOMICAL:

- Possible large effects on fisheries
- NW Passage clear for shipping

## CHEMICAL:

- Decreased ocean acidification rate
- CO<sub>2</sub> sink might become a source

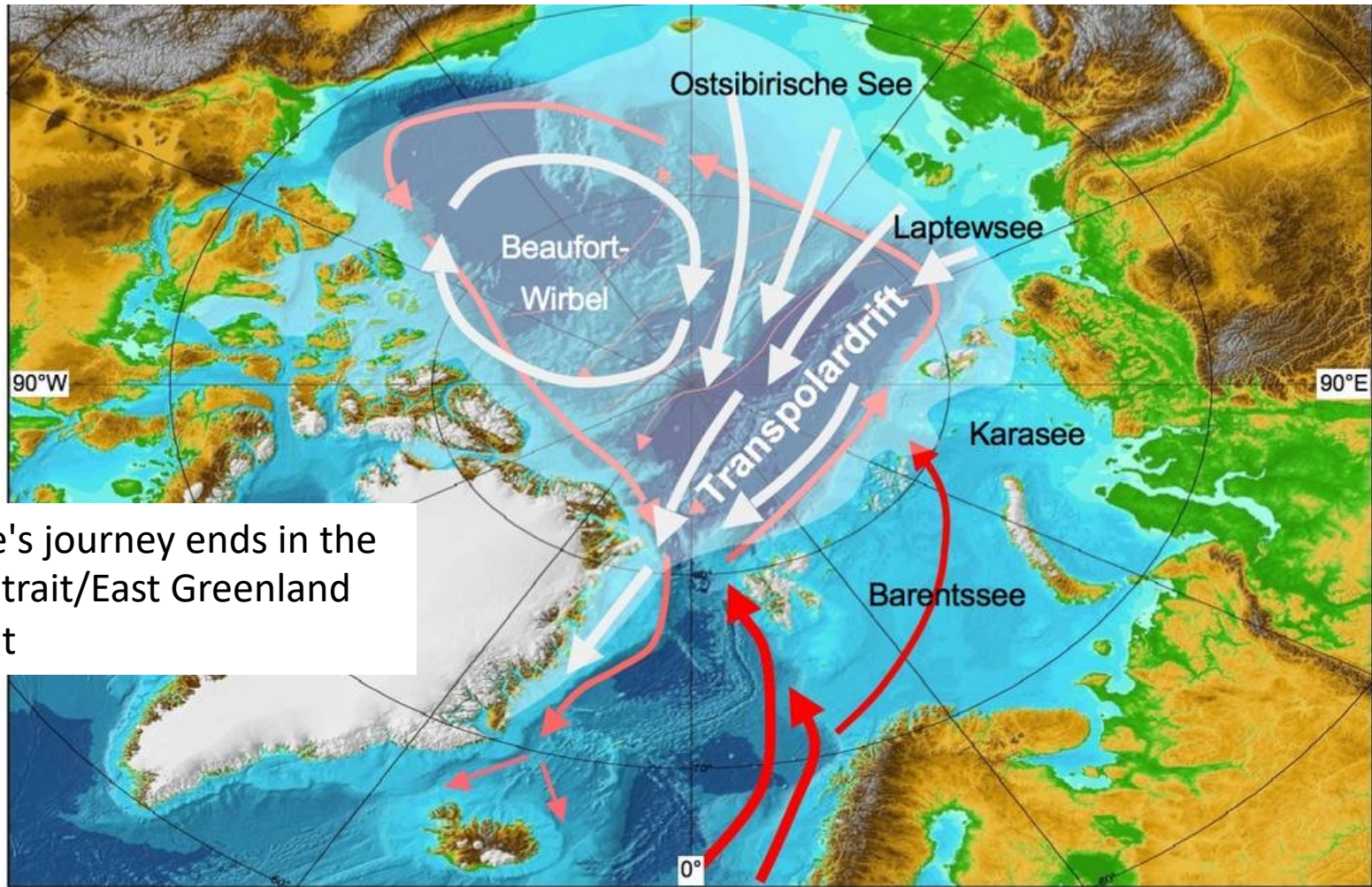
## PHYSICAL:

- Increased atmospheric temperature from CO<sub>2</sub>
- Glacial melting, sea level
- Vast freshwater input, effects on density-driven circulation



# The **transpolar drift** connects the Arctic shelf seas with the Fram Strait

(between Svalbard and Greenland)

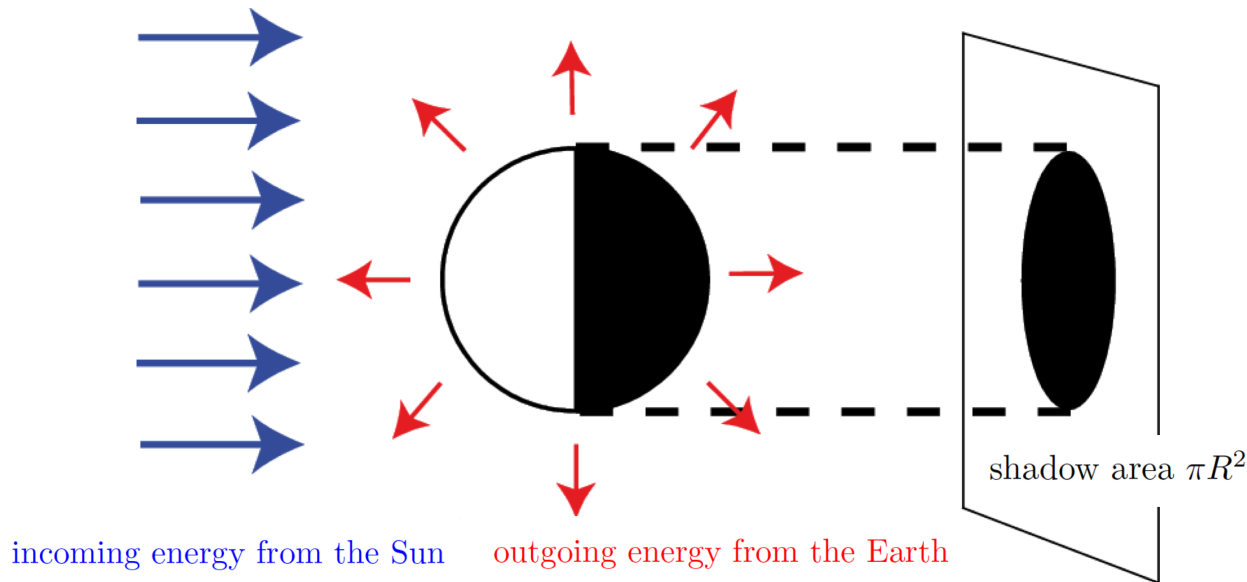


The ice's journey ends in the Fram Strait/East Greenland Current

# Energy balance model

$$(1 - \alpha)S\pi R^2 = 4\pi R^2 \epsilon \sigma T^4$$

$$T = \sqrt[4]{\frac{(1 - \alpha)S}{4\epsilon\sigma}}$$



Heat capacity

Heat transport

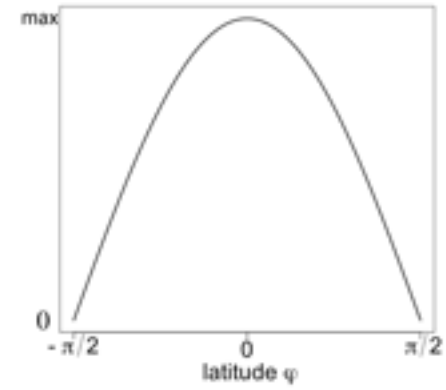
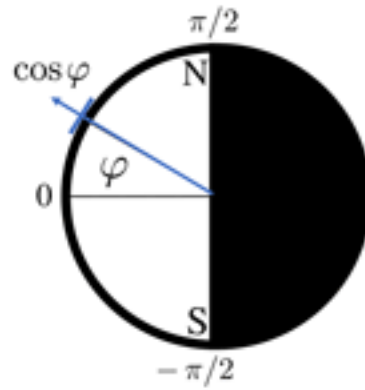
GCM experiment

Simple models are helpful for understanding of

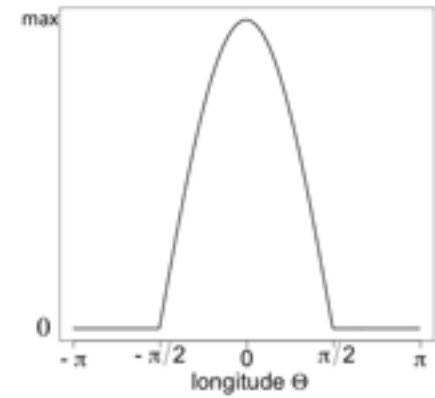
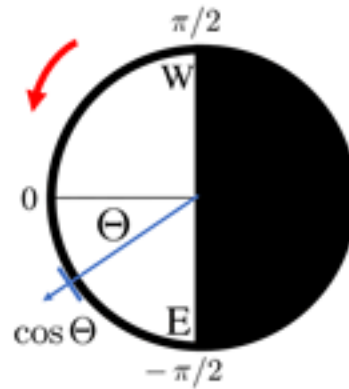
- Critical parameters
- Concepts of climate

# EBM

(a)



(b)



# What we really want to know ...

What we really want is the mean of the temperature  $\bar{T}$ . Therefore, we take the fourth root of (4):

$$T = \sqrt[4]{\frac{(1 - \alpha)S \cos \varphi \cos \Theta}{\epsilon \sigma}} \times 1_{[-\pi/2 < \Theta < \pi/2]}(\Theta) \quad . \quad (6)$$

If we calculate the zonal mean of (6) by integration at the latitudinal cycles we have

$$\begin{aligned} T(\varphi) &= \frac{1}{2\pi} \int_{-\pi/2}^{-\pi/2} \sqrt[4]{\frac{(1 - \alpha)S \cos \varphi \cos \Theta}{\epsilon \sigma}} d\Theta \\ &= \frac{\sqrt{2}}{2\pi} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos \Theta)^{1/4} d\Theta}_{2.700} \sqrt[4]{\frac{(1 - \alpha)S}{4\epsilon \sigma}} (\cos \varphi)^{1/4} \\ &= 0.608 \cdot \sqrt[4]{\frac{(1 - \alpha)S}{4\epsilon \sigma}} (\cos \varphi)^{1/4} \end{aligned} \quad (7)$$

as a function on latitude (Fig. 2).

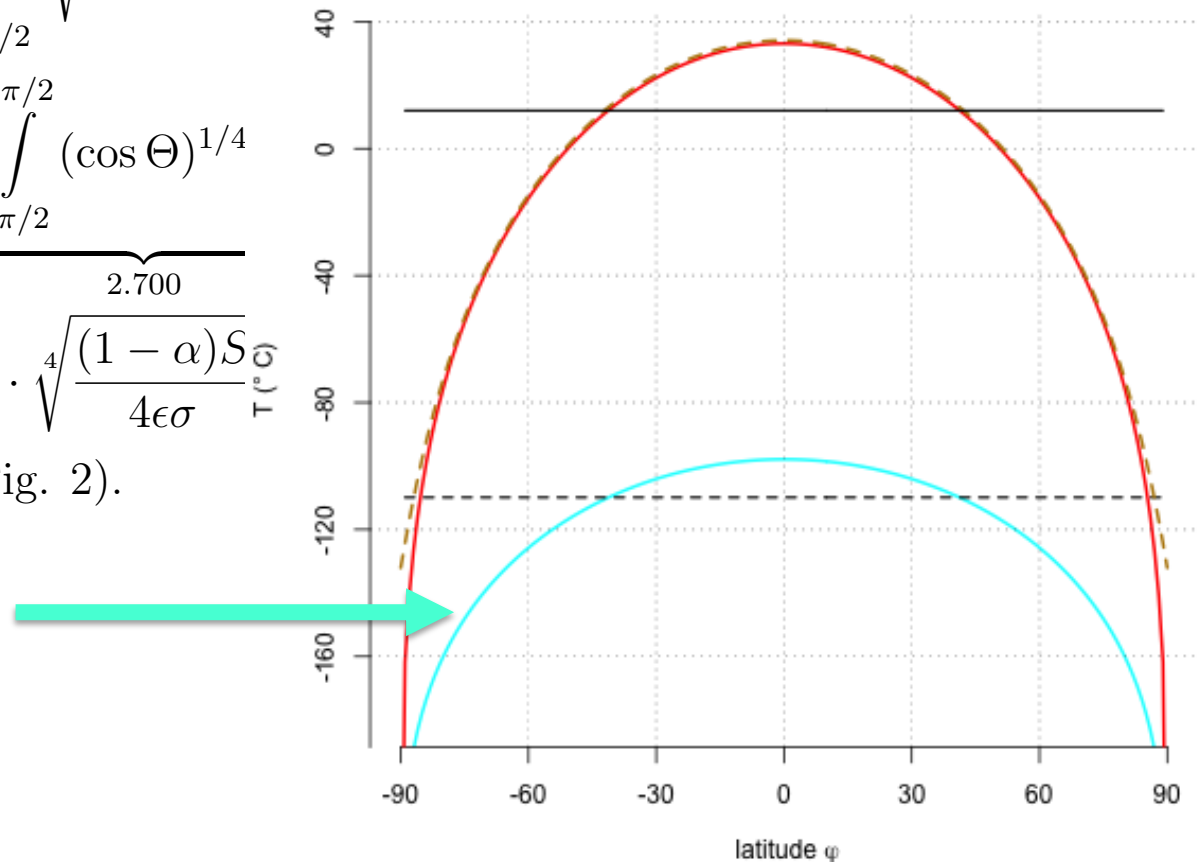
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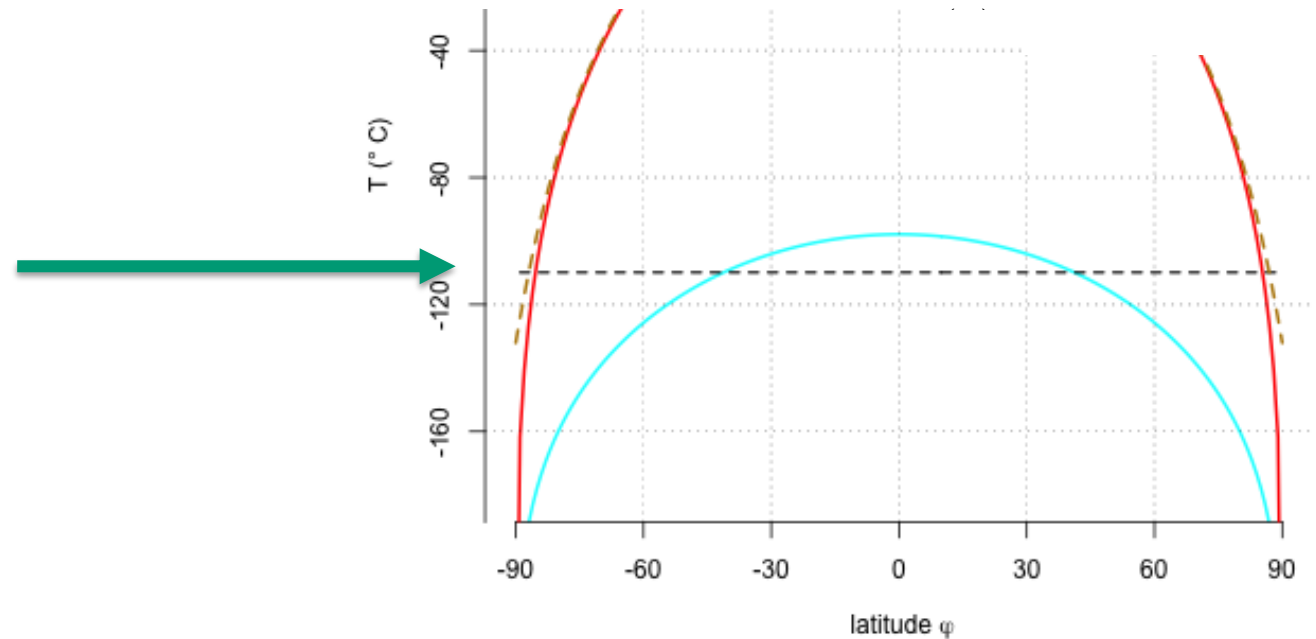






When we integrate this over the latitudes, we obtain

$$\begin{aligned}\bar{T} &= \frac{1}{2} \int_{-\pi/2}^{\pi/2} T(\varphi) \cos \varphi d\varphi \\ &= \frac{0.608}{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\int_{-\pi/2}^{\pi/2} (\cos \varphi)^{5/4} d\varphi}_{1.862} \\ &= 0.4\sqrt{2} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.566 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}\end{aligned}\quad (8)$$



# What is the difference ?

Let us have a closer look onto (1). The local radiative equilibrium of the Earth is

$$\epsilon\sigma T^4 = (1 - \alpha)S \cos \varphi \cos \Theta \quad \times 1_{[-\pi/2 < \Theta < \pi/2]}(\Theta) \quad (4)$$

where  $\varphi$  and  $\Theta$  are the latitude and longitude, respectively. Integration over the Earth surface is

$$\int_{-\pi/2}^{\pi/2} \left( \int_0^{2\pi} \epsilon\sigma T^4 R \cos \varphi d\Theta \right) R d\varphi = (1 - \alpha)S \int_{-\pi/2}^{\pi/2} R \cos^2 \varphi d\varphi \cdot \int_{-\pi/2}^{\pi/2} R \cos \Theta d\Theta$$
$$\epsilon\sigma R^2 \frac{4\pi}{4\pi} \int_{-\pi/2}^{\pi/2} \left( \int_0^{2\pi} T^4 \cos \varphi d\Theta \right) d\varphi = (1 - \alpha)SR^2 \int_{-\pi/2}^{\pi/2} \cos^2 \varphi d\varphi \cdot \int_{-\pi/2}^{\pi/2} \cos \Theta d\Theta$$

$$\epsilon\sigma 4\pi \overline{T^4} = (1 - \alpha)S \pi \quad (5)$$

giving a similar formula as (3) with the definition for the average  $\overline{T^4}$ .

# Heat capacity term

The energy balance shall take the heat capacity into account:

$$C_p \partial_t T = (1 - \alpha) S \cos \varphi \cos \Theta \times 1_{[-\pi/2 < \Theta < \pi/2]}(\Theta) - \epsilon \sigma T^4 \quad (9)$$

The energy balance (9) is integrated over the longitude and over the day

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi} \int_0^{2\pi} T(t) d\Theta \quad \text{with} \quad \tilde{T}^4 \approx \frac{1}{2\pi} \int_0^{2\pi} T^4 d\Theta$$

and therefore

$$\begin{aligned} C_p \partial_t \tilde{T} &= (1 - \alpha) S \cos \varphi \cdot \underbrace{\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cos \Theta d\Theta}_2 - \epsilon \sigma \tilde{T}^4 \\ &= (1 - \alpha) \frac{S}{\pi} \cos \varphi - \epsilon \sigma \tilde{T}^4 \end{aligned} \quad (10)$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1 - \alpha) S}{4\epsilon \sigma}} (\cos \varphi)^{1/4} \quad (11)$$

shown in Fig. 2 as the read line

# The new solution



The energy balance shall take the heat capacity

$$C_p \partial_t T = (1 - \alpha) S \cos \varphi \cos \Theta$$

longitude and over the day

The ene:

$$\tilde{T}(\tilde{t}) = \frac{1}{2\pi} \int_0^{2\pi} T(t) d\Theta \quad \text{with}$$

and therefore

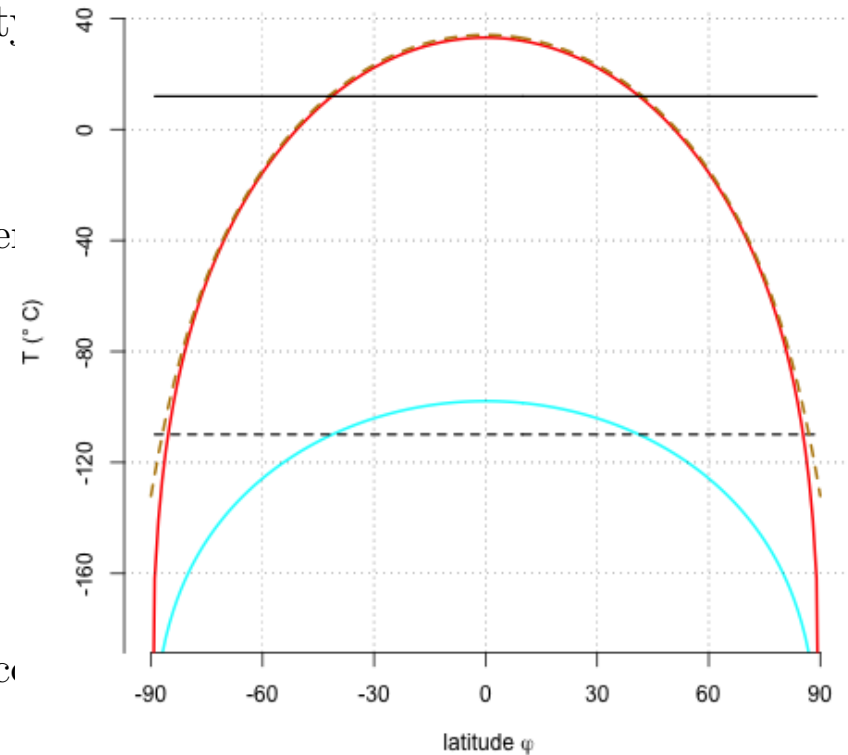
$$C_p \partial_t \tilde{T} = (1 - \alpha) S \cos \varphi \cdot \underbrace{\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cos \Theta d\Theta}_{=1} - \epsilon \sigma \tilde{T}^4$$

$$= (1 - \alpha) \frac{S}{\pi} \cos \varphi - \epsilon \sigma \tilde{T}^4$$

giving the equilibrium solution

$$\tilde{T}(\varphi) = \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1 - \alpha) S}{4\epsilon\sigma}} (\cos \varphi)^{1/4} \quad (11)$$

shown in Fig. 2 as the red line



**Figure 2.** Latitudinal temperatures of the EBM with zero heat capacity (7) in cyan (its mean as a dashed line), the global approach (3) as solid black line, and the zonal and time averaging (11) in red. The dashed brownish curve shows the numerical solution by taking the zonal mean of (9).



$$\begin{aligned}
\bar{T} &= \sqrt[4]{\frac{4}{\pi}} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \underbrace{\frac{1}{2} \int_{-\pi/2}^{\pi/2} (\cos \varphi)^{5/4} d\varphi}_{1.862} \\
&= \sqrt[4]{\frac{4}{\pi}} \frac{1.862}{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.989 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \tag{12}
\end{aligned}$$

Therefore,  $\bar{T} = 285 \approx 288$  K, very similar as in (1). A numerical solution of (9) is shown as the brownish dashed line in Fig. 2 where the diurnal cycle has been taken into account and  $C_p = C_p^a$  has been chosen as the atmospheric heat capacity

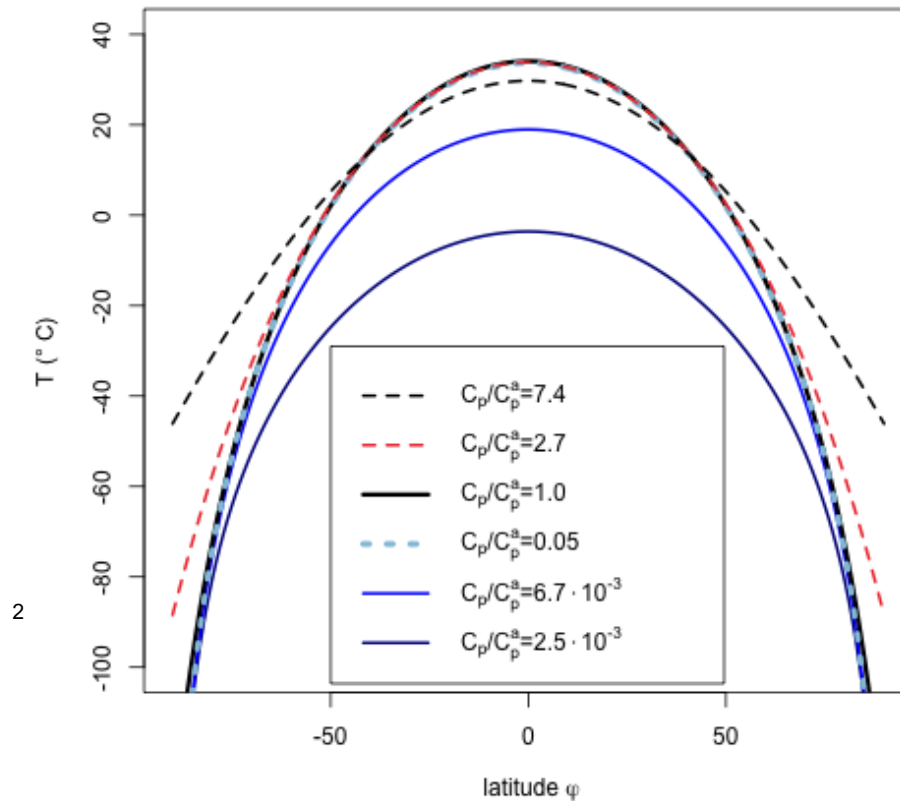
$$C_p^a = c_p p_s / g = 1004 \text{ JK}^{-1} \text{ kg}^{-1} \cdot 10^5 \text{ Pa} / (9.81 \text{ m s}^{-2}) = 1.02 \cdot 10^7 \text{ JK}^{-1} \text{ m}^{-2}$$

which is the specific heat at constant pressure  $c_p$  times the total mass  $p_s/g$ .  $p_s$  is the surface pressure and  $g$  the gravity. The temperature  $\bar{T}$  is 286 K, again close to 288 K.

# Heat capacity

$$C_p/C_p^a$$

The effect of heat capacity is systematically analyzed in Fig. 3. The temperatures are relative insensitive for a wide range of  $C_p$ . We find a severe drop in temperatures for heat capacities below 0.01 of the atmospheric heat capacity  $C_p^a$ .

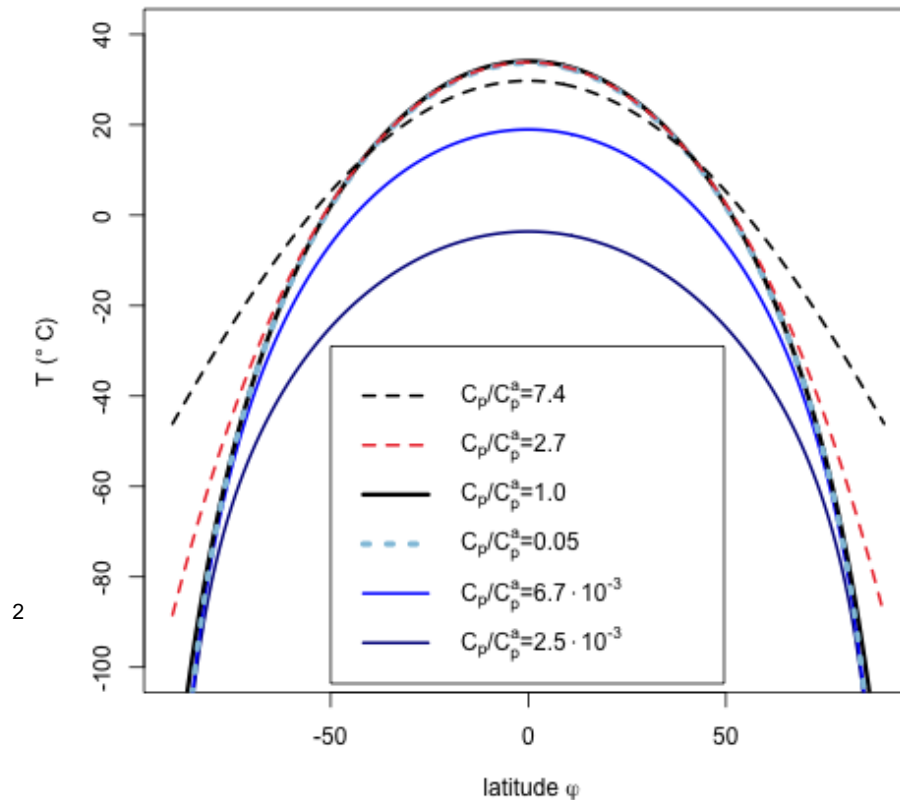


**Figure 3.** Temperature depending on  $C_p$  when solving (9) numerically. The reference heat capacity is the atmospheric heat capacity  $C_p^a = 1.02 \cdot 10^7 JK^{-1}m^{-2}$ . The climate is insensitive to changes in heat capacity  $C_p \in [0.05 \cdot C_p^a, 2.0 \cdot C_p^a]$ .

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Additionally: rotation rate  $\rightarrow 4.5^\circ$  colder for 240 h

Planetary boundary layer

Mixed layer depth

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A diffusive heat flux goes down the gradient of temperature and the convergence of this heat flux drives a ocean temperature tendency:

$$C_p^o \partial_t T = -\partial_z (k^o \partial_z T) \quad (13)$$

where  $k_v = k^o / C_p^o$  is the oceanic vertical eddy diffusivity in  $m^2 s^{-1}$ , and  $C_p^o$  the oceanic heat capacity relevant on the specific time scale. The vertical eddy diffusivity  $k_v$   $10^{-5}$   $10^{-4} m^2 s^{-1}$  depending on depth and region.

A scale analysis of (13) yields a characteristic depth scale  $h_T$  through

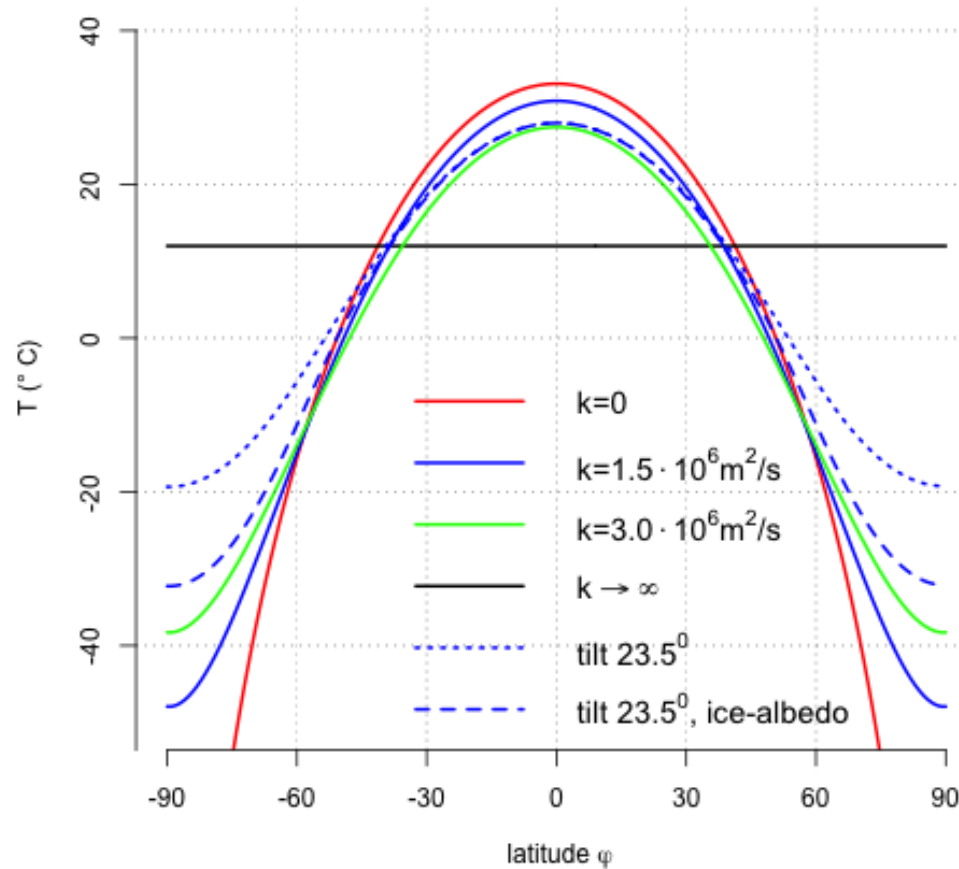
$$\frac{\Delta T}{\Delta t} = k_v \frac{\Delta T}{h_T^2} \quad \longrightarrow \quad h_T = \sqrt{k_v \Delta t} \quad (14)$$

For the diurnal cycle  $h_T$  is less than half a meter and the heat capacity generally less than that of the atmosphere.



heat transport across a latitude is  $HT = -k\nabla T$ . One can solve the EBM

$$C_p \partial_t T = \nabla \cdot HT + (1 - \alpha) \frac{S}{\pi} \cos \varphi - \epsilon \sigma T^4 \quad . \quad (15)$$



**Figure 4.** Equilibrium temperature of (15) using different diffusion coefficients. The blue lines use  $1.5 \cdot 10^6 \text{ m}^2/\text{s}$  with no tilt (solid line), a tilt of  $23.5^\circ$  (dotted line), and as the dashed line a tilt of  $23.5^\circ$  and ice-albedo feedback using the representation of Sellers (1969). Except for the dashed line, the global mean values are identical to the value calculated in (12). Units are  $^\circ\text{C}$ .

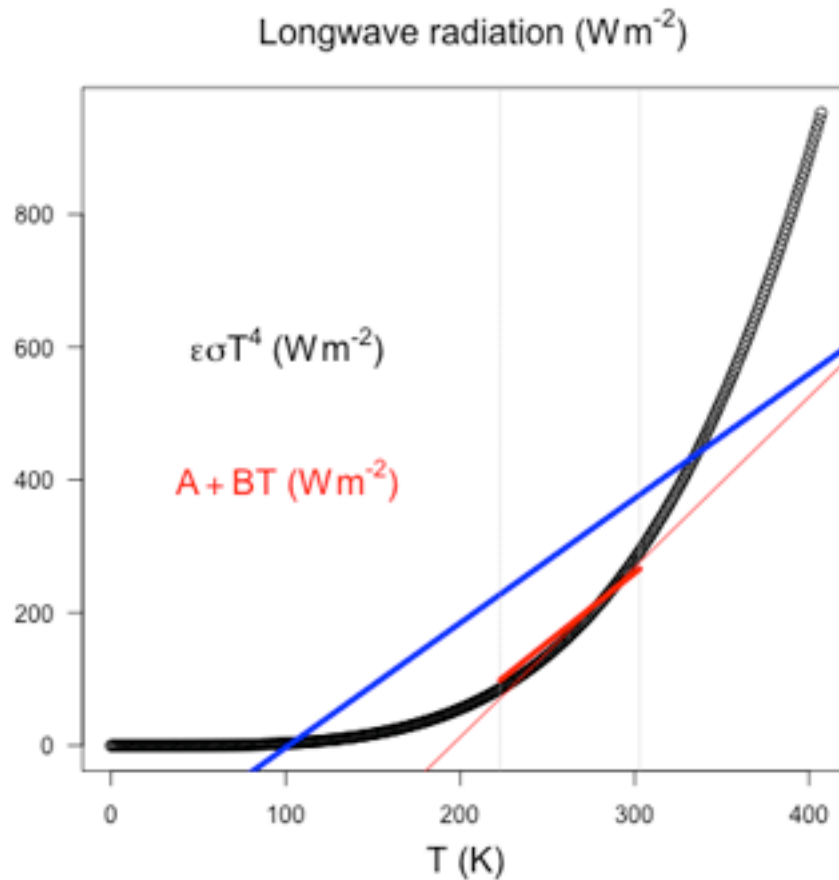
# EBM revisited

EBMs crucial tool in climate research, especially because they describe the processes essential for the genesis of the global climate (confirmed by complex climate models)

EBMs conceptual tools, due to the essentials "scientific understanding" : radiation balance on the ground and the absorption in the atmosphere are the essential factors, independent of the size of the Earth and the thermal characteristics, but depends on the albedo, emissivity and solar constant.

**Here: length of the day, effective heat capacity**

Ironically, the global mean in the revised EBM is very close to the original proposed value



Black dots show  $\epsilon\sigma T^4$ . The blue line shows the linear fit for the range of temperatures 0 to 407 K. The thin red line shows the linear fit for temperatures between  $-50$  to  $30$  °C (range is shown as vertical dotted lines). The thick red line shows Budyko's (1969) linearization with  $A=203.3 \text{ W m}^{-2}$  and  $B=2.09 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ .

The regression coefficients for the blue line are  $A=321.4 \text{ W m}^{-2}$  and  $B=1.88 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ , whereas for the thin red line they are  $A=199.5 \text{ W m}^{-2}$  and  $B=2.56 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$ .

# EBM with sea ice

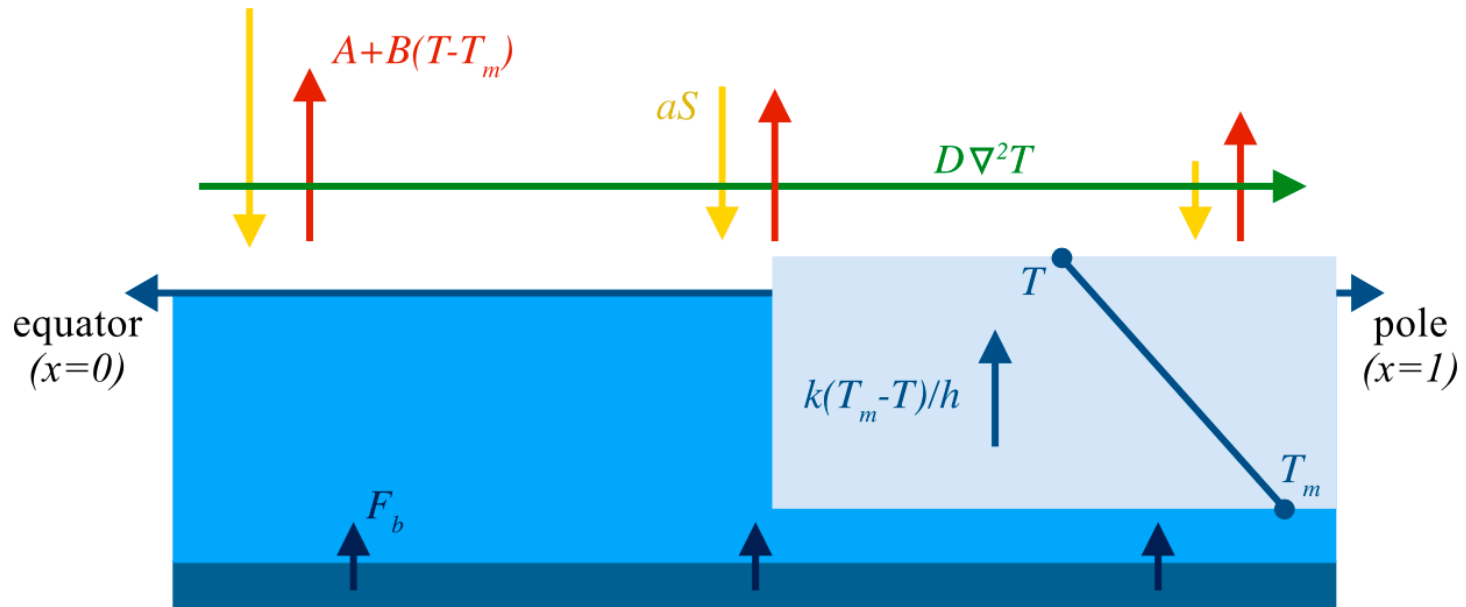


FIG. 1. Schematic of the global model of climate and sea ice described in section 2, showing the fluxes included in the model: insolation (yellow), OLR (red), horizontal heat transport (green), ocean heating (dark blue), and vertical heat flux through the ice (blue). The temperature of the ice is given by  $T$  at the surface and  $T_m$  at the base.

# The equations

$$\frac{\partial E}{\partial t} = \underbrace{aS}_{\text{solar}} - \underbrace{L}_{\text{OLR}} + \underbrace{D\nabla^2 T}_{\text{transport}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{forcing}}$$

$$E \equiv \begin{cases} -L_f h, & E < 0 \quad (\text{sea ice}), \\ c_w(T - T_m), & E \geq 0 \quad (\text{open water}), \end{cases}$$

$$S(t, x) = S_0 - S_1 x \cos \omega t - S_2 x^2$$

$$a(x, E) = \begin{cases} a_0 - a_2 x^2, & E > 0 \quad (\text{open water}), \\ a_i, & E < 0 \quad (\text{ice}), \end{cases}$$

$$D\nabla^2 T = D \frac{\partial}{\partial x} \left[ (1 - x^2) \frac{\partial T}{\partial x} \right]$$

$$L = A + B(T - T_m)$$

# EBM with sea ice

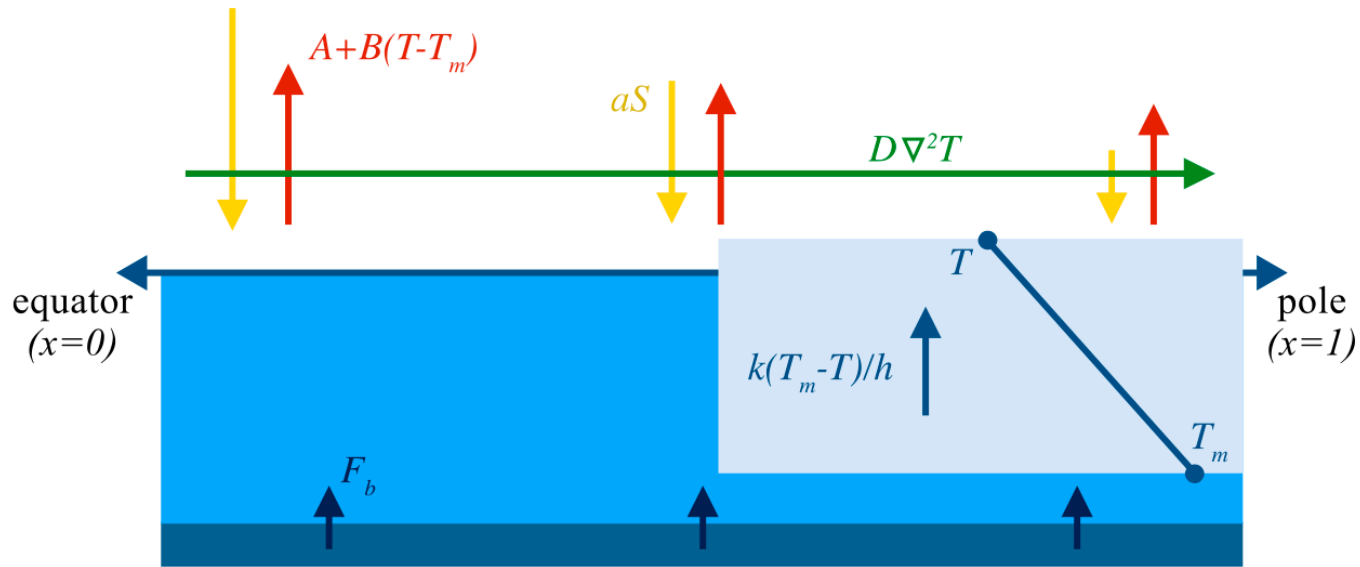


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$$k(T_m - T_0)/h = -aS + A + B(T_0 - T_m) - D\nabla^2 T - F.$$

$$T = \begin{cases} T_m + E/c_w, & E > 0 & \text{(open water),} \\ T_m, & E < 0, \quad T_0 > T_m & \text{(melting ice),} \\ T_0, & E < 0, \quad T_0 < T_m & \text{(freezing ice).} \end{cases}$$

# output

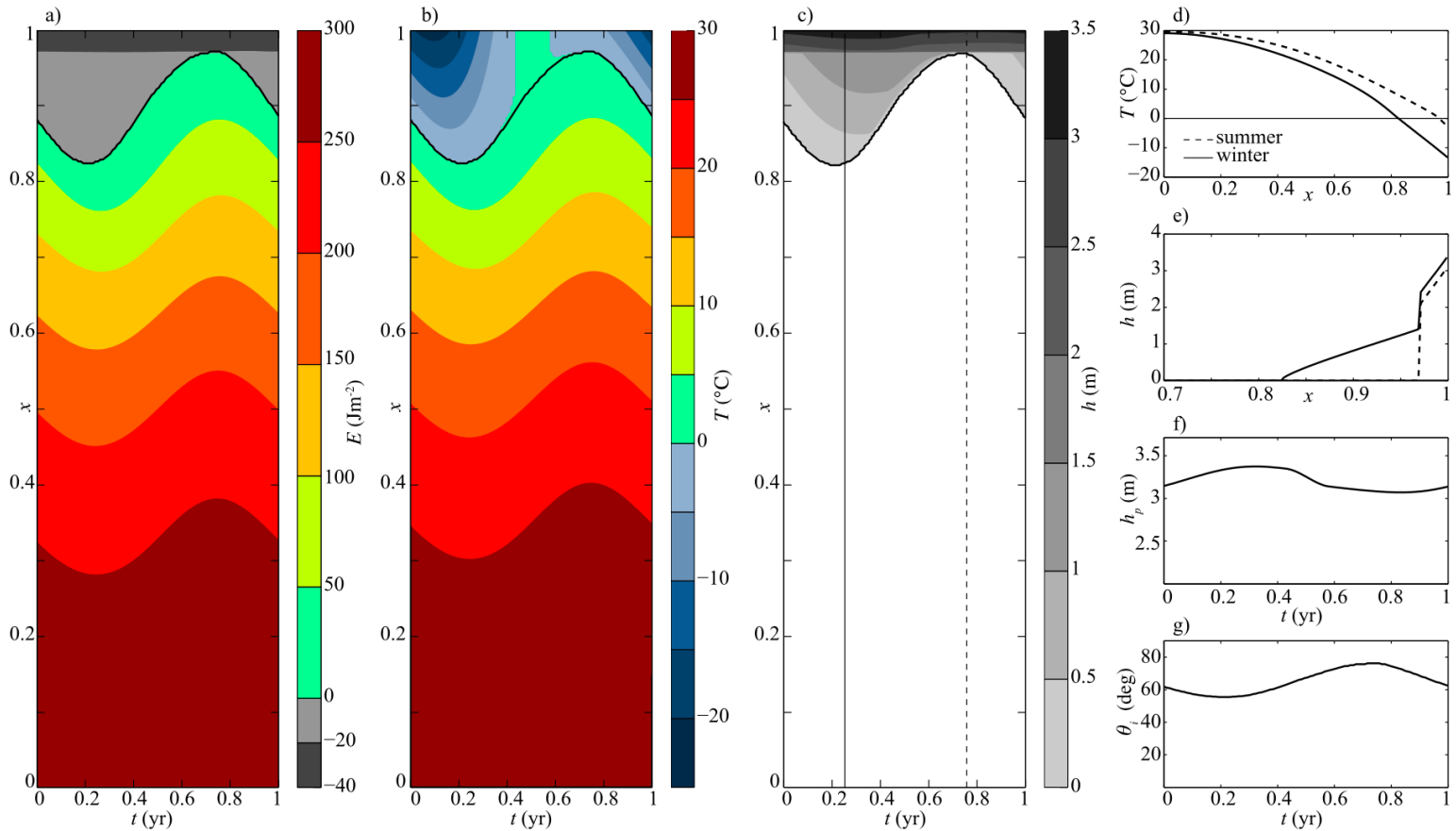


FIG. 2. Simulated climate in the default parameter regime. Contour plot of the seasonal cycle of (a) surface enthalpy  $E(x, t)$ , (b) surface temperature  $T(x, t)$ , and (c) sea ice thickness  $h(x, t)$ . The black curve in (a)–(c) indicates the ice edge. (d) Surface temperature  $T$  in summer and winter, corresponding to dashed and solid vertical lines in (c). (e) Ice thickness  $h$  in summer and winter where  $x > 0.7$ . (f) Seasonal cycle of ice thickness at the pole  $h_p$ . (g) Seasonal cycle of the latitude of the sea ice edge  $\theta_i$ .

# Exercise 1: EBM

$D^* = W m^{-2} K^{-1}$  Diffusivity for heat transport: 0.6

$A = W m^{-2}$  OLR: 193

$B = W m^{-2} K^{-1}$  OLR temperature dependence: 2.1

$c_w = W y r m^{-2} K^{-1}$  Ocean mixed layer heat capacity: 9.8

$S_0 = W m^{-2}$  Insolation at equator: 420

$S_2 = W m^{-2}$  Insolation spatial dependence: 240

$A_0$  Ice-free coalbedo at equator: 0.7

$A_2$  Ice-free coalbedo spatial dependence: 0.1

$A_i$  Coalbedo when there is sea ice: 0.4

$W m^{-2}$  Radiative forcing: 0

$\gamma$  Gamma: 1

- 1) What will happen if the  $CO_2$  content in the atmosphere is doubled? Radiative forcing =  $4 W/m^2$
- 2) What will happen if the factor  $\gamma$  is 1% higher/lower in the long-wave radiation?
- 3) Describe the effect if the diffusivity is enhanced by a factor of 2!
- 4) The coalbedo of sea ice can vary between 0.3 and 0.4. Describe the effect when varying the value!
- 5) Write down the numerical scheme (time stepping etc. from the source code) !
- 6) Show the evolution at one specific latitude and discuss it!

<https://paleodyn.uni-bremen.de/study/MES/ebm/>

Upper model



# Exercise 2: Aquaplanet EBM with seasonal cycle

- 1) What will happen if the  $\text{CO}_2$  content in the atmosphere is doubled? Radiative forcing =  $4 \text{ W/m}^2$  = lowering of A
- 2) Discuss the sea ice evolution during the year !
- 3) Reduce and enhance the sea ice thermal conductivity by 20% mimicking more /less snow on top of sea ice !
- 4) Write down the numerical scheme (time stepping) !

<https://paleodyn.uni-bremen.de/study/MES/ebm/>

Lower model



